B3 Variation of gravity with latitude and elevation

By measuring the subtle **changes in the acceleration of gravity** from one place to another, it is possible to learn about **changes in subsurface density**.

However, other factors can cause gravity to vary with position on the Earth. These effects must be removed from measurements in order to use gravity data to study the interior of the Earth.

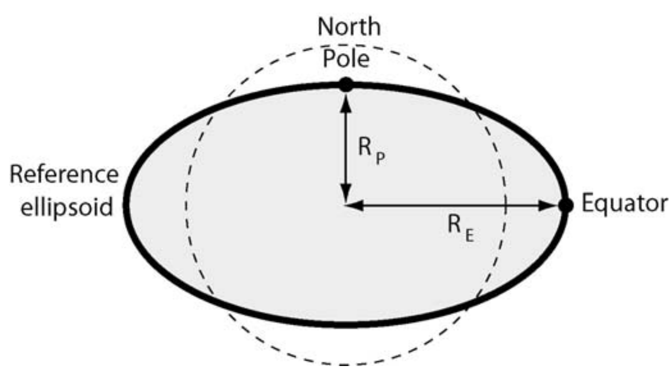
B3.1 Variation of gravity with latitude

It is observed that at the Equator, g_E = 978,033 mgal while at the poles g_P = 983,219 mgal

This difference is 5186 mgal, which is a lot larger than changes in gravity because of subsurface density.

Can this observation be explained by the fact that the Earth is a rotating ellipsoid?

(A)The Earth is distorted by rotation



The Earth is an oblate spheroid.

R_E = 6378 km

R_P = 6357 km.

Qualitative answer

Since a point on the Equator is **further** from the centre of the Earth than the poles, gravity will be **weaker at the Equator** and $g_E < g_P$

Quantitative answer

For a sphere $g(r) = \frac{GM_E}{r^2}$ where the mass of the Earth, $M_E = 5.957\ 10^{24}$ kg.

At the North Pole, R_P = 6357 km and g_P = 983,219 mgal.

If we move **up** 21 km to the equator, the **decrease** in gravity will be 6467 mgal

Thus $g_E = g_P$ - 6467 mgal, which is **too much** to explain the observed difference between the Equator and the Poles.

(B) - Centrifugal forces vary with latitude

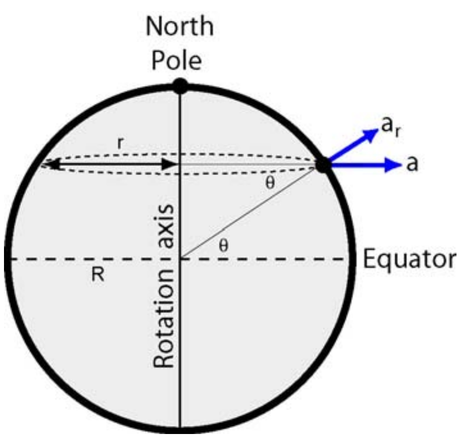
The **rotation** of the Earth also causes gravity to vary with latitude.

Qualitative answer

Imagine you are standing at the North Pole. The rotation of the Earth will not change **g**, all that will happen is that you rotate once a day.

Now imagine you are at the Equator. If we could increase the rotation rate of the Earth enough, you would be ultimately be thrown into space (i.e. become weightless). Thus rotation makes **gravity weaker** at the equator.

Quantitative answer



An observer is at a point with latitude θ. This observer travels around the rotation axis in circle with radius $r = R \cos \theta$.

The rotation rate is ω (radians per sec).

This corresponds to a radial acceleration $a = r\,\omega^2$ oriented towards the rotation axis.

Assume $R_E = 6378$ km

In a "vertical" direction (defined as pointing towards the centre of the Earth) this has a component :

$a_r = a \cos \theta = R\omega^2 \cos^2 \theta$

Now the Earth rotates once per day so $\omega = 2\pi / (24 \times 60 \times 60) = 7.27\ 10^{-5}$ rad s^{-1}

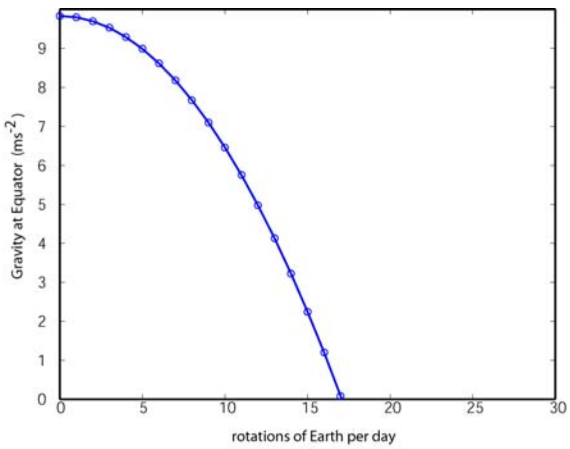
At the North Pole $\theta = 90^o$ $a_r = 0$

