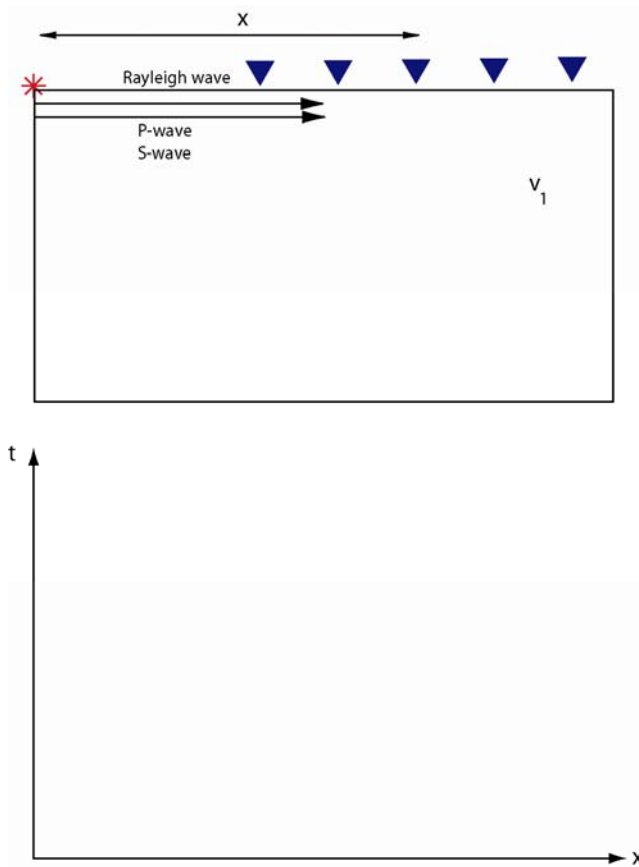


C3.1 Travel time curves

C3.1.1 Travel time curve for a flat Earth

C3.1.1.1 Uniform structure

Direct arrivals : P-wave, S-wave, Rayleigh wave



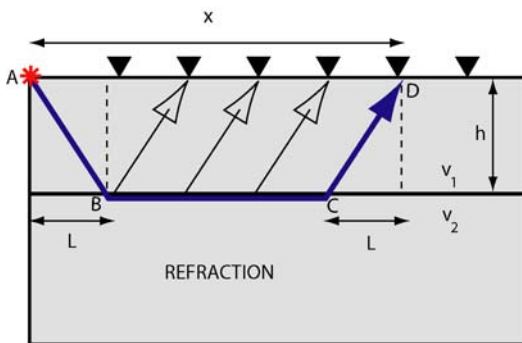
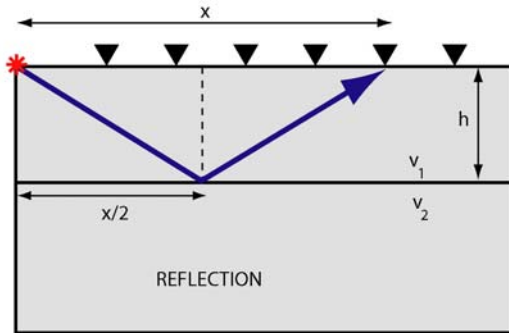
C3.1.1.2 Travel time curve for two-layers

Direct arrivals : P-wave, S-wave, Rayleigh wave

Reflected arrivals : P-wave

Refracted arrival P-wave

Consider the case of $v_1 < v_2$



Direct wave $t = \frac{x}{v_1}$

Reflected wave $t = \frac{\sqrt{x^2 + 4h^2}}{v_1}$

Derivation in class

Refracted wave

This is generated when the refracted wave travels horizontally, just below the interface

$$\frac{\sin \theta_c}{v_1} = \frac{\sin 90^\circ}{v_2}$$

This gives $\sin \theta_c = \frac{v_1}{v_2}$ and can show that $AB = CD = z / \cos \theta_c$

Can also show that $BC = x - 2h \tan \theta_c$

Total travel time

$$t = t_{AB} + t_{BC} + t_{CD}$$

$$t = \frac{z}{v_1 \cos \theta_c} + \frac{(x - 2h \tan \theta_c)}{v_2} + \frac{h}{v_1 \cos \theta_c}$$

$$t = \frac{x}{v_2} + \frac{2h}{v_1 \cos \theta_c} - \frac{2h \tan \theta_c}{v_2}$$

$$t = \frac{x}{v_2} + \frac{2hv_2 - 2hv_1 \sin \theta_c}{v_1 v_2 \cos \theta_c}$$

