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Higgs phenomenon for higher spin fields on AdS_3

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Abstract

In a previous work, a marginal deformation of 2d coset type model with $\mathcal{N} = 3$ superconformal symmetry was studied, and it was interpreted as a change of boundary conditions for bulk fields in the dual higher spin theory. The deformation breaks generic higher spin gauge symmetry, and the generated mass of a spin 2 field was computed. The deformation is expected to correspond to the introduction of finite string tension in a superstring theory. In this paper, we extend the analysis and compute the masses of generic higher spin fields at the leading order of $1/c$ (c is the CFT central charge) but at the full order of the deformation parameter. We find that the masses are not generated for $\mathrm{so}(3)_R$ singlet higher spin fields at this order and the spectrum is the Regge-like one for $\mathrm{so}(3)_R$ triplet higher spin-charged fields. This suggests that the deformed theories are related to superstring theory with pure NSNS-flux.

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1 Introduction

There exist many higher spin states in superstring theory, and a higher spin symmetry is expected to appear at the limit where the masses of these states vanish. Moreover, it was proposed that the broken phase of higher spin gauge theory can describe superstring theory [1]. Recently it became possible to discuss the direct relation between higher spin gauge theory and superstring theory but on AdS space. This is due to the developments on higher spin gauge theory on AdS space such as Vasiliev theory [2] and AdS/CFT correspondence. For examples, 4d Vasiliev theory is conjectured to be dual to 3d O(N) vector model [3] (see also [4]), and a lower dimensional version was proposed in [5] where 3d Vasiliev theory in [6] is dual to a 2d large N minimal model.

The first concrete proposal on the relation between higher spin gauge theory and superstring theory was made in [7] by extending the duality in [3]. There are two other proposals given by generalizing the lower dimensional version of duality in [5]. Lower dimensional models are generically more tractable than higher dimensional ones, so we expect to learn more about the relation. The proposal with large or small $\mathcal{N} = 4$ supersymmetry was made in [8, 9, 10], while that with $\mathcal{N} = 3$ supersymmetry was given in [11, 12, 13]. Compared to the proposal in [8, 9, 10], the relation between higher spin fields and strings is more transparent in [11, 12, 13] like in the duality in [7]. See [14, 15, 16, 17, 18] for related works.

Utilizing these features of the proposal in [12], we have examined the Higgs phenomenon due to the braking of higher spin symmetry in [13][1]. We deform the CFT such that generic higher spin symmetry would be broken except for the $\mathcal{N} = 3$ superconformal symmetry. The deformation is of the double-trace type and it can be interpreted as the change of boundary conditions for the bulk fields in the dual higher spin theory [20]. The breaking of higher spin gauge symmetry would make the higher spin fields massive. In [13], we have computed the mass of a spin 2 field both from the bulk and the boundary theories by making use of the previous works [21, 22, 23, 24]. Similar phenomenon was also discussed for higher spin fields in [25] but on AdS_4.

The aim of this paper is to extend the analysis in [13] to generic higher spin fields. We work at the leading order of $1/c$ with c the CFT central charge, where the classical gravity computation is reliable. Utilizing the holographic duality, we succeeded to obtain the mass formula for all higher spin fields and at the full order of the deformation parameter. The bulk higher spin theory has $\mathcal{N} = 3$ supersymmetry and there are $\mathrm{so}(3)_R$ singlet fields with spin $s = 2, 3, 4, \ldots$ and $\mathrm{so}(3)_R$ triplet fields with spin $s = 1, 2, 3, \ldots$. We observe that the masses are not generated at this order for the $\mathrm{so}(3)_R$ singlet fields. This result is actually consistent with that in [26], since their deformation is of the same type as ours. We also find the mass square of spin s field $M^2_{(s)}$ is proportional to the spin s as $M^2_{(s)} \propto (s-1)$ for $\mathrm{so}(3)_R$ triplet fields. This result suggests that the higher spin theory is closely related to a superstring theory with pure NSNS-flux.

This paper is organized as follows; In the next section, we review the $\mathcal{N} = 3$ holography proposed in [12] and introduce the marginal deformations of [13]. We also give the final result of the mass formula in (2.24) and (2.25) below. In section 3 we compute the Higgs masses for low spin $\mathrm{so}(3)_R$ singlet fields by using a brute force method. In section 4 we generalize the results for generic higher spin fields and also $\mathrm{so}(3)_R$ triplet fields by making use of a free ghost system. We adopt two different ways of computation. In section 5 we summarize the useful properties of embedding formulation for tensor fields on AdS_{d+1}. Using these known results, we interpret the boundary computation in terms of bulk theory. From the interpretation we deduce the correction terms at the higher

[1]The Higgs phenomenon for the proposal in [9] was examined in a quite recent paper [19].

order of the deformation parameter in section 6. We conclude this paper and discuss future problems in section 7. In appendix A we construct the free ghost realization of higher spin superalgebra and $\mathcal{N}=3$ superconformal subalgebra. In appendix B, we give integral formulas which are used in subsection 6.2.

2 $\mathcal{N}=3$ holography and a summary of results

The proposal in [12, 13] includes a duality between a 3d Prokushkin-Vasiliev theory in [6] and a large N limit of 2d coset type model. The special feature of the duality lies in the $\mathcal{N}=3$ supersymmetry. The 3d Prokushkin-Vasiliev theory has extended supersymmetry only if we choose a specific mass parameter [6, 27]. We associate U$(2M)$ Chan-Paton factor to the fields but with a U(M) invariant condition, and the higher spin theory with these conditions has $\mathcal{N}=3$ supersymmetry. The 2d coset model with $\mathcal{N}=3$ superconformal symmetry is given by the critical level model

$$\frac{\mathrm{su}(N+M)_{N+M}\oplus \mathrm{so}(2NM)_1}{\mathrm{su}(N)_{N+2M}\oplus \mathrm{su}(M)_{M+2N}\oplus \mathrm{u}(1)_\kappa}\,, \tag{2.1}$$

where $\kappa = 2NM(N+M)^2$ and the central charge is

$$c=\frac{3}{2}MN\,. \tag{2.2}$$

In order to compare the classical gravity theory, we need to take the large N limit. The proposal was confirmed by the comparison of one-loop partition function and symmetry algebra at low spins, see [28, 12, 17].

In [13] we have studied a marginal deformation of the coset model and its interpretation in dual higher spin theory. In particular, we computed the Higgs mass of a spin 2 field both from the CFT and the higher spin theory. The aim of this paper is to compute the Higgs masses of spin $s\geq 2$ fields in the higher spin gauge theory. In this section, we summarize the necessary information on the $\mathcal{N}=3$ holography and the mass formula obtained in this paper.

2.1 Higher spin superalgebra

We start from the gauge algebra of the Prokushkin-Vasiliev theory with extended supersymmetry. The higher spin theory can be defined by modifying the $\mathcal{N}=2$ higher spin gauge theory with U(M') Chan-Paton (CP) factor in [6]. Here the U(M') CP factor just means that the fields take $M'\times M'$ matrix values. The holography with the higher spin gauge theory was proposed in [29, 8, 11]. The theory includes gauge fields with spin $s=1,3/2,2,5/2,\cdots$, which can be described by a Chern-Simons gauge theory [30]. Moreover, there are also matter fields with masses parametrized by λ. The higher spin gauge theory in [12, 13] is then obtained by a $\mathbb{Z}_2$ truncation with $\lambda = 1/2$.[2] For the $\mathcal{N} = 3$ holography, we assign U($2M$) CP factor and the U(M) invariant condition as mentioned above.

The gauge algebra can be defined by using y_α ($\alpha = 1, 2$) and $\hat{k}$ satisfying

$$[y_\alpha, y_\beta] = 2i\epsilon_{\alpha\beta}(1 - (1 - 2\lambda)\hat{k}) \,, \quad \hat{k}^2 = 1 \,, \quad \{\hat{k}, y_\alpha\} = 0 \,. \tag{2.3}$$

We denote the algebra generated by these variables as $sB[\lambda]$. The gauge algebra $\text{shs}_{M'}[\lambda]$ for the $\mathcal{N} = 2$ higher spin gauge theory with U(M') CP factor is defined as

$$sB_{M'}[\lambda] \equiv sB[\lambda] \otimes \mathcal{M}_{M'} = \mathbb{C} \oplus \text{shs}_{M'}[\lambda] \,. \tag{2.4}$$

Here $\mathcal{M}_{M'}$ denotes the $M' \times M'$ matrix algebra and $\mathbb{C}$ represents the central element. The bosonic subalgebra with $M' = 1$ and in the $\hat{k} = 1$ subsector is known as hs[λ]. At $\lambda = 1/2$, the commutator of y_α does not involve $\hat{k}$ anymore, and the algebra can be truncated by assigning the invariance under $\hat{k} \to -\hat{k}$. The truncated algebra may be called as $\text{shs}^T_{M'}[1/2]$.

The matrix algebra $\mathcal{M}_{M'}$ with $M' = 2^n$ can be generated by the Clifford elements ϕ^I ($I = 1, 2, \cdots, 2n + 1$) satisfying $\{\phi^I, \phi^J\} = 2\delta^{IJ}$. This indicates that the truncated algebra $\text{shs}^T_{M'}[1/2]$ includes osp($2n + 1|2$) subalgebra [6, 27, 12, 17]

$$T_{\alpha\beta} = \{y_\alpha, y_\beta\} \,, \quad Q^I_\alpha = y_\alpha \otimes \phi^I \,, \quad M^{IJ} = [\phi^I, \phi^J] \,. \tag{2.5}$$

In our case, we associate U($2M$) CP factor and require U(M) invariant condition. The subalgebra $\text{shs}^T_2[1/2]$ survives the invariant condition, thus the theory can be seen to have $\mathcal{N} = 3$ supersymmetry. We consider the bosonic subsector of gauge fields based on $\text{shs}^T_2[1/2]$. Decomposing the U(2) of the CP factor as U(1)×SU(2), the trace part gives so$(3)_R$ singlet higher spin fields $A^0_{(s)}$ and the SU(2) part gives so$(3)_R$ triplet higher spin fields $A^i_{(s)}$ with $i = 1, 2, 3$. We would like to compute the masses of these fields after the deformation braking the higher spin symmetry. Notice that $A^0_{(2)}$ and $A^i_{(1)}$ should be kept massless since they are based on the algebra of unbroken $\mathcal{N} = 3$ supersymmetry.

2.2 Dual CFT

We are interested in a large N limit of the symmetry algebra in the dual CFT. Let us denote the gauge algebra of the higher spin theory as g_{hs}. Near the AdS boundary, the symmetry algebra is enhanced to be a W-algebra obtained by a Hamiltonian reduction of affine g_{hs} algebra as explained in [32, 33, 34]. A claim in [35] is that at the large N limit we can truncate the W-algebra into so called "wedge" subalgebra consistently, and the subalgebra is identical to the original higher spin algebra g_{hs}. We represent the CFT

[2]The holography with the truncation and $M' = 1$ was proposed in [28], where the higher spin theory has $\mathcal{N} = 1$ supersymmetry. A different $\mathcal{N} = 1$ holography was already conjectured in [31].

currents as $J^{(s,a)}(z)$ with $a = 0,1,2,3$ (or their mode expansions $J_n^{(s,a)}$ with $n \in \mathbb{Z}$), which are dual to the higher spin fields $A_s^{(a)}$ introduced above. From the above argument, the wedge subalgebra generated by $J_n^{(s,a)}$ with $|n| < s$ should be given by $\mathrm{shs}_2^T[1/2]$ at the large N limit.