At the Equator, $\theta = 90^o$ $a_r = 0.03370$ m s^{-2} = 3370 mgal

$g_P = g_E$ +3370 mgal

$g_E < g_P$

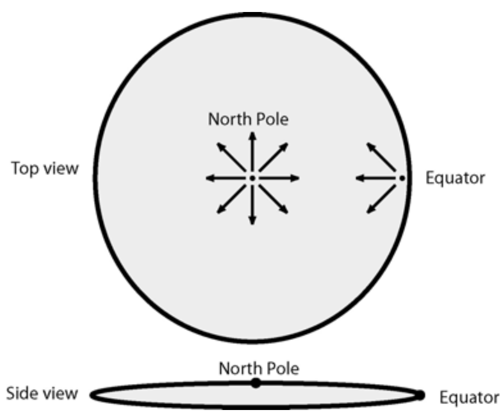
Question : You are standing on the Equator. How fast would the Earth need to rotate to throw you into space?



(C) Mass distribution of the Earth

These two factors both make $g_E < g_P$ so to get the observed difference we need to find a factor that has the **opposite** effect. The change in shape from a sphere to an ellipsoid redistributes the Earth's mass. Thus results in more mass between points on the Equator and the centre of the Earth, than between the poles and the centre of the Earth.

Qualitative answer



Consider the case shown above where the flattening is extreme. The observer at the pole experiences the pull of gravity in all directions, and this almost cancels out. An observer on the equator only experiences the pull of gravity due to mass located to the left.

Thus this effect will make $g_E > g_P$

Quantitative answer

Integration shows that $g_E \sim g_P + 4800$ mgal

Overall variation of g with latitude

Combining these three effects (A,B and C) gives

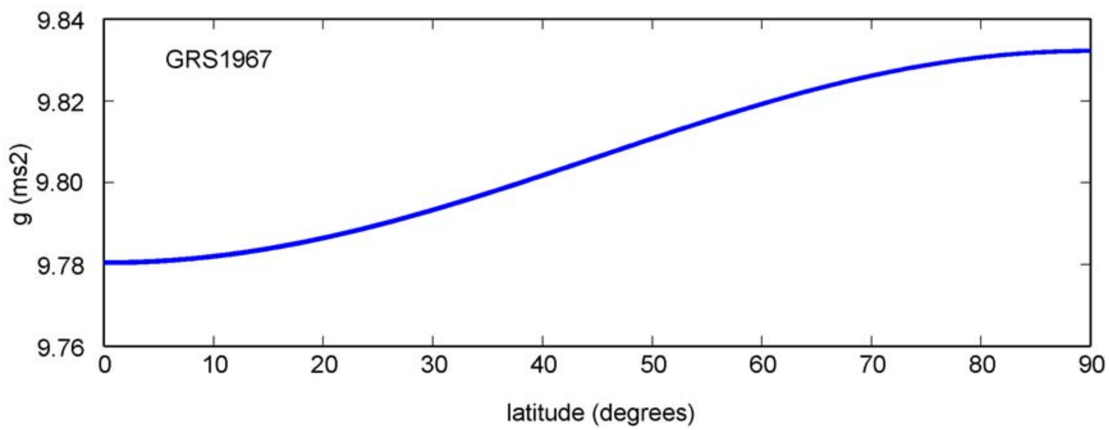
$g_P = g_E + 6467 + 3370 - 4800$ mgal $= g_E + 5037$ mgal (approximately as observed)

These factors are represented in the following equation, that defines the variation of g with latitude θ

$$g(\theta) = 9.78031846\,(1 + 0.0053024 \sin^2\theta - 0.0000058 \sin^2 2\theta)$$

This equation is called the **Geodetic Reference System** for 1967.

More recent revisions are essentially the same, but with ever more decimal places …..



Calculation 1 - What value does this equation predict for Edmonton?

In Edmonton θ = 53˚ 30’ 25” N and the GRS67 equation gives

$$g = 9.78031846\,(1 + 0.003417902 - 0.000005395)\ \text{m s}^{-2}$$

$$= 9.81369388\ \text{m s}^{-2}$$

Calculation 2 – How rapidly does gravity vary in a north-south direction?

The variation of g with latitude is important when a survey extends over a significant north-south distance. Differentiating the GRS67 equation with respect to θ yields

$$\frac{dg}{d\theta} = 9.78031846\ (0.0053024\ x\ 2 \sin\theta \cos\theta - 0.0000058\ x\ 4 \sin 2\theta \cos 2\theta)$$

$$= 0.049526 \quad \text{m s}^{-2} \text{ per radian}$$

$$= 0.0008655 \quad \text{m s}^{-2} \text{ per degree}$$

$$= 86.550 \quad \text{mgal per degree}$$

$$= 0.7868 \quad \text{mgal km}^{-1} \quad (\text{1 degree latitude} = 111 \text{ km})$$

All the these equations define the expected value of **theoretical gravity (or normal gravity)** at latitude θ. Differences between this value and what is actually measured are **anomalies** that we will analyse for information about subsurface density structure.