$$t = \frac{x}{v_2} + \frac{2h - 2h \sin^2 \theta_c}{v_1 v_2 \cos \theta_c}$$

$$t = \frac{x}{v_2} + \frac{2h \cos \theta_c}{v_1}$$

$$t = \frac{x}{v_2} + \frac{2h \sqrt{v_2^2 - v_1^2}}{v_1 v_2}$$

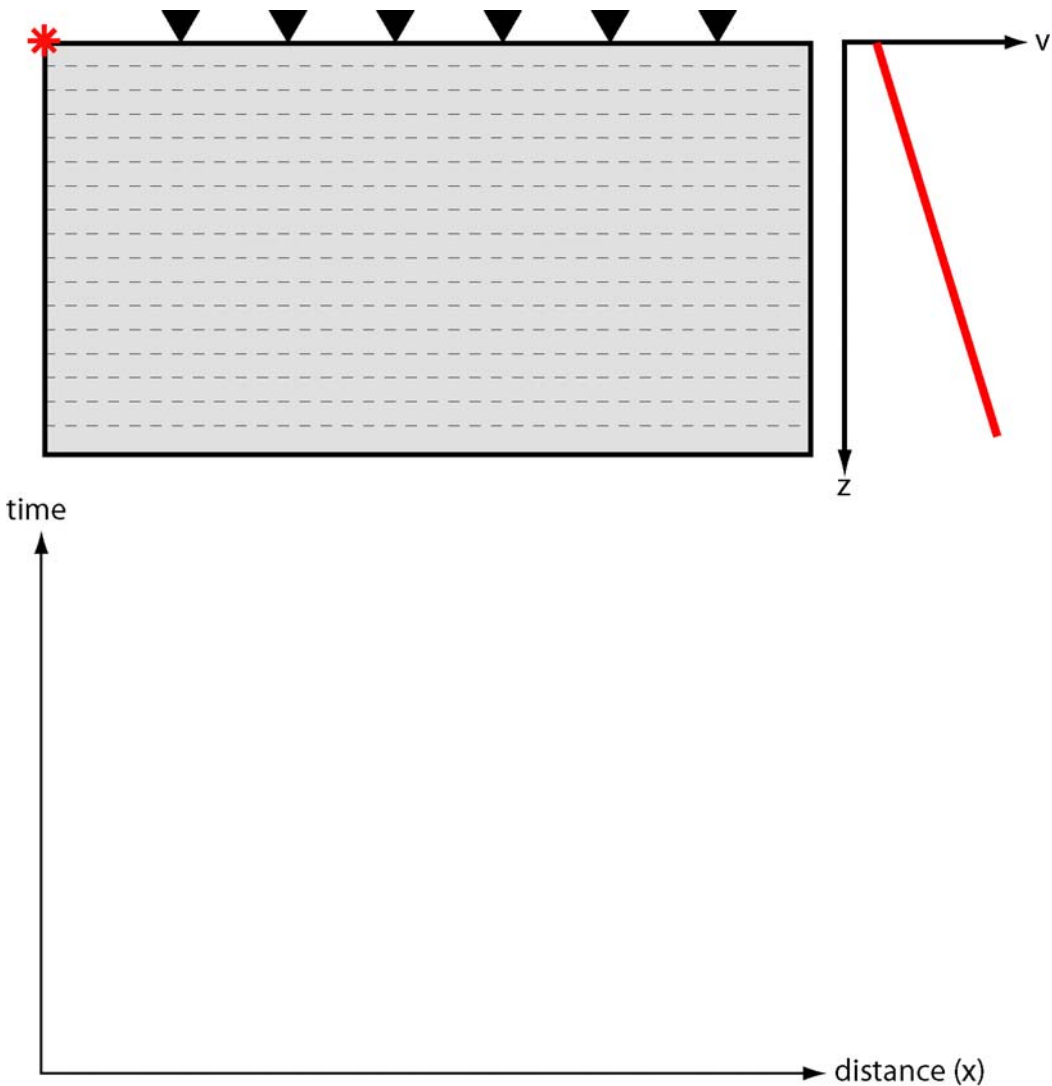
$$t = \frac{x}{v_2} + \text{constant}$$

- The travel time curve for the refracted wave is a **straight line** with slope = $1 / v_2$
- The refracted arrival is only observed when $x > x_{crit} = 2z \tan \theta_c$
- The refracted wave is the **first arrival** at $x > x_{cross}$
- When $x = x_{crit}$ the refracted and reflected waves are the same
- v_2 can be calculated from the **slope** of the refracted wave on the t - x plot
- The depth of the interface (z) can be found by extrapolating the travel time of the refracted wave to $x = 0$ where the travel time is