It will be useful to realize the higher spin algebra $\mathrm{shs}_2^T[1/2]$ by free ghost system as in appendix A. Since we are interested in the bosonic subsector, we only need to include one type of ghost system, say, (b_A, c_A) with $A = 1,2$. The operator product is

$$b_A(z)c_B(w) \sim \frac{\delta_{AB}}{z-w}\,, \tag{2.6}$$

and the conformal weights are

$$(h_+, h_-) = \left(\frac{1+\lambda}{2}, \frac{1-\lambda}{2}\right)\,. \tag{2.7}$$

We are interested in only the case with $\lambda = 1/2$, but we keep λ generic unless necessary. The integer spin s currents are then given as [36]

$$[V_\lambda^{(s)}(z)]_{AB} = \sum_{i=0}^{s-1} a^i(s, \lambda+1)\partial^{s-1-i}\{(\partial^i b_A)c_B\} \tag{2.8}$$

with

$$a^i(s,\lambda) = \binom{s-1}{i}\frac{(-\lambda-s+2)_{s-1-i}}{(s+i)_{s-1-i}} \quad (0 \le i \le s-1)\,. \tag{2.9}$$

Here we have used the following notation as

$$(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)} = a(a+1)\cdots(a+n-1)\,. \tag{2.10}$$

We examine two type of currents $J_+^{(s)}(z) \equiv J^{(s,0)}(z)$ and $J_-^{(s)}(z) \equiv J^{(s,3)}(z)$ since the properties of the other $\mathrm{so}(3)_R$ charged currents $J^{(s,a)}(z)$ with $a = 1,2$ can be deduced from those with $a = 3$. The properties of the wedge subalgebra for $J_\pm^{(s)}(z)$ at the large N limit can be obtained from those of

$$V_{\lambda,\pm}^{(s)}(z) = [V_\lambda^{(s)}(z)]_{11} \pm [V_\lambda^{(s)}(z)]_{22}\,. \tag{2.11}$$

Here the normalization of the currents is[3]

$$\langle J_\pm^{(s)}(z_1)J_\pm^{(s)}(z_2)\rangle = -\frac{(2s-1)cN_s}{6}\frac{1}{z_{12}^{2s}}\,, \tag{2.12}$$

$$N_s = \frac{3\sqrt{\pi}\Gamma(s)}{4^{s-1}(\lambda^2-1)\Gamma(s+\frac{1}{2})}\frac{\Gamma(s-\lambda)\Gamma(s+\lambda)}{\Gamma(1-\lambda)\Gamma(1+\lambda)}\,, \tag{2.13}$$

[3]This information is obtained from the classical gravity theory as in [37]. This is a property outside the wedge subalgebra, so the quantity cannot be computed with the currents introduced in (2.11). See, for instance, (3.4) below.

where the central charge is [32, 33]

$$c = \frac{3}{2G_N} \tag{2.14}$$

with Newton's constant G_N.

The higher spin theory includes two complex scalar fields and two Dirac spinor fields along with higher spin gauge fields. Due to the U($2M$) CP factor and the U(M) invariant condition, the single particle states of the matter fields take 2×2 matrix values. These matter fields are dual to operators denoted as $\mathcal{O}_\pm^{A\bar{B}}(z,\bar{z})$ and $\mathcal{F}_\pm^{A\bar{B}}(z,\bar{z})$ with $A, \bar{B} = 1, 2$.[4] The definition of these states in [13] leads to the following relation of complex conjugation as

$$\mathcal{O}_\pm^{11}(z) = \bar{\mathcal{O}}_\pm^{22}(z) \,, \quad \mathcal{O}_\pm^{12}(z) = \bar{\mathcal{O}}_\pm^{21}(z) \,, \quad \mathcal{F}_\pm^{11}(z) = \bar{\mathcal{F}}_\pm^{22}(z) \,, \quad \mathcal{F}_\pm^{12}(z) = \bar{\mathcal{F}}_\pm^{21}(z) \,. \tag{2.15}$$

We choose the boundary condition of the matter fields such that $\mathcal{O}_\pm^{A\bar{B}}(z,\bar{z})$ have the conformal weights $(h_\pm, h_\pm)$ and $\mathcal{F}_\pm^{A\bar{B}}(z)$ have $(h_\pm, h_\mp)$, where $h_\pm$ are defined in (2.7). For simplicity we denote the operators as

$$\mathcal{O}_+^1(z) \equiv \mathcal{O}_+^{11}(z) \,, \quad \mathcal{O}_+^2(z) \equiv \mathcal{O}_+^{12}(z) \,, \quad \mathcal{O}_-^1(z) \equiv \mathcal{O}_-^{22}(z) \,, \quad \mathcal{O}_-^2(z) \equiv \mathcal{O}_-^{21}(z) \,, \tag{2.16}$$

$$\mathcal{F}_+^1(z) \equiv \mathcal{F}_+^{11}(z) \,, \quad \mathcal{F}_+^2(z) \equiv \mathcal{F}_+^{12}(z) \,, \quad \mathcal{F}_-^1(z) \equiv \mathcal{F}_-^{22}(z) \,, \quad \mathcal{F}_-^2(z) \equiv \mathcal{F}_-^{21}(z) \,.$$

The three point functions are computed from the bulk higher spin gauge theory as [38, 37, 39, 40]

$$\langle \mathcal{O}_\pm^a(z_1)\bar{\mathcal{O}}_\pm^a(z_2) J_\eta^{(s)}(z_3)\rangle = A_\pm(s,\lambda) \left(\frac{z_{12}}{z_{13}z_{23}}\right)^s \langle \mathcal{O}_\pm^a(z_1)\bar{\mathcal{O}}_\pm^a(z_2)\rangle \,, \tag{2.17}$$

$$\langle \mathcal{F}_\pm^a(z_1)\bar{\mathcal{F}}_\pm^a(z_2) J_\eta^{(s)}(z_3)\rangle = A_\pm(s,\lambda) \left(\frac{z_{12}}{z_{13}z_{23}}\right)^s \langle \mathcal{F}_\pm^a(z_1)\bar{\mathcal{F}}_\pm^a(z_2)\rangle$$

with[5]

$$A_+(s,\lambda) = \frac{\Gamma(s)^2}{\Gamma(2s-1)} \frac{\Gamma(s+\lambda)}{\Gamma(1+\lambda)} \,, \quad A_-(s,\lambda) = \eta(-1)^s \frac{\Gamma(s)^2}{\Gamma(2s-1)} \frac{\Gamma(s-\lambda)}{\Gamma(1-\lambda)} \,. \tag{2.18}$$

These correlation functions can be reproduced by using a free ghost system as shown in [40]. In principal, it is possible to compute them directly using the coset model (2.1) with finite N and then taking the large N limit. However, the computation would be quite complicated, and it is convenient to use the classical gravity theory and the free ghost system at the limit.

[4] We suppress the argument of $\bar{z}$ in the following.

[5] The holography requires that either of A_+ or A_- has the factor $(-1)^s$, and here we choose to put the factor in A_-, see, e.g., [37]. Moreover, the so$(3)_R$ charges can be read from the action of the Pauli matrix σ^a. In our choice, $\mathcal{O}_+^a$ and $\bar{\mathcal{O}}_-^a$ has $+1$ eigenvalue of σ^3 and $\bar{\mathcal{O}}_+^a$ and $\mathcal{O}_-^a$ has -1 eigenvalue of σ^3. The so$(3)_R$ charge assignment is similar for the fermionic operators.

2.3 Marginal deformation and Higgs masses

In order to compare to superstring theory with finite string tension, we need to break the higher spin symmetry. As in [13] we deform the bulk higher spin theory by changing boundary conditions of the matter fields with keeping $\mathcal{N} = 3$ supersymmetry. We choose this deformation since a similar deformation was used in [7]. The change of boundary conditions is dual to the following double-trace type deformation of the dual CFT as [13]

$$\Delta S = -f \int d^2 w \mathcal{T}(w, \bar{w}) , \tag{2.19}$$

$$\mathcal{T} = \sum_{a=1}^{2} \frac{(-1)^{a-1}}{2} \left[\mathcal{O}^a_+ \mathcal{O}^a_- + \bar{\mathcal{O}}^a_- \bar{\mathcal{O}}^a_+ + \mathcal{F}^a_+ \mathcal{F}^a_- + \bar{\mathcal{F}}^a_- \bar{\mathcal{F}}^a_+ \right] . \tag{2.20}$$

The deformation is expected to break higher spin symmetry generically, and the corresponding currents are not conserved any more as

$$\bar{\partial} J^{(s)}_{\pm}(z) = \mathcal{K}^{(s-1)}_{\pm}(z) \tag{2.21}$$

with $\mathcal{K}^{(s-1)}_{\pm}(z)$ as spin $(s-1)$ operators.

The aim of this paper is to compute the masses of spin s fields, which are generated due to the symmetry breaking. A direct method is to compute the one-loop corrections of spin s propagators as was done in [21, 22, 24] for spin 2 fields. However the computation would be quite complicated. Instead of the direct way, we compute the masses from the viewpoint of dual CFT. In other words, we compute the masses by making use of the AdS isometry, which is the same as the conformal symmetry of the boundary CFT. The map is known between the mass of bulk spin s field and the conformal dimension $\Delta^{(s)}_{\pm}$ of dual spin s current $J^{(s)}_{\pm}(z)$ as

$$M^2_{(s,\pm)} = \Delta^{(s)}_{\pm}(\Delta^{(s)}_{\pm} - 2) - s(s-2) . \tag{2.22}$$

This formula reduces to

$$M^2_{(s,\pm)} = 2(s-1)(\Delta^{(s)}_{\pm} - s) \tag{2.23}$$

at the first order of the anomalous dimension $\Delta^{(s)}_{\pm} - s$. Using the map we can compute the mass of spin s field from the dual CFT.

In the rest of sections, we compute the anomalous dimensions using various methods. Before going into the details of the computation, we summarize our results on the mass formula here. Our results are at the first order of $1/c$ since we heavily use the classical bulk gravity theory and the free ghost system (2.6). However, we can obtain the Higgs masses at the full order of the deformation parameter f in (2.19). As we saw above, there are so$(3)_R$ singlet and triplet higher spin fields. The so$(3)_R$ singlet fields do not receive any corrections as

$$M^2_{(s,+)} = 0 \quad (s = 2, 3, 4, \ldots) . \tag{2.24}$$

The result may be expected because the deformation operator (2.20) is of the same type as the one in [26], where the authors considered the deformation preserving the higher spin symmetry at the leading order of $1/c$ and the deformation parameter f. For the $\mathrm{so}(3)_R$ triplet fields, the mass formula is obtained as

$$M^2_{(s,-)} = \frac{f^2}{(1+f^2)^2} \frac{12(s-1)}{c} \quad (s = 1, 2, 3, 4, \ldots) \,, \tag{2.25}$$

where the central charge is related to the Newton constant G_N as (2.14). It reproduces the result in [13] for $s = 2$. The results indicate that the deformed theory is related to a superstring theory with pure NSNS-flux. More detailed examination of these results will be given in the concluding section.

3 The examples of low spin currents

We start from the simple examples with $s = 2, 3, \ldots$ and of the $\mathrm{so}(3)_R$ singlet, which are given by $J^{(s)}_+(z)$. These higher spin currents generate the hs[λ] bosonic subalgebra. We would like to deform the theory as in (2.20), but here we consider a simpler version as

$$\mathcal{T}_\lambda = \frac{1}{2} \left[\mathcal{O}_+ \mathcal{O}_- + \bar{\mathcal{O}}_- \bar{\mathcal{O}}_+ \right] \,. \tag{3.1}$$

Denoting the corresponding state as $|\mathcal{T}_\lambda\rangle$, the eigenvalues of current zero modes are written as

$$J^{(s)}_{+,0} |\mathcal{T}_\lambda\rangle = (A_+(s,\lambda) + A_-(s,\lambda)) |\mathcal{T}_\lambda\rangle \,. \tag{3.2}$$

Here $A_+(s, \lambda)$ is defined in (2.18). For $\lambda = 1/2$, we can identify $\mathcal{O}_\pm$ as $\mathcal{O}^a_\pm$ $(a = 1, 2)$ in (2.16). We can use the same results for the fermionic operators $\mathcal{F}^a_\pm$ $(a = 1, 2)$ in (2.16).