<u>Calculation 3 – How much lighter would you feel after driving from Edmonton to Calgary?</u>

Assume the scales read 80 kg in Edmonton

Change in gravity = 300 x 0.7868 mgals

Fractional change = 300 x 0.7868 / 981369.388 = 2.34 10^{-4}

"Mass" in Edmonton = 80 kg   >   "Mass" in Calgary = 80 (1-2.34 10^{-4}) = 79.98 kg

Change =   19 grams!!!!

210 B3.2 Variation of gravity with elevation

(i) The Free air correction

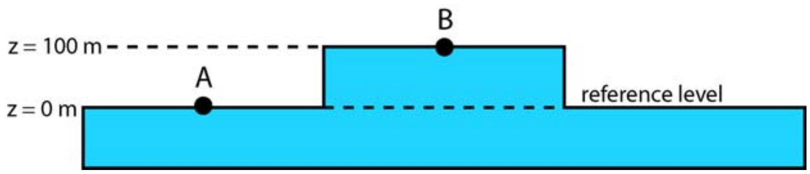
Newton's Theory of Gravitation states that at a distance, *r*, from the centre of the Earth

$$g(r) = \frac{GM}{r^2}$$

This means that as you move away from the centre of the Earth, the acceleration of gravity (g) decreases. In Edmonton, g = 9.81 ms $^{-2}$ and if you move **up** a distance, Δh, the acceleration of gravity will **decrease** by

$$\Delta g = 3.086\ \Delta h \times 10^{-6} \quad \text{m s}^{-2}$$
$$= 0.3086\ \Delta h \quad \text{mgal}$$

Consider the exciting topography of a flat topped mountain:



Gravity measurements are made at points A and B. The difference in elevation means that g_B will be less than g_B by an amount

$$\Delta g = 0.3086 \times 100 = 30.86 \text{ mgal}$$

When collecting gravity data, our real interest is to determine the density of the rocks below ground. The change in elevation from 'A' to 'B' will thus contaminate the data. The **Free Air correction** is a mathematical way of undoing the effect of elevation. It allows us to correct the data collected at 'B' in order to make it equivalent to data collected at the same elevation as 'A'.

In gravity surveys, we always define a **reference level** for the survey. Free Air corrections are made relative to this level. In general, any reference level could be chosen, but sea level is commonly chosen in coastal areas. In Alberta, the average level of the prairies would be a good choice.

If a gravity measurement was made Δh **above** the reference level, we must **add**

$$C_{FA} = 0.3086\ \Delta h \quad mgal$$

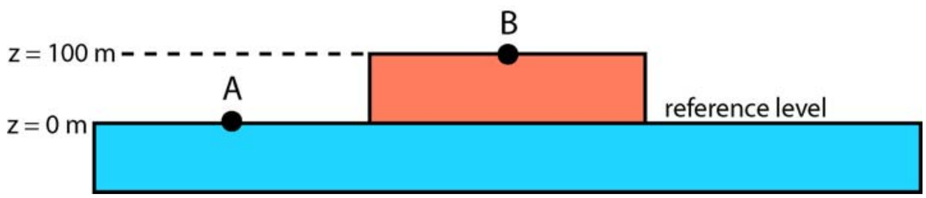
C_{FA} is called the Free Air correction for a given gravity measurement.

Similarly, if a gravity measurement was made Δh **below** the reference level, we must **subtract**

$$C_{FA} = 0.3086\ \Delta h \quad mgal$$

Question : to keep data accurate to 0.1 mgal, how accurately must we know the elevation?

(ii) The Bouguer correction



Unfortunately, this is not the end of story! Compare the gravity measurements at 'A' and 'B'. At point A, the gravity measurement is solely due to structure below the reference level (blue). At 'B' the gravity measurement is due to structure below the reference level, **plus** the gravitational pull of the 100 metres of mountain (red). The net result is that $g_B > g_A$

From section B2.3 the magnitude of this extra gravitational attraction is approximately

$$g_B - g_A = 2\pi G \rho \Delta h$$

where ρ is the density of the mountain.

Thus to remove this effect we need to **subtract** $C_B = 2\pi G \rho \Delta h$ from the observed gravity measurement at 'B'. This is called the **Bouguer correction** and

$$C_B = 0.00004193 \ \rho \Delta h \quad \text{mgal}$$

Note that to apply the Bouguer correction we need to estimate ρ, the density that lies between 'B' and the reference level. Using the value $\rho = 2670$ kg m^{-3} this gives

$$C_B = -0.1119 \ \Delta h \quad \text{mgal}$$

This value represents an average density for crustal rocks. Other information (*e.g* borehole gravity data or Nettleton's method) may be used to give a better estimate of the density.


Pierre Bouguer

Summary

Measurement **above** reference level	**Add** Free Air correction	**Subtract** Bouguer correction
Measurement **below** reference level	**Subtract** Free Air correction	**Add** Bouguer correction