$$t_i = \frac{2z \sqrt{v_2^2 - v_1^2}}{v_1 v_2}$$

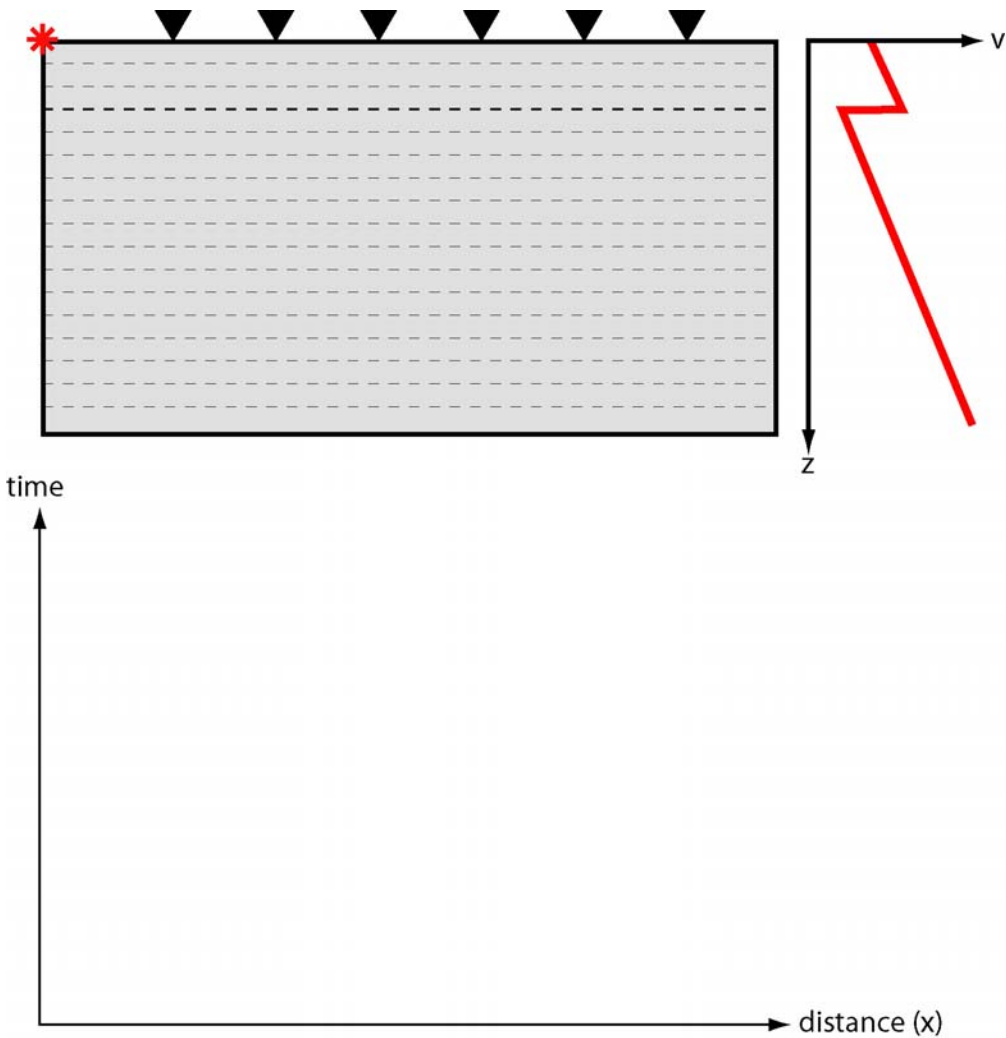
Rearranging gives $z = \frac{v_1 v_2 t_i}{2 \sqrt{v_2^2 - v_1^2}}$

C3.1.1.3 Velocity gradient



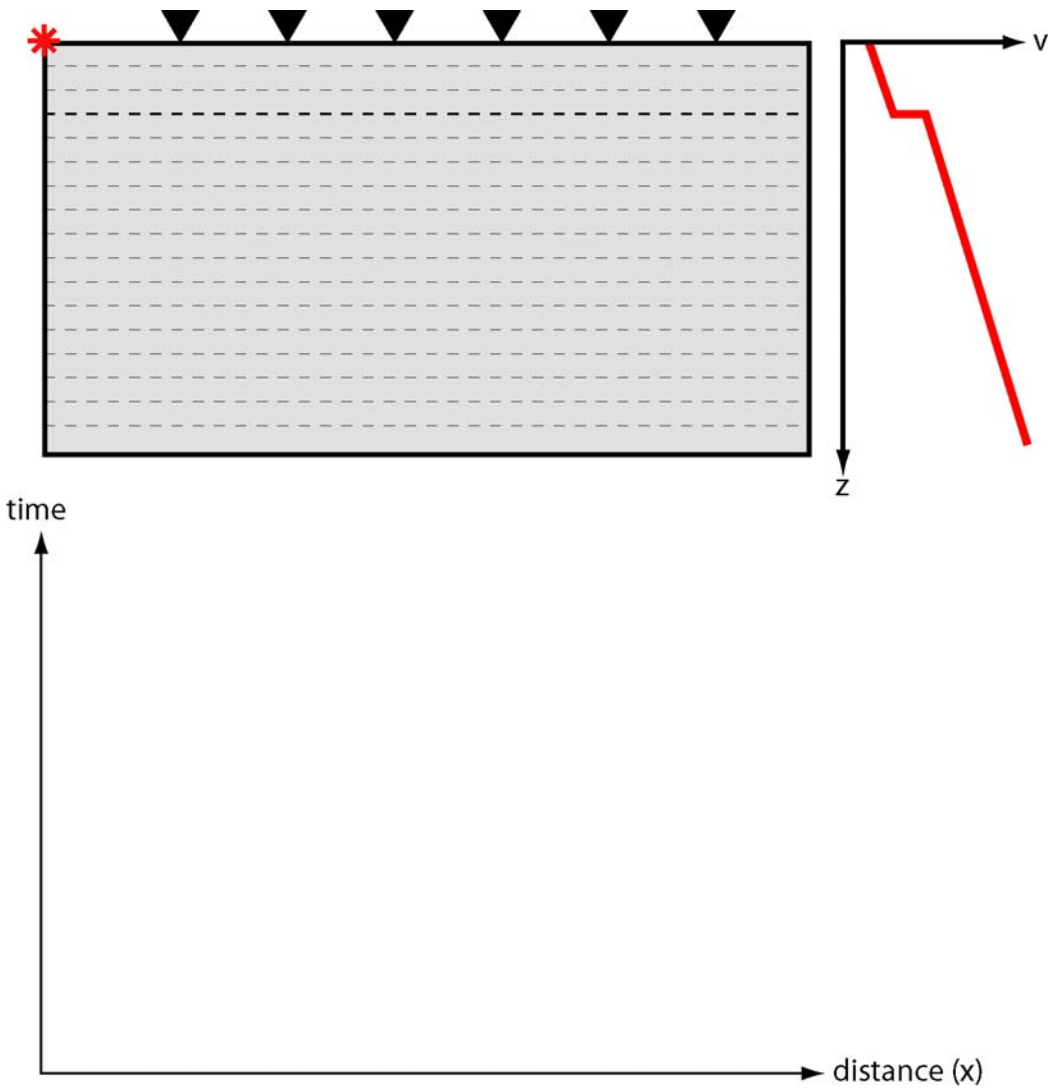
- Snells Law requires that the ray parameter, p , is constant.
- Thus $\sin \theta / v$ is constant along the ray. As v increases, so does θ and the ray travels closer to the horizontal.
- Ultimately $\sin \theta = 1$ which requires $\theta = 90^\circ$ and the ray travels horizontally.
- The uniform increase in velocity causes **curved** ray paths
- Sketch the direct wave.
- Can think of this wave as a superposition of many refracted arrivals