After the deformation, the divergence of currents can be written as (see, e.g., [26, 13])

$$\bar\partial J^{(s)}_+(z) = 2\pi f \sum_{l=0}^{s-1} \frac{(-1)^l}{l!} (L_{-1})^l J^{(s)}_{+,-s+l+1} \mathcal{T}_\lambda(z) \tag{3.3}$$

with $L_n = J^{(2)}_{+,n}$. The right hand side vanishes for $s = 2$ since the eigenvalue of $J^{(2)}_{+,0}$ is one in our case. This implies that the mass is not generated for the usual graviton field as in (2.24). The square root of the left hand side of (3.3) can be computed as

$$|\bar\partial J^{(s)}_+|^2 = \langle 0| J^{(s)}_{+,s} \bar{L}_1 \bar{L}_{-1} J^{(s)}_{+,-s} |0\rangle = -(\Delta^{(s)}_+ - s) \frac{(2s-1)N_s c}{6} \,, \tag{3.4}$$

where (2.12) is used. Therefore, if we can compute the square root of the right hand side of (3.3), then we can read off the anomalous dimension for $J^{(s)}_+(z)$ from this expression. In this section, we compute the square root of the right hand side explicitly for low spin examples with $s = 3, 4$.

3.1 Spin 3 current

For the spin 3 current $W(z) \equiv J_+^{(3)}(z)$, the divergence of current in (3.3) becomes

$$\bar{\partial} W = 2\pi f \left(W_{-2} - L_{-1} W_{-1} + \frac{1}{2} L_{-1}^2 W_0 \right) \mathcal{T}_\lambda \, . \tag{3.5}$$

The problem is now to compute the square root of the right hand side explicitly. For the purpose we need the commutation relations among the mode expansions of higher spin currents, which are given as[6]

$$\begin{aligned}
&[L_m, L_n] = (m-n) L_{n+m} + \frac{c}{12} m(m^2-1)\delta_{m+n} \, , \quad [L_m, W_n] = (2m-n) W_{m+n} \, , \\
&[W_m, W_n] = 2(m-n) U_{m+n} - \frac{N_3}{12}(m-n)(2m^2+2n^2-mn-8) L_{m+n} \\
&\qquad - \frac{8N_3}{(c+\frac{22}{5})}(m-n)\Lambda^{(4)}_{m+n} - \frac{N_3 c}{144} m(m^2-1)(m^2-4)\delta_{m+n}
\end{aligned} \tag{3.6}$$

with U as the spin 4 field $J_+^{(4)}$ and $\Lambda_m^{(4)}$ as a composite operator made with L_n. The terms proportional to $\Lambda_m^{(4)}$ are subleading for large c and hence are neglected, and the expression of N_3 is given in (2.13).

We compute the anomalous dimension by comparing the norm of the both side of (3.5) as

$$\begin{aligned}
|\bar{\partial} W|^2 = (2\pi f)^2 \langle \mathcal{T}_\lambda | \left(W_2 - W_1 L_1 + \frac{1}{2} W_0 (L_1)^2 \right) \\
\cdot \left(W_{-2} - L_{-1} W_{-1} + \frac{1}{2}(L_{-1}) W_0 \right) |\mathcal{T}_\lambda\rangle \, .
\end{aligned} \tag{3.7}$$

The following eigenvalues are introduced as

$$L_0 |\mathcal{T}_\lambda\rangle = h |\mathcal{T}_\lambda\rangle \, , \quad W_0 |\mathcal{T}_\lambda\rangle = w |\mathcal{T}_\lambda\rangle \, , \quad U_0 |\mathcal{T}_\lambda\rangle = u |\mathcal{T}_\lambda\rangle \, . \tag{3.8}$$

Then we have

$$\begin{aligned}
&\frac{1}{4}\langle \mathcal{T}_\lambda | W_0 (L_1)^2 (L_{-1})^2 W_0 |\mathcal{T}_\lambda\rangle = (1+2h) h w^2 \langle \mathcal{T}_\lambda | \mathcal{T}_\lambda \rangle \, , \\
&-\frac{1}{2}\langle \mathcal{T}_\lambda | W_0 (L_1)^2 L_{-1} W_{-1} |\mathcal{T}_\lambda\rangle = -\frac{1}{2}\langle \mathcal{T}_\lambda | W_1 L_1 (L_{-1})^2 W_0 |\mathcal{T}_\lambda\rangle = -3w^2(1+2h)\langle \mathcal{T}_\lambda | \mathcal{T}_\lambda \rangle \, , \\
&\frac{1}{2}\langle \mathcal{T}_\lambda | W_2 (L_{-1})^2 W_0 |\mathcal{T}_\lambda\rangle = \frac{1}{2}\langle \mathcal{T}_\lambda | W_0 (L_1)^2 W_{-2} |\mathcal{T}_\lambda\rangle = 6w^2 \langle \mathcal{T}_\lambda | \mathcal{T}_\lambda \rangle \, ,
\end{aligned} \tag{3.9}$$

[6] We replaced N_3 in the commutation relations of the paper [41] by $-N_3$, effectively this amounts to a rescaling of the spin four field compared to [41]. After this replacement, the current-current two point function becomes (2.12) with positive coefficient for $0 < \lambda < 1$.

$$\langle\mathcal{T}_\lambda|W_1L_1L_{-1}W_{-1}|\mathcal{T}_\lambda\rangle = \left(2(h+1)\left(4u+\frac{N_3}{2}h\right)+9w^2\right)\langle\mathcal{T}_\lambda|\mathcal{T}_\lambda\rangle\,,$$

$$\langle\mathcal{T}_\lambda|W_2W_{-2}|\mathcal{T}_\lambda\rangle = 4(2u-N_3h)\langle\mathcal{T}_\lambda|\mathcal{T}_\lambda\rangle\,,$$

$$-\langle\mathcal{T}_\lambda|W_2L_{-1}W_{-1}|\mathcal{T}_\lambda\rangle = -\langle\mathcal{T}_\lambda|W_1L_1W_{-2}|\mathcal{T}_\lambda\rangle = -4\left(4u+\frac{N_3}{2}h\right)\langle\mathcal{T}_\lambda|\mathcal{T}_\lambda\rangle\,.$$

Thus the right hand side of (3.7) leads to

$$|\bar\partial W|^2 = [u(8h-16)+N_3h(h-7)+w^2(2h^2-11h+15)](2\pi f)^2\langle\mathcal{T}_\lambda|\mathcal{T}_\lambda\rangle = 0\,. \tag{3.10}$$

Here we have used

$$h=1\,,\quad w=\lambda\,,\quad u=\frac{3}{5}(1+\lambda^2)\,,\quad N_3=\frac{1}{5}(\lambda^2-4)\,, \tag{3.11}$$

which come from (3.2). Therefore at the leading order of $1/c$ and f^2 we find as in (2.24)

$$M^2_{(3)} = 4(\Delta-3) = 0\,, \tag{3.12}$$

where we have used (2.23) and (3.4).

3.2 Spin 4 current

If we want to compute the deformation for the spin 4 current we need to know its commutation relations again. The operator product of the field $U(z)$ with itself has only poles of even order and the coefficients can only be normally ordered polynomials in $T(z), \partial T(z), \partial^2 T(z), \partial^3 T(z), \partial^4 T(z), W(z), U(z), \partial U(z), \partial^2 U(z), Y(z)$ where $Y(z)$ is the spin 6 current $J^{(6)}_+$. As explained in [42] there are various relations between structure constants. Especially the one of a normally ordered polynomials of type $:\partial^j T(z)X(z):$ for some primary field $X(z)$ is related to the structure constant for $X(z)$. In [42] an explicit formula for this relation is given, it is very lengthy, but one can easily extract that they behave as $1/c$ for large c and are thus negligible for our considerations. In [43] terms of the commutator of the spin four field modes with themselves were computed, the term corresponding to $:W(z)W(z):$ is also subleading for large c and hence their results imply that

$$\begin{aligned}[U_m,U_n] &= 3(m-n)Y_{m+n} - f_U(m,n)U_{m+n} + f_L(m,n)L_{m+n} + \\ &\quad -\frac{cN_4}{4320}m(m^2-1)(m^2-4)(m^2-9)\delta_{m+n,0} + \\ &\quad + P(m,n,T,T',T'',T''',T'''',W,U,U',U'')\,, \end{aligned}\tag{3.13}$$

$$f_U(m,n) = n_{44}(m-n)(m^2-mn+n^2-7)\,,$$

$$f_L(m,n) = -\frac{N_4}{360}(m-n)\left(108 - 39m^2 + 3m^4 + 20mn - 2m^3n - 39n^2 + \right.$$

$$\left. +4m^2n^2 - 2mn^3 + 3n^4\right) ,$$

where P denotes modes of a normally ordered polynomial in the indicated fields. It is subleading and can be neglected for our computations. For general λ, the involved constants can be read off by comparing to [43]. They are

$$n_{44} = \frac{(\lambda^2 - 19)}{30} , \quad N_4 = -\frac{3}{70}(\lambda^2 - 4)(\lambda^2 - 9) \tag{3.14}$$

and the charges of the zero modes Y_0 and U_0 on $|\mathcal{T}_\lambda\rangle$ are obtained from (3.2). They are

$$y = \frac{5}{42}(8 + 15\lambda^2 + \lambda^4) , \quad u = \frac{3}{5}(1 + \lambda^2) . \tag{3.15}$$

We need to compute to leading order in $1/c$ of

$$\begin{aligned} \text{rhs} := \langle\mathcal{T}_\lambda| &\left(U_3 - U_2L_1 + \frac{1}{2}U_1(L_1)^2 - \frac{1}{6}U_0(L_1)^3\right) \\ &\cdot \left(U_{-3} - L_{-1}U_{-2} + \frac{1}{2}(L_{-1})^2U_{-1} - \frac{1}{6}(L_{-1})^3U_0\right)|\mathcal{T}_\lambda\rangle . \end{aligned} \tag{3.16}$$

This computation is straightforward and lengthy. First define

$$Z_n := \langle\mathcal{T}_\lambda|U_nU_{-n}|\mathcal{T}_\lambda\rangle = \begin{cases} (6ny - f_U(n,-n)u + f_L(n,-n)h)\langle\mathcal{T}_\lambda|\mathcal{T}_\lambda\rangle & (n \neq 0) , \\ u^2\langle\mathcal{T}_\lambda|\mathcal{T}_\lambda\rangle & (n = 0) . \end{cases} \tag{3.17}$$

We get

$$\begin{aligned} \langle\mathcal{T}_\lambda|U_3U_{-3}|\mathcal{T}_\lambda\rangle &= Z_3 , \\ \langle\mathcal{T}_\lambda|U_3L_{-1}U_{-2}|\mathcal{T}_\lambda\rangle &= 6Z_2 , \\ \langle\mathcal{T}_\lambda|U_3(L_{-1})^2U_{-1}|\mathcal{T}_\lambda\rangle &= 30Z_1 , \\ \langle\mathcal{T}_\lambda|U_3(L_{-1})^3U_0|\mathcal{T}_\lambda\rangle &= 120Z_0 , \\ \langle\mathcal{T}_\lambda|U_2L_1L_{-1}U_{-2}|\mathcal{T}_\lambda\rangle &= 25Z_1 + 2(h+2)Z_2 , \\ \langle\mathcal{T}_\lambda|U_2L_1(L_{-1})^2U_{-1}|\mathcal{T}_\lambda\rangle &= 10(2h+3)Z_1 + 80Z_0 , \\ \langle\mathcal{T}_\lambda|U_2L_1(L_{-1})^3U_0|\mathcal{T}_\lambda\rangle &= 120(h+1)Z_0 , \\ \langle\mathcal{T}_\lambda|U_1(L_1)^2(L_{-1})^2U_{-1}|\mathcal{T}_\lambda\rangle &= 4(2h+3)(h+1)Z_1 + 128(h+1)Z_0 , \end{aligned} \tag{3.18}$$

$$\langle \mathcal{T}_\lambda | U_1 \left(L_1\right)^2 \left(L_{-1}\right)^3 U_0 | \mathcal{T}_\lambda \rangle = 48(2h^2+3h+1) Z_0 \,,$$

$$\langle \mathcal{T}_\lambda | U_0 \left(L_1\right)^3 \left(L_{-1}\right)^3 U_0 | \mathcal{T}_\lambda \rangle = 24h(2h^2+3h+1) Z_0 \,.$$

So that with $h = 1$

$$\text{rhs} = Z_3 - 6Z_2 + 15Z_1 - 20Z_0 \,. \tag{3.19}$$

Plugging n_{44} and N_4 into the expresions for $f_L(n,m)$ and $f_U(n,m)$, we get

$$\text{rhs} = 36y + 4(\lambda^2-19)u + \frac{18}{35}(\lambda^2-4)(\lambda^2-19) - 20u^2 \tag{3.20}$$

and it turns out that independent of λ this expression vanishes identically

$$\text{rhs} = 0 \,. \tag{3.21}$$

Therefore at the leading order of $1/c$ and f^2 we again find as in (2.24)

$$M^2_{(4)} = 6(\Delta^{(4)}_+ - 4) = 0 \tag{3.22}$$

with the use of (2.23) and (3.4).

4 Generic spin s currents

As above, we can compute the anomalous dimension of $J^{(s)}_+$ by comparing the two ways to express $|\bar\partial J^{(s)}_+|^2$. One way can be found in (3.4). The other way is to compute $|\mathcal{K}^{(s-1)}|^2$ using (2.21). The computation with the expression in (3.3) becomes complicated rapidly when we increase s as seen in the examples with $s = 3, 4$. Fortunately, the right hand side in (3.3) involves only the wedge subalgebra with $J^{(s)}_{+,n}$ ($|n| < s$), so we can utilize the free ghost system (2.6). There are at least two merits to use the free ghost system. One is that the other expression of $|\bar\partial J^{(s)}_+|^2$ can be dealt with more easily even for generic s. Another is that the analysis can be generalized to the so$(3)_R$ triplet currents simply by inserting a phase factor -1 as in (2.11). In this section, we obtain the mass formula (2.24) and (2.25) at the first order of $1/c$ and f^2 in two ways. The analysis on the higher order of f^2 is postponed to later sections.

4.1 Direct computation

As explained above, we can utilize the generators $V^{(s)}_{\lambda,\pm}(z)$ defined in (2.11). The generators have the following operator products like

$$V^{(s)}_{\lambda,\pm}(z) b_1(w) \sim \sum_{i=0}^{s-1} a^i(s, \lambda+1) \partial_z^{s-1-i} \left(\frac{1}{z-w}\right) \partial^i b_1(w) \,, \tag{4.1}$$

$$V^{(s)}_{\lambda,\pm}(z) c_2(w) \sim \pm(-1)^s \sum_{i=0}^{s-1} a^i(s, 1-\lambda) \partial_z^{s-1-i} \left(\frac{1}{z-w}\right) \partial^i c_2(w) \,. \tag{4.2}$$

We can reproduce (2.17) and (2.18) if we identify $V^{(s)}_{\lambda,\pm}$ as $J^{(s)}_{\pm}$ and b_1, c_2 as $\mathcal{O}_+, \mathcal{O}_-$, see [40]. Therefore, up to total derivative, we have

$$\bar{\partial} V^{(s)}_{\lambda,\pm}(z)(b_1 c_2)(w) \tag{4.3}$$

$$= 2\pi\delta^{(2)}(z-w)\sum_{i=0}^{s-1}[a^i(s,\lambda+1)\partial^{s-1-i}(\partial^i b_1 c_2)(w) \pm (-1)^s a^i(s,1-\lambda)\partial^{s-1-i}(b_1\partial^i c_2)(w)]$$

$$= 2\pi\delta^{(2)}(z-w)\sum_{i=0}^{s-1}(1\mp 1)\tilde{a}^i(s,\lambda+1)(\partial^i b_1\partial^{s-1-i}c_2)(w)\,.$$

Here $\tilde{a}^i(s,\lambda)$ is given as

$$\tilde{a}^i(s,\lambda) = \begin{pmatrix} s-1 \\ i \end{pmatrix}\frac{(-1)^i}{(s)_{s-1}}(\lambda-s)_i(2-\lambda-s)_{s-1-i}\,, \tag{4.4}$$

and the identities (see [36])

$$\sum_{i=0}^{s-1} a^i(s,\lambda)\partial^{s-1-i}((\partial^i A)B) = \sum_{i=0}^{s-1}\tilde{a}(s,\lambda)(\partial^i A)(\partial^{s-1-i}B)\,, \tag{4.5}$$

$$\tilde{a}^i(s,\lambda) = (-1)^{s-1}\tilde{a}^{s-1-i}(s,2-\lambda) \tag{4.6}$$

are used.

With these preparations, we can compute the divergence of current as

$$\bar{\partial} J^{(s)}_{\pm}(z) = \pi f\sum_{i=0}^{s-1}[(1\mp 1)\tilde{a}^i(s,\lambda+1)](\partial^{s-1-i}\mathcal{O}_+\partial^i\mathcal{O}_-)(z) \tag{4.7}$$

after the deformation with $\frac{1}{2}\mathcal{O}_+\mathcal{O}_-(w)$. Let us define $|\mathcal{O}_\pm\rangle$ as the eigenstate of L_0 with

$$L_0|\mathcal{O}_\pm\rangle = h_\pm|\mathcal{O}_\pm\rangle \tag{4.8}$$

and $\langle\bar{\mathcal{O}}_\pm|$ as its conjugate state. Then we can show that

$$\langle\bar{\mathcal{O}}_\pm|(L_1)^s(L_{-1})^s|\mathcal{O}_\pm\rangle = F(s,h_\pm)\langle\bar{\mathcal{O}}_\pm|\mathcal{O}_\pm\rangle\,, \quad F(s,h_\pm) = \Gamma(s+1)\frac{\Gamma(2h_\pm+s)}{\Gamma(2h_\pm)} \tag{4.9}$$

by repeatedly using the commutation relations among L_n ($n = 0, \pm 1$). With the formula, we have

$$\langle\partial^{s-1-i}\bar{\mathcal{O}}_+\partial^i\bar{\mathcal{O}}_-|\partial^{s-1-i}\mathcal{O}_+\partial^i\mathcal{O}_-\rangle = F(s-1-i,h_+)F(i,h_-)C_+C_-$$

with $C_\pm = \langle\bar{\mathcal{O}}_\pm|\mathcal{O}_\pm\rangle$. Adding the conjugate deformation operator $\frac{1}{2}\bar{\mathcal{O}}_-\bar{\mathcal{O}}_+(w)$, we find

$$|\bar{\partial} J^{(s)}_{\pm}|^2 = (2\pi f)^2\sum_{i=0}^{s-1}\left\{[(1\mp 1)\tilde{a}^i(s,\lambda+1)]^2 F(s-1-i,h_+)F(i,h_-)\right\}\langle\mathcal{T}_\lambda|\mathcal{T}_\lambda\rangle$$

$$= (2\pi f)^2\left[-\frac{1\mp 1}{2}\frac{2^{4-2s}\pi^{3/2}\mathrm{Csc}\,(\lambda\pi)\Gamma(s)}{\lambda\Gamma(1-\lambda-s)\Gamma(1+\lambda-s)\Gamma(-1/2+s)}\right]\langle\mathcal{T}_\lambda|\mathcal{T}_\lambda\rangle\,. \tag{4.10}$$

With (2.23) and (3.4) we obtain

$$M^2_{(s,\pm)} = \frac{12(s-1)}{(2s-1)N_s c}(2\pi f)^2 \left[\frac{1\mp 1}{2}\frac{2^{4-2s}\pi^{3/2}\mathrm{Csc}\,(\lambda\pi)\Gamma(s)}{\lambda\Gamma(1-\lambda-s)\Gamma(1+\lambda-s)\Gamma(-1/2+s)}\right]\langle\mathcal{T}_\lambda|\mathcal{T}_\lambda\rangle$$

$$= \frac{1\mp 1}{2}\frac{8(1-\lambda^2)}{c}(s-1)(2\pi f)^2\langle\mathcal{T}_\lambda|\mathcal{T}_\lambda\rangle\,. \tag{4.11}$$

This reproduces our findings for the so$(3)_R$ singlet fields with $s = 3, 4$.

For the original problem we have to set $\lambda = 1/2$ and multiply the factor 4. Furthermore, the standard kinetic term for the dual matter fields fixes the normalization as $C_+ = C_- = 1/(2\pi)$ [13]. Thus the mass for the spin s field dual to $J^{(s)}_\pm$ can be computed as

$$M^2_{(s,\pm)} = \frac{1\mp 1}{2}\frac{12(s-1)}{c}(2\pi f)^2 \tag{4.12}$$

at the leading order of $1/c$ and f^2. This is the term at the order of f^2 in the mass formula of (2.24) and (2.25).

4.2 Alternative computation

In this subsection, we reproduce (4.10) in a different way of computation for the following two purposes. One is to check the computation obtained above. Another is for a preparation of later analysis. We will see that this way of computation has the dual gravity interpretation in terms of Witten diagram. Relying on the interpretation, we will include the corrections at the higher order of f^2 to the mass formula as in (2.24) and (2.25).

Here we utilize the standard method of conformal perturbation theory as

$$\langle\bar\partial J^{(s)}_\pm(z)\bar\partial J^{(s)}_\pm(w)e^{f\int d^2x\mathcal{T}_\lambda}\rangle = \langle\bar\partial J^{(s)}_\pm(z)\bar\partial J^{(s)}_\pm(w)\rangle + f\int d^2x\langle\bar\partial J^{(s)}_\pm(z)\bar\partial J^{(s)}_\pm(w)\mathcal{T}_\lambda(x)\rangle$$

$$+\frac{f^2}{2}\int d^2x\int d^2y\langle\bar\partial J^{(s)}_\pm(z)\bar\partial J^{(s)}_\pm(w)\mathcal{T}_\lambda(x)\mathcal{T}_\lambda(y)\rangle + \cdots\,. \tag{4.13}$$

The correlation functions in the above expression are evaluated by using the non-perturbed theory. We can see that the first two terms in the right hand side vanishes. In the third term, there are two types of contribution as

$$I_1(x,w) = \int d^2x\int d^2y\langle\bar\partial J^{(s)}_\pm(z)\mathcal{O}_+(x)\bar{\mathcal{O}}_+(y)\rangle\langle\bar\partial J^{(s)}_\pm(w)\mathcal{O}_-(x)\bar{\mathcal{O}}_-(y)\rangle\,, \tag{4.14}$$

and

$$I_2(x,w) = \int d^2x\int d^2y\langle\bar\partial J^{(s)}_\pm(z)\bar\partial J^{(s)}_\pm(w)\mathcal{O}_+(x)\bar{\mathcal{O}}_+(y)\rangle\langle\mathcal{O}_-(x)\bar{\mathcal{O}}_-(y)\rangle\,. \tag{4.15}$$

We first consider the contribution of the type in (4.14). Taking derivative of (2.17) with respect to $\bar{z}_3$, we have

$$\frac{\partial}{\partial \bar{z}_3}\left(\frac{1}{z_{13}z_{23}}\right)^s = -\frac{2\pi}{(s-1)!}\left[(\partial_{z_3}^{s-1}\delta^{(2)}(z_{13}))\frac{1}{(z_{23})^s} + (\partial_{z_3}^{s-1}\delta^{(2)}(z_{23}))\frac{1}{(z_{13})^s}\right] . \qquad (4.16)$$