C3.1.1.4 Velocity gradient and low-velocity zone



- The uniform increase in velocity causes curved ray paths
- Sketch the direct wave
- Sketch the reflected wave from interface
- Sketch the refracted wave that travels below the interface.
- As wave strikes interface it is refracted towards the normal. This makes it travel further, causing a **shadow zone**.

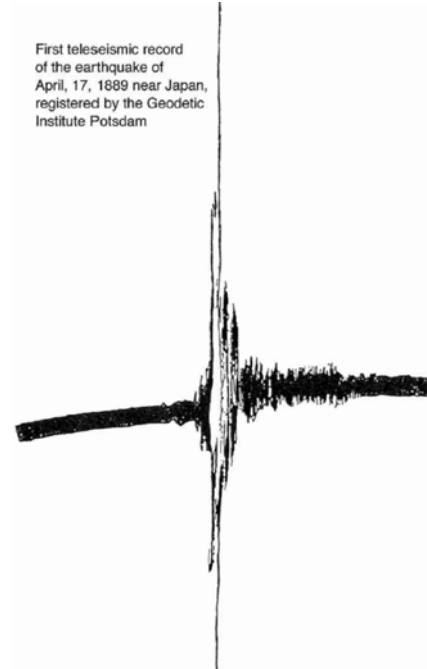
C3.1.1.5 Velocity gradient and high-velocity zone



- direct wave is not observed at large values of x
- reflected wave is not observed at large values of x
- At largest offset, the direct wave and reflected wave take the same path
- refracted wave produced because we have an increase in velocity
- example of triplication

C3.1.2 Travel time curve for uniform velocity, spherical Earth

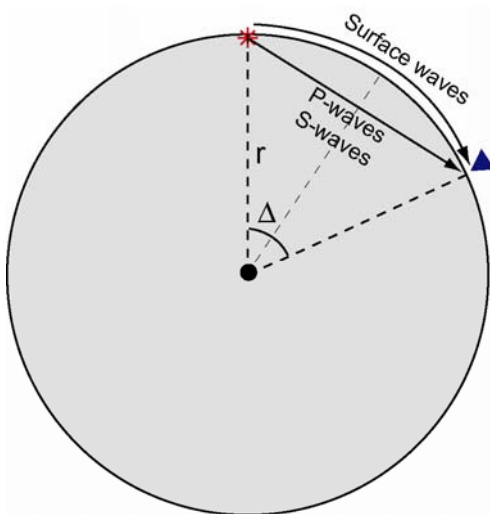
- 1883 John Milne speculated that *“it is not unlikely that every large earthquake might with proper appliances be recorded at any point of the globe”*
- First **teleseismic** signal observed in 1889 when waves from an earthquake in Japan were recorded in Germany.
- In 1897 Richard Oldham showed that earthquakes consisted of *“preliminary tremors”* and *“large waves”*. Time difference between them increases with distance and can be used to locate the earthquake. See C2.2



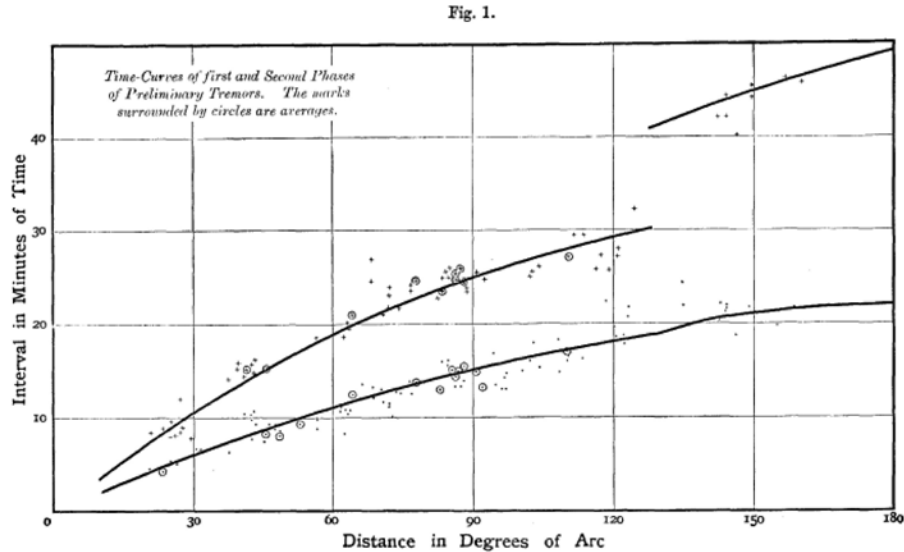
- 1900 Oldham realized that the *“preliminary tremors”* travelled through the centre of Earth (body waves) while the *“large waves”* travelled along the surface (surface waves).