Since there is no contribution from the terms proportional to $\delta^{(2)}(z-x)\delta^{(2)}(w-x)$ for $z \neq w$, we obtain

$$I_1(z,w) = \pm 2A_+(s,\lambda)A_-(s,\lambda)C_+C_-\frac{(2\pi)^2}{[(s-1)!]^2} \qquad (4.17)$$

$$\times \int d^2x \int d^2y \delta^{(2)}(x-z)\delta^{(2)}(y-w)\partial_x^{s-1}\partial_y^{s-1}\left[\frac{1}{(y-z)^s}\frac{1}{(x-w)^s}\frac{1}{(x-y)^{2-2s}}\right]\frac{1}{(\bar{x}-\bar{y})^2} ,$$

where we have used the invariance under the exchange of x and y. Evaluating the action of derivatives, we find[7]

$$I_1(z,w) = \pm 2A_+(s,\lambda)A_-(s,\lambda)C_+C_-\frac{(2\pi)^2}{[(s-1)!]^2}\frac{1}{(z-w)^{2s}(\bar{z}-\bar{w})^2} \qquad (4.18)$$

$$\times \sum_{k,\ell=0}^{s-1}\binom{s-1}{k}\binom{s-1}{\ell}(s)_{s-1-k}(2-2s)_k(2-2s+k)_\ell(s)_{s-1-\ell}$$

$$= \pm(2\pi)^2\frac{2^{3-2s}\pi^{3/2}\mathrm{Csc}\,(\lambda\pi)\Gamma(s)}{\lambda\Gamma(1-\lambda-s)\Gamma(1+\lambda-s)\Gamma(-1/2+s)}C_+C_-\frac{1}{(z-w)^{2s}(\bar{z}-\bar{w})^2} .$$

For the total contribution, we need to multiply a pre-factor. First we have $f^2/2$ in (4.13). Moreover, the exchange of x, y yields a factor 2. From $\mathcal{T}(x)\mathcal{T}(y)$, we have two terms as

$$\mathcal{T}(x)\mathcal{T}(y) = \frac{1}{4}\mathcal{O}_+(x)\mathcal{O}_-(x)\bar{\mathcal{O}}_-(y)\bar{\mathcal{O}}_+(y) + \frac{1}{4}\bar{\mathcal{O}}_-(x)\bar{\mathcal{O}}_+(x)\mathcal{O}_+(y)\mathcal{O}_-(y) + \cdots . \qquad (4.19)$$

Totally, we have

$$\frac{f^2}{2}\cdot 2\cdot\frac{1}{4}\cdot 2 = \frac{f^2}{2} . \qquad (4.20)$$

For the contribution of the type in (4.15), we need

$$\bar{\partial}J_\pm^{(s)}(z)\mathcal{O}_+(x) = 2\pi\sum_{i=0}^{s-1}a^i(s,\lambda+1)\partial_z^{s-1-i}\delta^{(2)}(z-x)\partial^i\mathcal{O}_+(x) ,$$

which comes from the free ghost computation in (4.1). With this expression, we have

$$2(2\pi)^2\sum_{i=0}^{s-1}\tilde{a}^i(s,\lambda+1)^2\langle\partial^i\mathcal{O}_+(z)\partial^i\bar{\mathcal{O}}_+(w)\rangle\langle\partial^{s-1-i}\mathcal{O}_-(z)\partial^{s-1-i}\bar{\mathcal{O}}_-(w)\rangle \qquad (4.21)$$

$$= -(2\pi)^2\frac{2^{3-2s}\pi^{3/2}\mathrm{Csc}\,(\lambda\pi)\Gamma(s)}{\lambda\Gamma(1-\lambda-s)\Gamma(1+\lambda-s)\Gamma(-1/2+s)}C_+C_-\frac{1}{(z-w)^{2s}(\bar{z}-\bar{w})^2} ,$$

[7] The second equality is checked for $s = 2, 3, \ldots, 300$.

where we have used (4.9). The result does not change even by exchanging $\pm$ of $\mathcal{O}_\pm$ and $\bar{\mathcal{O}}_\pm$ in (4.15). As in (4.20) the total contribution is with the pre-factor $f^2/2$. The sum of the two types of contribution reproduces (4.10) if we use $\langle \mathcal{T}_\lambda | \mathcal{T}_\lambda \rangle = \frac{1}{2} C_+ C_-$.

5 Basics for bulk analysis

In the next section we study the Higgs phenomenon in the viewpoint of the bulk theory. Using the gravity interpretation, we deduce how to include the corrections at higher order of f^2. As a preparation for it, we introduce basic tools in this section. In this and the next sections, we mainly work on arbitrary dimensional AdS$_{d+1}$ space. First we review the embedding formalism for tensor fields on AdS$_{d+1}$ or conformal fields on d-dimensional flat space-time, see, for examples, [44, 45, 46, 47, 48]. Then we give several important properties of AdS propagators, in particular, the split representation of them in [49], see also [50, 51].

5.1 Embedding formalism

Euclidean AdS$_{d+1}$ space can be described by a hypersurface in $d+2$ Minkowski space defined as

$$X^2 = -1 \,, \quad X^0 > 0 \,. \tag{5.1}$$

The isometry group is SO$(d+1,1)$. In the light-cone coordinates X^A ($A = +, -, 1, \ldots, d$), the metric is given by

$$X^2 = \eta_{AB} X^A X^B = -X^+ X^- + \delta_{ab} X^a X^b \tag{5.2}$$

with $a = 1, \ldots d$. The Poincaré coordinates are parametrized as

$$X = \frac{1}{z}(1, z^2 + y^2, y^a) \,. \tag{5.3}$$

Near the AdS boundary, the hypersurface approaches the light-cone $X^2 = 0$. Thus the boundary can be represented by the light rays, which may be described by P^A satisfying

$$P^2 = 0 \,, \quad P \sim \lambda P \tag{5.4}$$

with $\lambda \in \mathbb{R}$. In the Poincaré patch, the boundary is parametrized as

$$P = (1, y^2, y^a) \,. \tag{5.5}$$

A totally symmetric traceless tensor $h_{\mu_1 \ldots \mu_s}(x)$ on AdS$_{d+1}$ can be described by a SO$(d+1,1)$ tensor $H_{A_1 \ldots A_s}(X)$ on the embedding space. These tensors are related as

$$h_{\mu_1 \ldots \mu_s}(x) = \frac{\partial X^{A_1}}{\partial x^{\mu_1}} \cdots \frac{\partial X^{A_s}}{\partial x^{\mu_s}} H_{A_1 \ldots A_s}(X) \,, \tag{5.6}$$

which implies that

$$X^{A_1} H_{A_1 \dots A_s}(X) = 0 \tag{5.7}$$

due to $X^2 = -1$. Introducing auxiliary variables W^A, we define

$$H(X, W) = W^{A_1} \cdots W^{A_s} H_{A_1 \dots A_s}(X) \,. \tag{5.8}$$

We assign $W^2 = 0$ from the traceless condition and $W \cdot X = 0$ from the transverse one (5.7). On the boundary given by the light-cone $P^2 = 0$, we define a totally symmetric traceless tensor $F_{A_1 \dots A_s}(P)$. We require $F(\lambda P) = \lambda^{-\Delta} F(P)$ for $\lambda > 0$, where we denote the scaling dimension as Δ. Moreover, the condition to be tangent to the light-cone $P = 0$ requires $P^{A_1} F_{A_1 \dots A_s}(X) = 0$. Introducing Z^A we define

$$F(P, Z) = Z^{A_1} \cdots Z^{A_s} P_{A_1 \dots A_s}(X) \,, \tag{5.9}$$

where we assign $Z^2 = 0$ and $Z \cdot P = 0$. The transverse condition can be encoded by requiring $F(P, Z + \alpha P) = F(P, Z)$ for all α.

5.2 AdS propagators

We consider a spin s field with dual scaling dimension Δ, which propagates from X_1 and X_2 and with polarization vectors W_1 and W_2, respectively. We represent the bulk-to-bulk propagator as $\Pi_{\Delta,s}(X_1, X_2; W_1, W_2)$. The bulk-to-boundary propagator may be then represented as $\Pi_{\Delta,s}(X, P; W, Z)$. The structure of the bulk-to-boundary propagator can be fixed by conformal symmetry. In [49], it was claimed that the bulk-to-bulk propagator can be written in terms of the following AdS harmonic function as[8]

$$\begin{aligned} &\Omega_{\nu,s}(X_1, X_2; W_1, W_2) \\ &= \frac{1}{16\pi s! (\frac{d}{2} - 1)_s \nu^2} \int_\partial dP \Pi_{\frac{d}{2}+i\nu,s}(X_1, P; W_1 . D_Z) \Pi_{\frac{d}{2}-i\nu,s}(X_2, P; W_2, Z) \,, \end{aligned} \tag{5.10}$$

Here $(a)_n$ is defined in (2.10) and the operator D_Z is given by

$$D_Z^A = \left(\frac{d}{2} - 1 + Z \cdot \frac{\partial}{\partial Z} \right) \frac{\partial}{\partial Z_A} - \frac{1}{2} Z^A \frac{\partial^2}{\partial Z \cdot \partial Z} \,. \tag{5.11}$$

The harmonic function is found to satisfy [50, 49]

$$\begin{aligned} &\Omega_{\nu,s}(X_1, X_2; W_1, W_2) \\ &= \frac{1}{8 i \pi \nu} \Big(\Pi_{\frac{d}{2}+i\nu,s}(X_1, X_2; W_1, W_2) - \Pi_{\frac{d}{2}-i\nu,s}(X_1, X_2; W_1, W_2) \Big) \,. \end{aligned} \tag{5.12}$$

This relation will be important for later analysis.

In the CFT computation we have used the three point functions in (2.17). With the above propagators we can express them as Witten diagrams in terms of bulk theory as in figure 1. We introduce a spin s field $h_{\mu_1 \dots \mu_s}$ and a complex scalar ϕ, $\bar{\phi}$. As argued in [49]

[8] We changed the normalization of boundary-to-bulk operator by $-4\nu^2$.

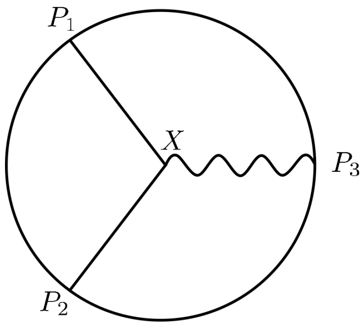

Figure 1: Witten diagram for a three point function

the generic three point coupling may be set in the form of

$$g \int_{\mathrm{AdS}} dx \sqrt{g} (\bar{\phi} \nabla_{\mu_1} \cdots \nabla_{\mu_s} \phi) h^{\mu_1 \dots \mu_s} . \tag{5.13}$$

Here we have assumed the transverse condition $\nabla^{\mu_1} h_{\mu_1 \dots \mu_s} = 0$. Let us denote Δ_0, Δ_s as the dual scaling dimensions for the complex scalar and the spin s field. Then the Witten diagram can be evaluated as

$$\begin{aligned} &\langle \mathcal{O}_{\Delta_0}(P_1) \bar{\mathcal{O}}_{\Delta_0}(P_2) J^{(s)}(P_3, Z) \rangle \\ &= g \int_{\mathrm{AdS}} dX \Pi_{\Delta_0,0}(P_1, X) \frac{\Pi_{\Delta_s,s}(X, P_3, K, Z)(W \cdot \nabla)^J \Pi_{\Delta_0,0}(X, P_2)}{s!((d-1)/2)_s} . \end{aligned} \tag{5.14}$$

Here K is a projector operator, whose expression may be found in [49] as

$$\begin{aligned} K_A = {} & \frac{d-1}{2} \left[\frac{\partial}{\partial W^A} + X_A \left(X \cdot \frac{\partial}{\partial W} \right) \right] + \left(W \cdot \frac{\partial}{\partial W} \right) \frac{\partial}{\partial W^A} \\ & + X_A \left(W \cdot \frac{\partial}{\partial W} \right) \left(X \cdot \frac{\partial}{\partial W} \right) - \frac{1}{2} W_A \left[\frac{\partial^2}{\partial W \cdot \partial W} + \left(X \cdot \frac{\partial}{\partial W} \right)^2 \right] . \end{aligned} \tag{5.15}$$

However, we will not use the details of the expression later on.

6 Dual bulk interpretation

In section 4 we have computed the Higgs masses of spin s fields using the CFT technique. In principle the mass can be computed directly from the bulk higher spin theory. In fact, it was pointed out in [25] that the mass term would arise from the one-loop corrections of spin s propagator when we assign non-standard boundary conditions to bulk fields. However, it is technically difficult to extract the information of the mass from the one-loop computations. For the simple example with $s = 2$, the explicit value has been computed in [13] by following the previous works [21, 22, 24]. In principle, we can generalize their method to the case with $s > 2$, but it seems to be quite complicated.

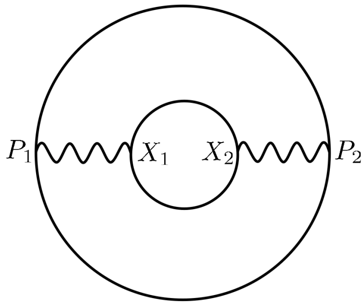

Figure 2: One-loop diagram for a current-current two point function $\langle J_\pm^{(s)}(P) J_\pm^{(s)}(Q)\rangle$

We take a different route to extract the information of mass from the one-loop effects on spin s propagator. Instead of bulk-to-bulk propagator, we consider boundary-to-boundary one, which is equivalent to the two point function of boundary spin s current $\langle J_\pm^{(s)}(z) J_\pm^{(s)}(w)\rangle$. Acting $\partial_{\bar z}\partial_{\bar w}$, we can compute $|\bar\partial J_\pm^{(s)}|^2$ from the bulk higher spin theory. With (3.4) we can read off the anomalous dimension of dual spin s current and the Higgs mass of the spin s field can be obtained from (2.22). As mentioned above, the mass term arises from the one-loop correction on spin s propagator. Therefore, in case of boundary-to-boundary propagator, the one-loop diagram we need to compute is the one in figure 2. As was pointed out, e.g., in [24], we can easily see how the corrections at the higher order of f^2 enter from the bulk gravity viewpoints. Comparing the CFT computations, we obtain the higher order corrections in the mass formula of (2.24) and (2.25).

6.1 One loop contributions

In the d-dimensional CFT, we introduce two complex single-trace operators $\mathcal{O}_\pm$ with scaling dimensions $\Delta_\pm$ satisfying $\Delta_+ + \Delta_- = d$. These operators are dual to bulk complex scalars $\phi^\pm$ with the same mass but different boundary conditions. As in (3.1), we consider the deformation by the double-trace operator as[9]

$$\Delta S = -f \int d^d x \mathcal{T}_d(x) \,, \quad \mathcal{T}_d = \mathcal{O}_+ \mathcal{O}_- + \bar{\mathcal{O}}_- \bar{\mathcal{O}}_+ \,. \tag{6.1}$$

It was shown in [20] that the deformation corresponds to the change of boundary condition for $\phi^\pm$. After the deformation the boundary conditions for the two bulk fields $\phi^\pm$ are mixed, and the effects can removed by the rotation of the fields. Utilizing the rotation, the propagators $\langle \phi^\alpha(X)\phi^\beta(Y)\rangle = \Pi^{\alpha\beta}(X,Y)$ $(\alpha, \beta = \pm)$ can be obtained as [52, 13]

$$\Pi^{\alpha\beta}(X,Y) = \frac{1}{1+\tilde f^2} \begin{pmatrix} \Pi_{\Delta_+,0} + \tilde f^2 \Pi_{\Delta_-,0} & \tilde f \Pi_{\Delta_-,0} - \tilde f \Pi_{\Delta_+,0} \\ \tilde f \Pi_{\Delta_-,0} - \tilde f \Pi_{\Delta_+,0} & \Pi_{\Delta_-,0} + \tilde f^2 \Pi_{\Delta_+,0} \end{pmatrix} \tag{6.2}$$

[9]Here we do not include a factor 1/2 in (2.20). The factor 1/2 is introduced there to cancel the Jacobian arising from the change of worldsheet coordinates as $z = \sigma_1 + i\sigma_0, \bar z = \sigma_1 - i\sigma_0$.

with $\tilde{f} = 2(\Delta_+ - d/2)$. We are interested in the case with $\Delta_\pm = (d \pm 1)/2$, which lead to $\tilde{f} = f$ for $\Delta_\pm = (d \pm 1)/2$. If we take $f \to \infty$, then the boundary conditions of $\phi^\pm$ are exchanged as we can see in (6.2).

We would like to examine the loop effects of the Witten diagram in the figure 2. The effects can be read off from the two point function of the bulk currents $\langle \hat{J}^{(s)}_\pm(x) \hat{J}^{(s)}_\pm(y) \rangle$. Here the bulk currents can be obtained from the interaction term in (5.13) as $\hat{J}^{(s)}_\pm = \hat{J}^{(s)}_1 \pm \hat{J}^{(s)}_2$, where

$$\hat{J}^{(s)}_1 = \bar{\phi}^+ \nabla_{\mu_1} \cdots \nabla_{\mu_s} \phi^+ \, , \quad \hat{J}^{(s)}_2 = \bar{\phi}^- \nabla_{\mu_1} \cdots \nabla_{\mu_s} \phi^- \, . \tag{6.3}$$

Since the bulk currents are written in terms of bilinears of scalar fields, the two point functions can be evaluated by the product of two bulk-to-bulk propagators for the scalar fields in (6.2). The two point function is given by the sum over the four contributions as (see [13] for spin 2 case)

$$\begin{aligned} &\langle \hat{J}^{(s)}_\pm(x) \hat{J}^{(s)}_\pm(y) \rangle \\ &= \langle \hat{J}^{(s)}_1(x) \hat{J}^{(s)}_1(y) \rangle \pm \langle \hat{J}^{(s)}_1(x) \hat{J}^{(s)}_2(y) \rangle \pm \langle \hat{J}^{(s)}_2(x) \hat{J}^{(s)}_1(y) \rangle + \langle \hat{J}^{(s)}_2(x) \hat{J}^{(s)}_2(y) \rangle \, . \end{aligned} \tag{6.4}$$

Since we know that there is no contribution to the scaling dimension from f independent term, the non-trivial contribution from each term satisfies

$$\langle \hat{J}^{(s)}_1(x) \hat{J}^{(s)}_1(y) \rangle_f = \langle \hat{J}^{(s)}_2(x) \hat{J}^{(s)}_2(y) \rangle_f = -\langle \hat{J}^{(s)}_1(x) \hat{J}^{(s)}_2(y) \rangle_f = -\langle \hat{J}^{(s)}_2(x) \hat{J}^{(s)}_1(y) \rangle_f \, , \tag{6.5}$$

which can be seen from the explicit form of the scalar propagators in (6.2). This expression implies that the conformal dimension for so$(3)_R$ singlet current does not receive any correction at the leading order of $1/c$. Therefore, the result in (2.24) can be easily obtained from the viewpoint of bulk theory. Moreover, the propagators in (6.2) implies the following important fact. Once we have expression in the first order of f^2, the final result is obtained simply replacing f^2 by $f^2/(1+f^2)^2$. From the results in [13], the same conclusion can be obtained for the deformations of fermionic operators. The full order expression of f^2 in (2.25) can be obtained in this way. If we take $f \to \infty$, then these masses vanish. This is consistent with the fact that the effect of the deformation at the $f \to \infty$ limit is just exchanging the boundary conditions of $\phi^\pm$ and we know that the higher spin gauge symmetry is not broken there.

6.2 Relation to the CFT computation

As argued above, the Higgs masses should be read off from the one-loop Witten diagram for the current-current two point functions $\langle J^{(s)}_\pm(P) J^{(s)}_\pm(Q) \rangle$ as in figure 2. These quantities have been computed by the CFT method at the leading order of f^2, therefore we could relate the two ways of computation. With the relation, we can see how the higher order corrections of f^2 would be computed in the CFT language.

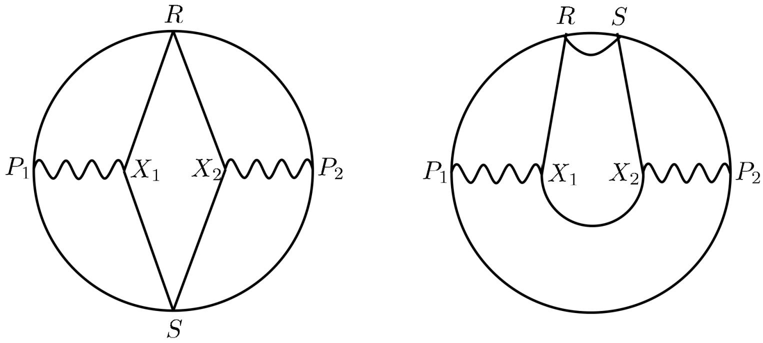

Figure 3: Contributions to anomalous dimension with the extra insertion of boundary operators

In the one-loop diagram of figure 2, there are one scalar propagator along the upper solid line and the other along the lower solid line. The contributions to the anomalous dimension come from the shift in propagators as in (6.2). Let us first examine the following term at the first order of f as

$$\Pi^{\pm\mp}(X,Y) = \tilde{f}(\Pi_{\Delta_-,0}(X,Y) - \Pi_{\Delta_+,0}(X,Y))\,. \tag{6.6}$$

This effect must be due to the insertion of a boundary deformation operator in (6.1). In fact, the relation (5.12) implies that

$$f\int_\partial dR\Pi_{\Delta,0}(X_1,R)\Pi_{d-\Delta,0}(X_2,R) = f(d-2\Delta)(\Pi_{\Delta,0}(X_1,X_2) - \Pi_{d-\Delta,0}(X_1,X_2))\,. \tag{6.7}$$

Thus with $\Delta = \Delta_\pm$ the deformed propagator $\Pi^{\pm\mp}(X_1,X_2)$ in (6.2) can be written in terms of two boundary-to-bulk propagators at the first order of f as expected. With this expression, this type of contribution can be written in terms of Witten diagram as in the left one of figure 3, which correspond to the product of CFT three point functions in (4.14).

In a similar way, we can see that the right diagram of figure 3 can be regarded as the contribution to the propagator $\Pi^{\pm\pm}(X_1,X_2)$ in (6.2) at the order of f^2. The diagram corresponds to the product of four and two point functions in (4.15). After the two insertions of boundary operators, a propagator between X_1 and X_2 becomes

$$\begin{aligned} & f^2\int dRdS\Pi_{\Delta,0}(X_1,R)\Pi_{d-\Delta,0}(R,S)\Pi_{\Delta,0}(S,X_2) \\ &= f^2(d-2\Delta)\int dS\Pi_{d-\Delta,0}(X_1,S)\Pi_{\Delta,0}(S,X_2) \\ &= -\tilde{f}^2(\Pi_{\Delta,0}(X_1,X_2) - \Pi_{d-\Delta,0}(X_1,X_2))\,, \end{aligned} \tag{6.8}$$

where we have used (B.6) and (6.7). They are the contribution to $\Pi^{\pm\pm}(X_1,X_2)$ in (6.2) for $\Delta = \Delta_\pm$ at the order of f^2 as expected.