What will the travel time curve look like for this Earth structure?

Direct arrivals: P-wave, S-wave, Rayleigh wave



- Measurements require a more complicated model!
- Oldham (1906) gave evidence that Earth had some internal structure with a core. Hindsight has shown that his analysis was only partially correct.



Observations

- P-waves did not appear to travel effectively beyond $\Delta = 105^\circ$
- Beyond $\Delta = 130^\circ$ the P-waves were observed again, but delayed by 2 minutes.
- S-waves apparently delayed by 10 minutes beyond $\Delta = 130^\circ$

Oldham's Explanation

- Low velocity core, radius ~ 2550 km

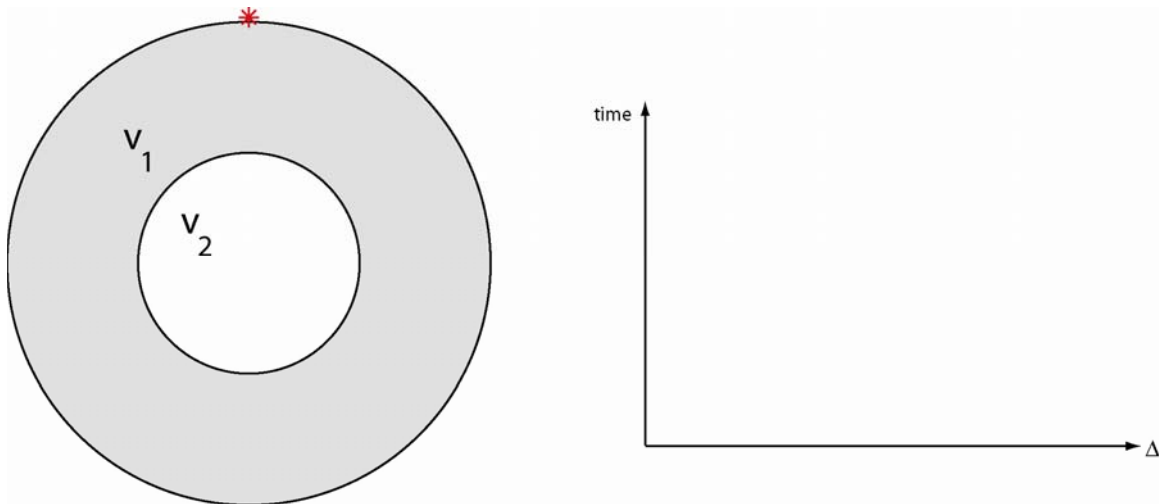
Hindsight

- Analysis of P-waves was correct. These are now called PKP and PKiKP phases
- We now know that S-waves do not travel in the liquid outer core.
- The S-waves reported by Oldham from $\Delta = 130^\circ$ to 180° are SS waves that are multiple bounces in the mantle

C3.1.3 Travel time curve for uniform Earth with a uniform core

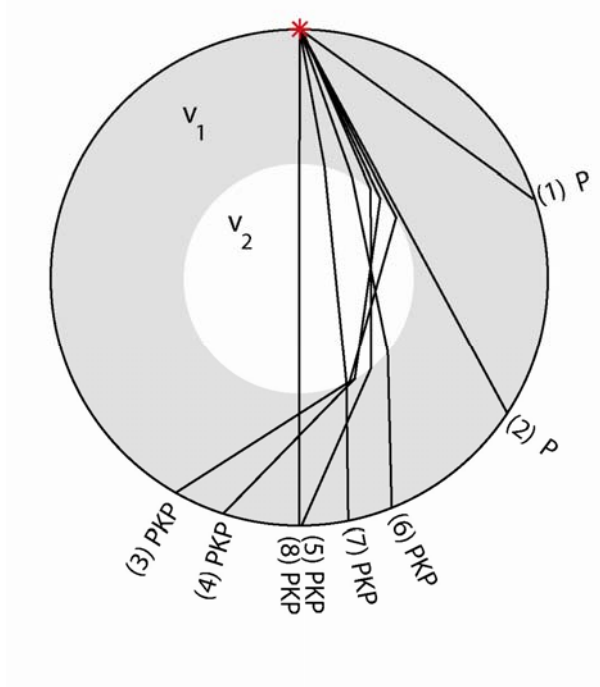
Sketch the following waves for the case for a low velocity core ($v_1 > v_2$)