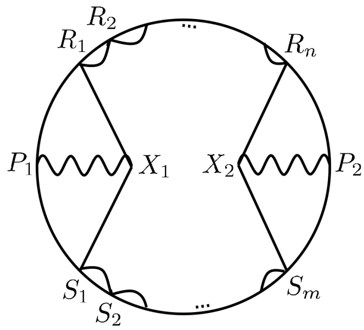

Figure 4: A contribution at the order of f^{n+m} from the insertion of $(n+m)$ boundary operators

From the experience with a few boundary insertions, we can guess that the contributions at the order of f^{n+m} come from the diagrams with the insertion of $n+m$ boundary operators as in figure 4. They are the Witten diagrams corresponding to the $(n+m)$-th order contributions in the conformal perturbation theory. Let us examine how the bulk scalar propagators change due to the insertions of many boundary operators. Since the insertion of the boundary interaction changes the boundary scaling dimension from Δ to $d-\Delta$, the propagator $\Pi^{\pm\mp}(X_1, X_2)$ can include odd number of insertions and the propagator $\Pi^{\pm\pm}(X_1, X_2)$ can include even number of insertions. Using (B.6) we can see that the $2n$ number of extra boundary insertions only give factor $(-\tilde{f}^2)^n$ after the integration over the insertion points. From the expansion of the boundary insertions as

$$\exp\left(f\int_\partial dQ\mathcal{T}_d(Q)\right) = 1 + f\int_\partial dQ\mathcal{T}_d(Q) + \frac{f^2}{2}\int_\partial dQ_1 dQ_2 \mathcal{T}_d(Q_1)\mathcal{T}_d(Q_2) + \cdots\,, \qquad (6.9)$$

we obtain the factor $1/(n!)$ for the term of the order f^n. However, we need to consider all possible permutations of the boundary points Q_i, which yields the factor $n!$. Thus totally, we have $1/(n!)\cdot n! = 1$ for the term of the order f^n. After summing over the all order contributions, the propagators with even number insertions become

$$\Pi_{\Delta,0}(X_1, X_2) - \frac{\tilde{f}^2}{1+\tilde{f}^2}\left(\Pi_{\Delta,0}(X_1, X_2) - \Pi_{d-\Delta,0}(X_1, X_2)\right), \qquad (6.10)$$

which reproduces $\Pi_{\pm\pm}(X_1, X_2)$ in (6.2). Similarly, the propagators with odd number insertions become

$$\frac{1}{1+\tilde{f}^2}\left(\tilde{f}\Pi_{\Delta_-,0}(X_1, X_2) - \tilde{f}\Pi_{\Delta_+,0}(X_1, X_2)\right), \qquad (6.11)$$

which reproduces $\Pi_{\pm\mp}(X_1, X_2)$ in (6.2). In this way, we have confirmed that the full order results in (2.24) and (2.25) can be obtained also by summing over all order corrections in the conformal perturbation theory from the boundary viewpoints.

7 Conclusion

In [12] it was proposed a holographic duality between $\mathcal{N}=3$ coset model (2.1) and a 3d Prokushkin-Vasiliev theory with extended supersymmetry [6]. In [13] the generic higher spin symmetry is broken by adding the deformation term (2.19) to the CFT in order to see the relation to superstring theory. The symmetry breaking induces the mass to the higher spin fields, and the explicit value of the mass was computed for a spin 2 field in [13]. The aim of this paper is to extend the analysis to the case with generic spin $s > 2$. We have computed the anomalous dimensions of higher spin currents in the boundary theory in various ways. Using the results, we have obtained the mass formula for higher spin fields in the bulk theory as in (2.24) and (2.25). The results are at the first order of $1/c$. At this order, we can utilize the free ghost system (2.6) and the classical dual bulk theory. Combining these techniques, we can obtain the expressions at the full order of the perturbation parameter f in (2.19).

There are two types of higher spin fields. The first type are the $\mathrm{so}(3)_R$ singlets and the other one are $\mathrm{so}(3)_R$ triplets. We found that the $\mathrm{so}(3)_R$ singlet fields do not receive any corrections as in (2.24). The result is actually expected since our deformation is of the same type as the one in [26]. They considered the deformation which does not break the higher spin symmetry at the leading order of $1/c$ and f. The deformation operator is characterized by the eigenvalue of $J_0^{(3)}$ and our deformation operator in (2.20) has the required eigenvalue. It was also argued that the higher spin currents are no longer conserved if we consider the higher order of $1/c$ and f. Therefore we expect that the masses would be generated for $s = 3, 4, \ldots$ once we include the higher order effects of $1/c$. It is an important task to study these $1/c$ corrections, but for the purpose we may need to deal directly with the $\mathcal{N}=3$ coset model (2.1) with finite N.

Interesting results may be found for the $\mathrm{so}(3)_R$ triplet fields. The mass formula for these fields is obtained in (2.25). The formula resembles the Regge spectrum on flat space-time, and this may indicate that the deformed theory is related to a superstring theory with pure NSNS-flux. In the pure NSNS-backgrounds, generic arguments in [53] say that only three types of target space of superstring theory are consistent with the boundary $\mathcal{N}=3$ superconformal symmetry. Comparing the BPS spectrum of superstring theory in [54], it was conjectured in [13] that the target space of the related superstring theory should be $\mathrm{AdS}_3\times\mathrm{M}^7$ with $\mathrm{M}^7 = \mathrm{SU}(3)/\mathrm{U}(1)$ or $\mathrm{SO}(5)/\mathrm{SO}(3)$. Since both cases lead to the same BPS spectrum, we can argue that they are related by a marginal deformation. As a future problem, we would like to compute the string spectrum with these target spaces and compare them with the results obtained here. It was found in [13] that there are other types of marginal deformations preserving $\mathcal{N}=3$ superconformal symmetry. These deformations may be useful to compare to the string spectra.

In order to obtain the contributions at the higher order of f^2, we have used the classical bulk theory results both for bosonic and fermionic deformation operators in subsection 6.1. We have tried to interpret the corrections from the boundary viewpoints only for the bosonic ones in subsection 6.2. It would be nice if we could extend the analysis for the fermionic ones, but for the purpose we need to generalize the embedding formalism also for the spinor tensor fields on AdS_{d+1}.

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A Higher spin superalgebra and free ghost system

The higher spin superalgebra $\mathrm{shs}_{M'}[\lambda]$ introduced in (2.4) can be realized by the symmetry of the free ghost system [36] (see also [40, 11])

$$S = \frac{1}{\pi} \int d^2 z \sum_{A=1}^{M'} \left\{ \beta_A \bar{\partial} \gamma_A + b_A \bar{\partial} c_A \right\} \,, \tag{A.1}$$

which lead to the operator products as

$$\gamma_A(z) \beta_B(w) \sim \frac{\delta_{AB}}{z-w} \,, \quad c_A(z) b_B(w) \sim \frac{\delta_{AB}}{z-w} \,. \tag{A.2}$$

Here $A, B = 1, 2, \ldots, M'$ and the conformal weights for b_A, c_A, β_A and γ_A are $(1+\lambda)/2$, $(1-\lambda)/2, \lambda/2$ and $1 - \lambda/2$.

The truncated algebra $\mathrm{shs}_2^T[1/2]$ can be generated by the free ghosts b_A, c_A, β_A and γ_A $(A = 1, 2)$ with the conformal weights $3/4, 1/4, 1/4$ and $3/4$, respectively. In [36], the explicit expression by free ghost system is given for higher spin currents with $\mathrm{shs}[\lambda]$ as the wedge subalgebra, and the truncation from $\mathrm{shs}[1/2]$ to $\mathrm{shs}_1^T[1/2]$ was also argued. Using the results we can realize $\mathrm{shs}_2^T[1/2]$ as the wedge subalgebra of the one generated by

$$[V^{(s)}(z)]_{AB} = \sum_{i=0}^{s-1} \tilde{a}^i(s, 1/2)(\partial^i \beta_A)(\partial^{s-1-i} \gamma_B) + \sum_{i=0}^{s-1} \tilde{a}^i(s, 3/2)(\partial^i b_A)(\partial^{s-1-i} c_B) \,, \tag{A.3}$$

$$[Q^{(s)}(z)]_{AB} = \sum_{i=1}^{s-1} \tilde{\alpha}^i(s, 1/2)(\partial^i \beta_A)(\partial^{s-1-i} c_B) + \sum_{i=0}^{s-2} \tilde{\beta}^i(s, 1/2)(\partial^i b_A)(\partial^{s-2-i} \gamma_B) \,,$$

where $\tilde{a}^i(s,\lambda)$ is defined in (4.4) and

$$\tilde{\alpha}^i(s,\lambda) = 2\begin{pmatrix} s-1 \\ i \end{pmatrix}\frac{(-1)^i}{(s)_{s-1}}(\lambda-s+1)_i(2-\lambda-s)_{s-1-i}\,, \tag{A.4}$$

$$\tilde{\beta}^i(s,\lambda) = \begin{pmatrix} s-2 \\ i \end{pmatrix}\frac{(-1)^i}{(s)_{s-2}}(\lambda-s+1)_i(2-\lambda-s)_{s-2-i}\,.$$

The $\mathcal{N}=3$ superconformal algebra can be obtained as a subalgebra generated by the low spin currents defined in (A.3). The explicit expressions for the low spin generators in (A.3) can be written as

$$T = \frac{1}{4}\sum_{A=1}^{2}\left[3(\partial\beta_A)\gamma_A - \beta_A\partial\gamma_A + (\partial b_A)c_A - 3b_A\partial c_A\right]\,,$$

$$J^+ = \beta_1\gamma_2 + b_1c_2\,, \quad J^- = \beta_2\gamma_1 + b_2c_1\,, \quad J^3 = \frac{1}{2}\sum_{A=1}^{2}(-1)^{A-1}\left[\beta_A\gamma_A + b_Ac_A\right]\,, \tag{A.5}$$

$$G^+ = (\partial\beta_1)c_2 - \beta_1\partial c_2 + 2b_1\gamma_2\,, \quad G^- = (\partial\beta_2)c_1 - \beta_2\partial c_1 + 2b_2\gamma_1\,,$$

$$G^3 = \frac{1}{2}\sum_{A=1}^{2}(-1)^{A-1}\left[(\partial\beta_A)c_A - \beta_A\partial c_A + 2b_A\gamma_A\right]\,, \quad \Psi = \frac{1}{2}\sum_{A=1}^{2}\beta_Ac_A\,.$$

We can check that these generators satisfy the operator product expansions for the $\mathcal{N}=3$ superconformal algebra with $c=k=0$. See [55] for some details of the $\mathcal{N}=3$ algebra.

B Integral formulas

The expression for the boundary-to-bulk propagators in the embedding formulation may be found in [49]. For a spin 0 field, it is given by

$$\Pi_{\Delta,0}(X,P) = \mathcal{C}_\Delta\frac{1}{(-2P\cdot X)^\Delta}\,, \quad \mathcal{C}_\Delta = \frac{(2\Delta-d)\Gamma(\Delta)}{\pi^{d/2}\Gamma(\Delta-d/2)}\,. \tag{B.1}$$

The boundary-to-boundary propagator is then obtained by replacing X by Q with $Q^2=0$.