- Direct P-wave
- P-wave that reflects from core



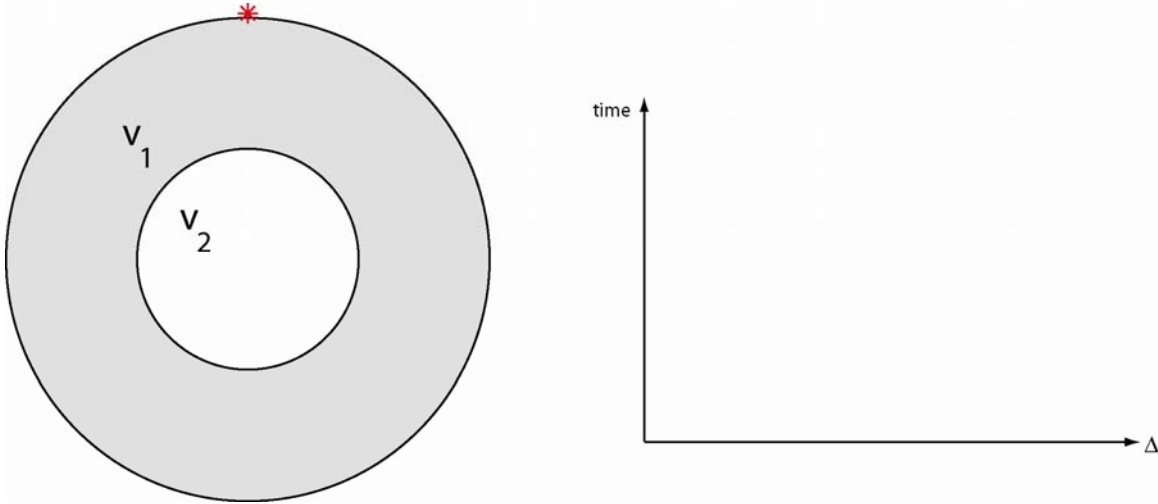
Now consider:

P-wave in mantle, P-wave in outer core, P-wave in mantle back to surface

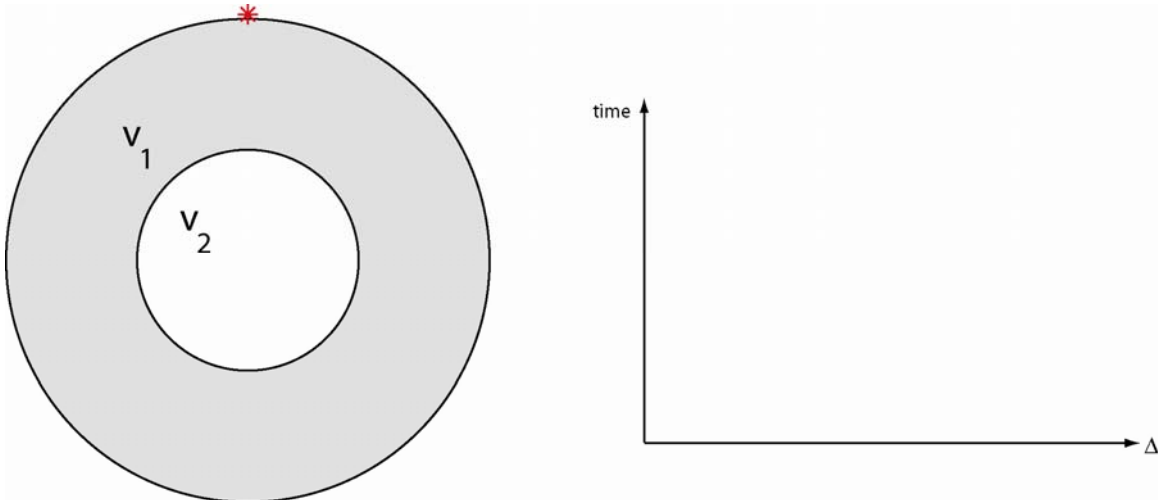


- Note that at points 4-8 there are two possible ray paths for the P-wave.
- This results in two PKP arrivals on the travel time curve.
- Amplitudes strongest at the cusp (6)

- The core acts as a powerful magnifying glass, distorting the seismic waves.
How would things be different if $v_1 < v_2$?



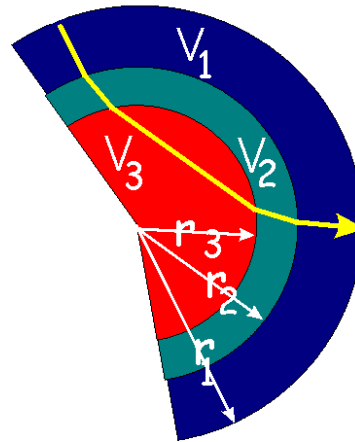
Multiples



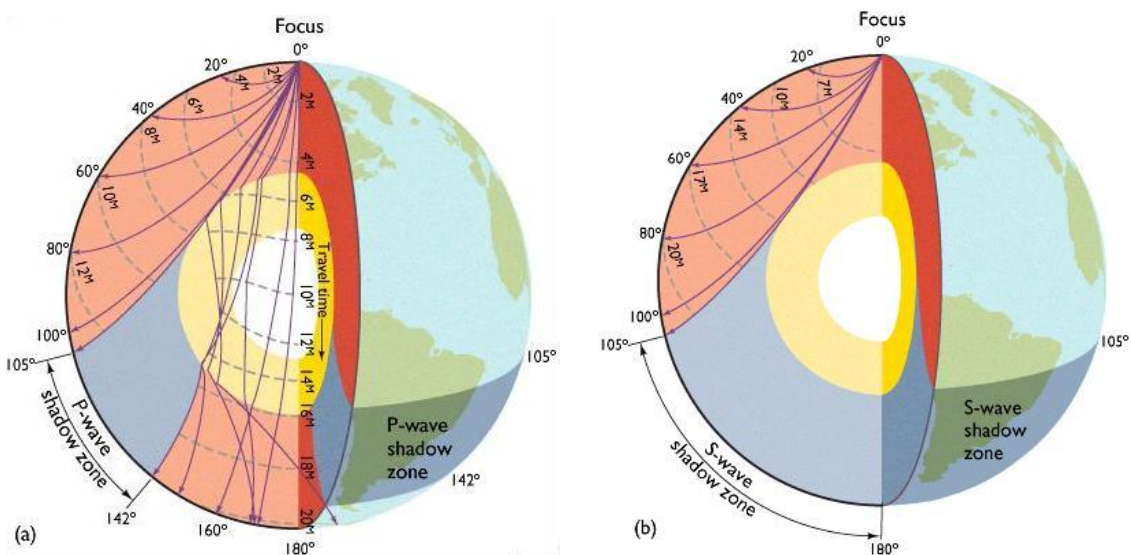
- Can compute the reflection co-efficient for a wave striking the surface of the Earth. In this case need to include both velocity and density in equation. $R = -1$
- **PP** is a P-wave that bounces from surface of Earth
- **SS** is a S-wave that bounces from surface of Earth
- How do travel times for P and PP compare?

C3.1.4 Velocity gradient in a spherical Earth with a core

$$p = \frac{r_1 \sin \theta_1}{v_1} = \frac{r_2 \sin \theta_2}{v_2} = \frac{r_3 \sin \theta_3}{v_3}$$

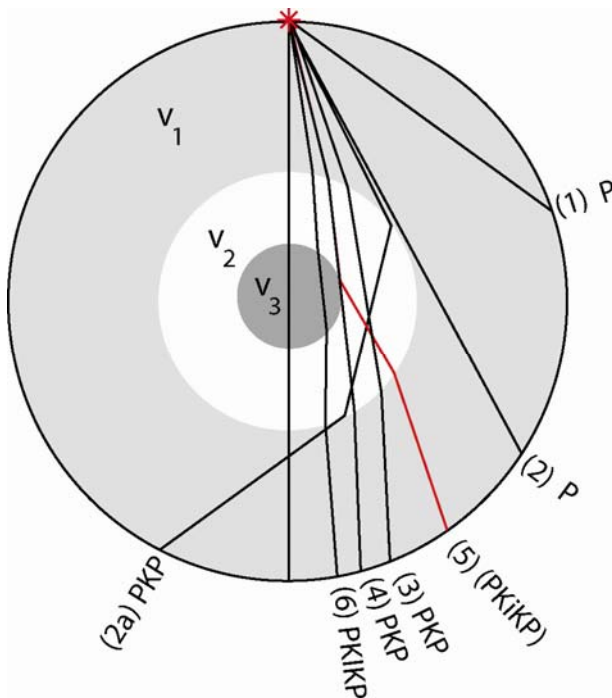


- Show that along the ray path, the ray parameter, p is constant.
- The angle θ is between the ray and the normal to each interface.
- Called the Benndorf relationship. See derivation in class
- Fowler Figure 8-2 : Refracted and reflected arrivals in a spherical earth when the core has a higher / lower velocity than the mantle.
- Fowler Figure 8-3 : Shows PKP arrivals with velocity gradient (analog to C3.1.3)
- P-wave shadow zone ($\Delta = 103^\circ$ to $\Delta = 143^\circ$). This geometry allows the radius of the core to be computed
- S-wave shadow zone ($\Delta = 103^\circ$ to $\Delta = 180^\circ$). Implies outer part of core is liquid with shear modulus, $\mu = 0$



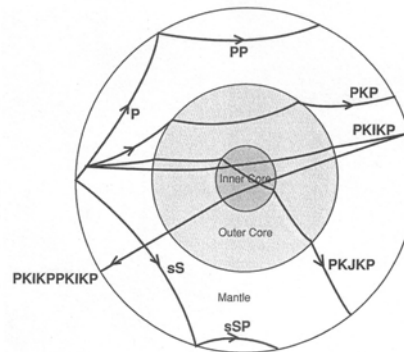
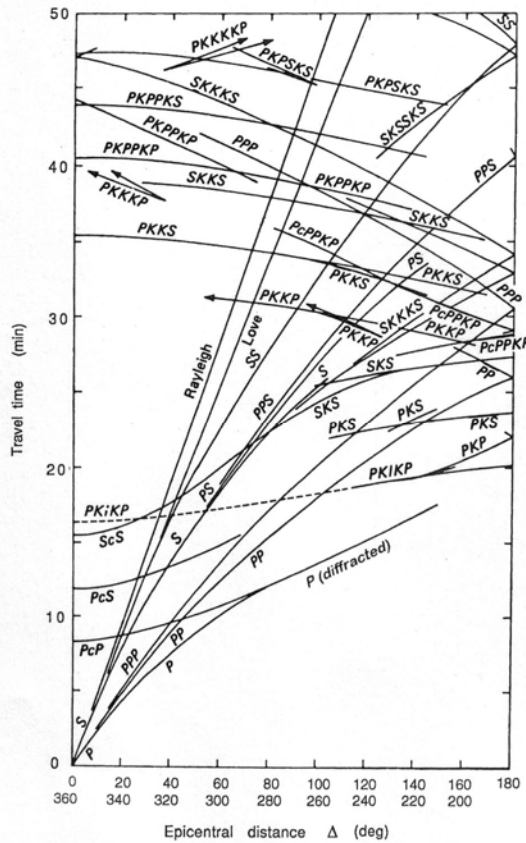
Some P-waves are observed in the shadow zone

- Diffractions (dashed lines in Fowler Figure 8-3). These waves travel along the core-mantle boundary, and arrive in places not predicted by ray theory. However, their location is consistent with Huyghens principle.
- Other P-waves observed in the shadow zone were shown to be due to a **solid inner core** with an increase velocity compared to outer core. In 1936 Inge Lehman suggested that these waves are reflections from the inner core. Called PKiKP in modern notation.