In the main context, we need to evaluate the following integral as

$$\begin{aligned} I_\Delta(X,Q) &= \int_\partial dP\,\Pi_{d-\Delta,0}(X,P)\Pi_{\Delta,0}(Q,P) \\ &= \mathcal{C}_{d-\Delta}\mathcal{C}_\Delta\int_\partial dP\frac{1}{(-2P\cdot X)^{d-\Delta}}\frac{1}{(-2P\cdot Q)^\Delta}\,. \end{aligned} \tag{B.2}$$

Using the Feynman parametrization

$$\frac{1}{\prod_i A_i^{a_i}} = \frac{\Gamma(\sum_i a_i)}{\prod_i \Gamma(a_i)} \int_0^\infty \prod_{i=2}^n dq_i q_i^{a_i-1} \frac{1}{(A_1 + \sum_{i=2}^n q_i A_i)^{\sum_i a_i}}\,, \tag{B.3}$$

the integral can be written as

$$I_\Delta(X,Q) = \mathcal{C}_{d-\Delta}\mathcal{C}_\Delta \frac{\Gamma(d)}{\Gamma(d-\Delta)\Gamma(\Delta)} \int_\partial dP \int_0^\infty dq \frac{q^{\Delta-1}}{(-2P\cdot Y)^d} \tag{B.4}$$

with $Y = X + qQ$. In [48] a useful integral formula was derived as

$$\int_\partial dP \frac{1}{(-2P\cdot Y)^d} = \frac{\pi^{d/2}\Gamma(d/2)}{\Gamma(d)} \frac{1}{(-Y^2)^{d/2}} \tag{B.5}$$

with $Y^2 < 0$. Applying the formula we have

$$\begin{aligned} I_\Delta(X,Q) &= \mathcal{C}_{d-\Delta}\mathcal{C}_\Delta \frac{\pi^{d/2}\Gamma(d/2)}{\Gamma(d-\Delta)\Gamma(\Delta)} \int_0^\infty dq \frac{q^{\Delta-1}}{(1-2qQ\cdot X)^{d/2}} \\ &= (d-2\Delta)\Pi_{\Delta,0}(X,Q)\,. \end{aligned} \tag{B.6}$$

In the last equality we have used

$$\int_0^\infty dq q^{\Delta-1}(1+q)^{-d/2} = \frac{\Gamma(d/2-\Delta)\Gamma(\Delta)}{\Gamma(d/2)}\,. \tag{B.7}$$

References

[1] D. J. Gross, *High-energy symmetries of string theory*, *Phys.Rev.Lett.* **60** (1988) 1229.

[2] M. Vasiliev, *Nonlinear equations for symmetric massless higher spin fields in (A)dS$_{(d)}$*, *Phys.Lett.* **B567** (2003) 139–151, [hep-th/0304049].

[3] I. Klebanov and A. Polyakov, *AdS dual of the critical $O(N)$ vector model*, *Phys.Lett.* **B550** (2002) 213–219, [hep-th/0210114].

[4] E. Sezgin and P. Sundell, *Massless higher spins and holography*, *Nucl.Phys.* **B644** (2002) 303–370, [hep-th/0205131].

[5] M. R. Gaberdiel and R. Gopakumar, *An AdS$_3$ dual for minimal model CFTs*, *Phys.Rev.* **D83** (2011) 066007, [arXiv:1011.2986].

[6] S. Prokushkin and M. A. Vasiliev, *Higher spin gauge interactions for massive matter fields in 3-D AdS space-time*, *Nucl.Phys.* **B545** (1999) 385, [hep-th/9806236].

[7] C.-M. Chang, S. Minwalla, T. Sharma, and X. Yin, *ABJ triality: From higher spin fields to strings*, *J.Phys.* **A46** (2013) 214009, [arXiv:1207.4485].

[8] M. R. Gaberdiel and R. Gopakumar, *Large $\mathcal{N} = 4$ holography*, *JHEP* **1309** (2013) 036, [arXiv:1305.4181].

[9] M. R. Gaberdiel and R. Gopakumar, *Higher spins & strings*, *JHEP* **1411** (2014) 044, [arXiv:1406.6103].

[10] M. R. Gaberdiel and R. Gopakumar, *Stringy symmetries and the higher spin square*, arXiv:1501.0723.

[11] T. Creutzig, Y. Hikida, and P. B. Rønne, *Extended higher spin holography and Grassmannian models*, *JHEP* **1311** (2013) 038, [arXiv:1306.0466].

[12] T. Creutzig, Y. Hikida, and P. B. Rønne, *Higher spin AdS_3 holography with extended supersymmetry*, *JHEP* **1410** (2014) 163, [arXiv:1406.1521].

[13] Y. Hikida and P. B. Ronne, *Marginal deformations and the Higgs phenomenon in higher spin AdS_3 holography*, arXiv:1503.0387.

[14] C. Candu and C. Vollenweider, *On the coset duals of extended higher spin theories*, *JHEP* **1404** (2014) 145, [arXiv:1312.5240].

[15] M. R. Gaberdiel and C. Peng, *The symmetry of large $\mathcal{N} = 4$ holography*, *JHEP* **1405** (2014) 152, [arXiv:1403.2396].

[16] M. Beccaria, C. Candu, and M. R. Gaberdiel, *The large $\mathcal{N} = 4$ superconformal W_∞ algebra*, *JHEP* **1406** (2014) 117, [arXiv:1404.1694].

[17] C. Candu, C. Peng, and C. Vollenweider, *Extended supersymmetry in AdS_3 higher spin theories*, *JHEP* **1412** (2014) 113, [arXiv:1408.5144].

[18] M. Baggio, M. R. Gaberdiel, and C. Peng, *Higher spins in the symmetric orbifold of K3*, arXiv:1504.0092.

[19] M. R. Gaberdiel, C. Peng, and I. G. Zadeh, *Higgsing the stringy higher spin symmetry*, arXiv:1506.0204.

[20] E. Witten, *Multitrace operators, boundary conditions, and AdS/CFT correspondence*, hep-th/0112258.

[21] M. Porrati, *Higgs phenomenon for 4-D gravity in anti-de Sitter space*, *JHEP* **0204** (2002) 058, [hep-th/0112166].

[22] M. Duff, J. T. Liu, and H. Sati, *Complementarity of the Maldacena and Karch-Randall pictures*, *Phys.Rev.* **D69** (2004) 085012, [hep-th/0207003].

[23] E. Kiritsis, *Product CFTs, gravitational cloning, massive gravitons and the space of gravitational duals*, *JHEP* **0611** (2006) 049, [hep-th/0608088].

[24] O. Aharony, A. B. Clark, and A. Karch, *The CFT/AdS correspondence, massive gravitons and a connectivity index conjecture*, *Phys.Rev.* **D74** (2006) 086006, [hep-th/0608089].

[25] L. Girardello, M. Porrati, and A. Zaffaroni, *3-D interacting CFTs and generalized Higgs phenomenon in higher spin theories on AdS*, *Phys.Lett.* **B561** (2003) 289–293, [hep-th/0212181].

[26] M. R. Gaberdiel, K. Jin, and W. Li, *Perturbations of W_∞ CFTs*, *JHEP* **1310** (2013) 162, [arXiv:1307.4087].

[27] M. Henneaux, G. Lucena Gómez, J. Park, and S.-J. Rey, *Super-W_∞ asymptotic symmetry of higher-spin AdS_3 supergravity*, *JHEP* **1206** (2012) 037, [arXiv:1203.5152].

[28] M. Beccaria, C. Candu, M. R. Gaberdiel, and M. Groher, $\mathcal{N}=1$ *extension of minimal model holography*, *JHEP* **1307** (2013) 174, [arXiv:1305.1048].

[29] T. Creutzig, Y. Hikida, and P. B. Rønne, *Higher spin AdS_3 supergravity and its dual CFT*, *JHEP* **1202** (2012) 109, [arXiv:1111.2139].

[30] M. Blencowe, *A consistent interacting massless higher spin field theory in $D=(2+1)$*, *Class.Quant.Grav.* **6** (1989) 443.

[31] T. Creutzig, Y. Hikida, and P. B. Rønne, $\mathcal{N}=1$ *supersymmetric higher spin holography on AdS_3*, *JHEP* **1302** (2013) 019, [arXiv:1209.5404].

[32] M. Henneaux and S.-J. Rey, *Nonlinear W_∞ as asymptotic symmetry of three-dimensional higher spin anti-de Sitter gravity*, *JHEP* **1012** (2010) 007, [arXiv:1008.4579].

[33] A. Campoleoni, S. Fredenhagen, S. Pfenninger, and S. Theisen, *Asymptotic symmetries of three-dimensional gravity coupled to higher-spin fields*, *JHEP* **1011** (2010) 007, [arXiv:1008.4744].

[34] A. Campoleoni, S. Fredenhagen, and S. Pfenninger, *Asymptotic W-symmetries in three-dimensional higher-spin gauge theories*, *JHEP* **1109** (2011) 113, [arXiv:1107.0290].

[35] M. R. Gaberdiel and T. Hartman, *Symmetries of holographic minimal models*, *JHEP* **1105** (2011) 031, [arXiv:1101.2910].

[36] E. Bergshoeff, B. de Wit, and M. A. Vasiliev, *The structure of the super $W_\infty(\lambda)$ algebra*, *Nucl.Phys.* **B366** (1991) 315–346.

[37] M. Ammon, P. Kraus, and E. Perlmutter, *Scalar fields and three-point functions in D=3 higher spin gravity*, *JHEP* **1207** (2012) 113, [arXiv:1111.3926].

[38] C.-M. Chang and X. Yin, *Higher spin gravity with matter in AdS_3 and its CFT dual*, *JHEP* **1210** (2012) 024, [arXiv:1106.2580].

[39] T. Creutzig, Y. Hikida, and P. B. Rønne, *Three point functions in higher spin AdS_3 supergravity*, *JHEP* **1301** (2013) 171, [arXiv:1211.2237].

[40] H. Moradi and K. Zoubos, *Three-point functions in $\mathcal{N}=2$ higher-spin holography*, *JHEP* **1304** (2013) 018, [arXiv:1211.2239].

[41] M. R. Gaberdiel and R. Gopakumar, *Triality in minimal model holography*, *JHEP* **1207** (2012) 127, [arXiv:1205.2472].

[42] R. Blumenhagen, M. Flohr, A. Kliem, W. Nahm, A. Recknagel, et al., *W algebras with two and three generators*, *Nucl.Phys.* **B361** (1991) 255–289.

[43] M. R. Gaberdiel, T. Hartman, and K. Jin, *Higher Spin Black Holes from CFT*, *JHEP* **1204** (2012) 103, [arXiv:1203.0015].

[44] L. Cornalba, M. S. Costa, and J. Penedones, *Deep inelastic scattering in conformal QCD*, *JHEP* **1003** (2010) 133, [arXiv:0911.0043].

[45] S. Weinberg, *Six-dimensional methods for four-dimensional conformal field theories*, *Phys.Rev.* **D82** (2010) 045031, [arXiv:1006.3480].

[46] M. S. Costa, J. Penedones, D. Poland, and S. Rychkov, *Spinning conformal correlators*, *JHEP* **1111** (2011) 071, [arXiv:1107.3554].

[47] M. S. Costa, J. Penedones, D. Poland, and S. Rychkov, *Spinning conformal blocks*, *JHEP* **1111** (2011) 154, [arXiv:1109.6321].

[48] D. Simmons-Duffin, *Projectors, shadows, and conformal blocks*, *JHEP* **1404** (2014) 146, [arXiv:1204.3894].

[49] M. S. Costa, V. Gonçalves, and J. Penedones, *Spinning AdS propagators*, *JHEP* **1409** (2014) 064, [arXiv:1404.5625].

[50] T. Leonhardt, R. Manvelyan, and W. Ruhl, *The group approach to AdS space propagators*, *Nucl.Phys.* **B667** (2003) 413–434, [hep-th/0305235].

[51] T. Leonhardt, W. Ruhl, and R. Manvelyan, *The group approach to AdS space propagators: A fast algorithm*, *J.Phys.* **A37** (2004) 7051, [hep-th/0310063].

[52] O. Aharony, M. Berkooz, and B. Katz, *Non-local effects of multi-trace deformations in the AdS/CFT correspondence*, *JHEP* **0510** (2005) 097, [hep-th/0504177].

[53] R. Argurio, A. Giveon, and A. Shomer, *Superstring theory on $AdS_3 \times G/H$ and boundary $\mathcal{N} = 3$ superconformal symmetry*, *JHEP* **0004** (2000) 010, [hep-th/0002104].

[54] R. Argurio, A. Giveon, and A. Shomer, *The spectrum of $\mathcal{N} = 3$ string theory on $AdS_3 \times G/H$*, *JHEP* **0012** (2000) 025, [hep-th/0011046].

[55] K. Miki, *The representation theory of the SO(3) invariant superconformal algebra*, *Int.J.Mod.Phys.* **A5** (1990) 1293.