C3.1.5 Actual travel time curves

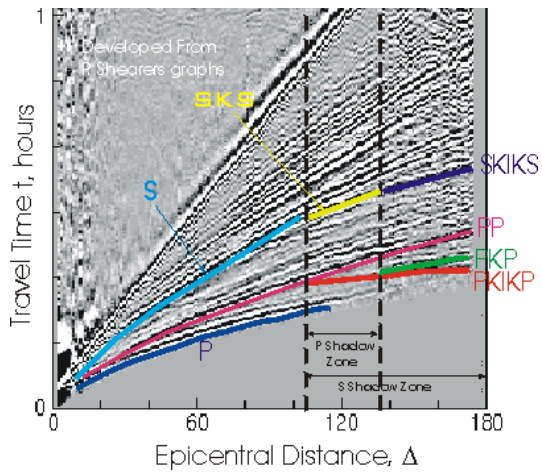
- Observations of many earthquakes led to the compilation of the Jeffreys-Bullen travel time curves.
- These are for an earthquake at the surface of the Earth and assume radial symmetry.
- We now know that both the core and mantle are not exactly symmetric.
- Departure from symmetry contains valuable information about structure (e.g. mantle convection, slab location etc).
- J-B travel times accurate to within a few seconds.
- P, PcP and PKP show a good example of a shadow zone due low velocity layer
- S, Scs, SKS give an example of triplications since the CMB represents an **increase in velocity** with depth for these waves.



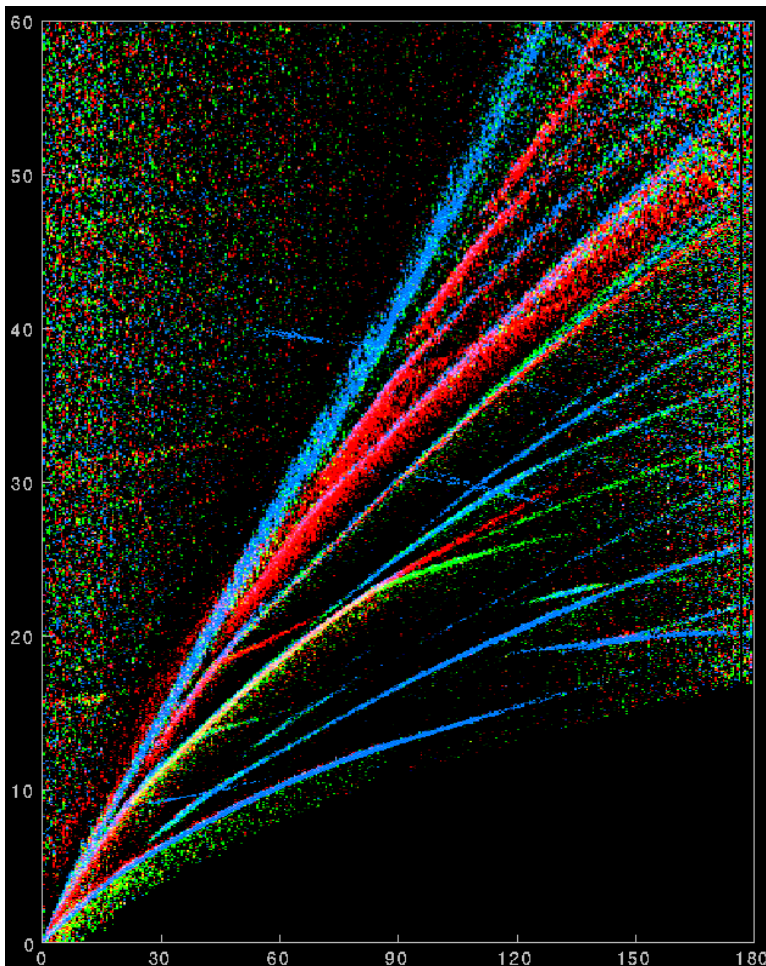
- P P-wave in mantle
- K P-wave in outer core
- c Reflection from outer core
- i Reflection from inner core
- I P-wave in inner core
- J S-wave in inner core

- Set of ray paths for all possible phases:
http://garnero.asu.edu/research_images/index.html#raypaths

- P and PcP Direct wave through the mantle and reflection from CMB
- P and P_{diff} Diffraction means that seismic energy travels to a region that is not predicted by ray theory
- PKP Note multiple paths and the cusp
- PKiKP Reflection of a P-wave from inner core gives P wave arrivals in the shadow zone
- SKS Used to study upper mantle anisotropy. Can only acquire splitting (polarization) on final leg through the mantle.
- SKKKS Can travel both sides of inner core
- PKiIKP Complicated! Note that certain teleseismic phases are only observed in very narrow ranges of Δ. Seismologists who study certain parts of the core and inner core must look for earthquakes and seismic stations with very specific separation (Δ)



- Fowler 4-18 shows a compilation of 60000 seismograms from 2995 earthquakes recorded from 1980 to 1987. From Earle and Shearer (1994)



- More details shown in Fowler 4-16 Fowler 4-15
- Need to account for earthquake depth
- Exhaustive list <http://www.iris.edu/data/vocab.htm>