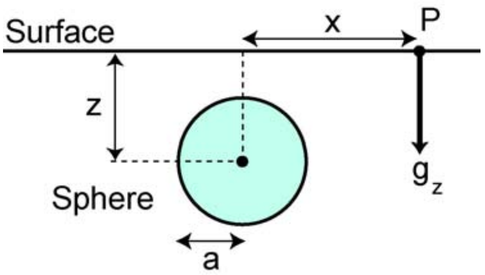
Geophysics 224
B2. Gravity anomalies of some simple shapes

B2.1 Buried sphere

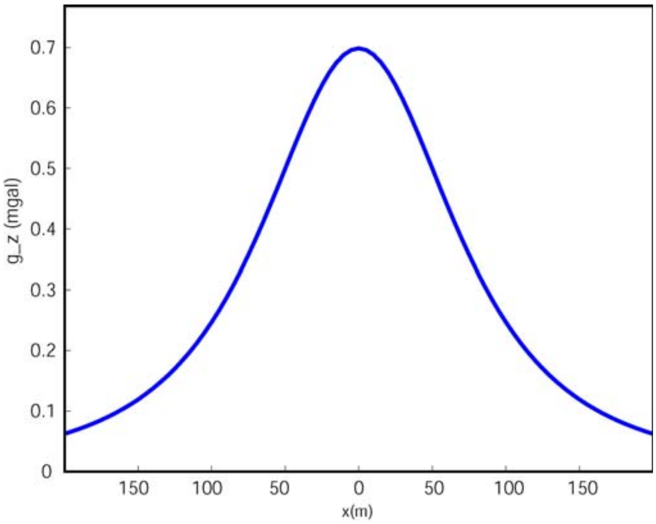


Gravity measurements are made on a surface profile across a buried sphere. The sphere has an excess mass M_S and the centre is at a depth z. At a distance x, the vertical component of **g** is given by

$$g_z = \frac{GM_S z}{(x^2 + z^2)^{\frac{3}{2}}}$$

This curve is drawn below for a sphere with:

Radius, a	= 50 m	Depth, z	= 100 m
Density contrast , $\Delta\rho$	= 2000 kg m^{-3}	Excess mass, M_S	= 10^9 kg



Note that::

- g_z has it's **maximum** value directly above the sphere at $x = 0$ m.
- The maximum value is $g_z^{max} = \frac{GM_S}{z^2}$
- The value of x where $g_z = (g_z^{max})/2$ is called the **half-width** of the curve ($x_{½}$).
  Can show that $x_{½} = 0.766$ z
- Far away from the sphere, g_z becomes very small

Gravity measurements are rarely made on a **single profile**. Usually they are made on a **grid** of points. This allows us to make a **map of g_z**.

Question: What will the map look like for the buried sphere

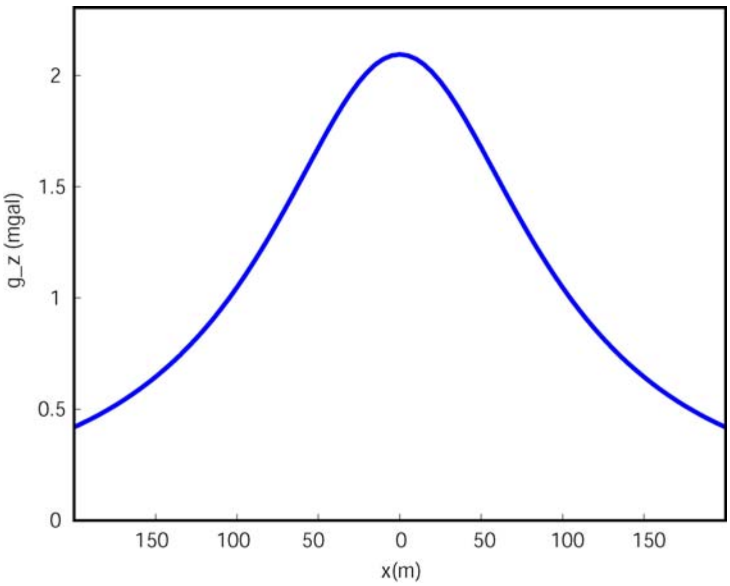
B2.2 Buried horizontal cylinder

When gravity measurements are made across a buried cylinder, it can be shown that the variation in g_z will be.

$$g_z = \frac{2G\pi a^2 z\Delta\rho}{(x^2 + z^2)}$$

This curve is drawn below for a cylinder with

radius, a	= 50 m	depth of axis, z	= 100 m
density contrast , Δρ	= 2000 kg m^{-3}	horizontal location, x	= 0 m



Note that :

- the maximum value of g_z is located directly above the axis of the cylinder.

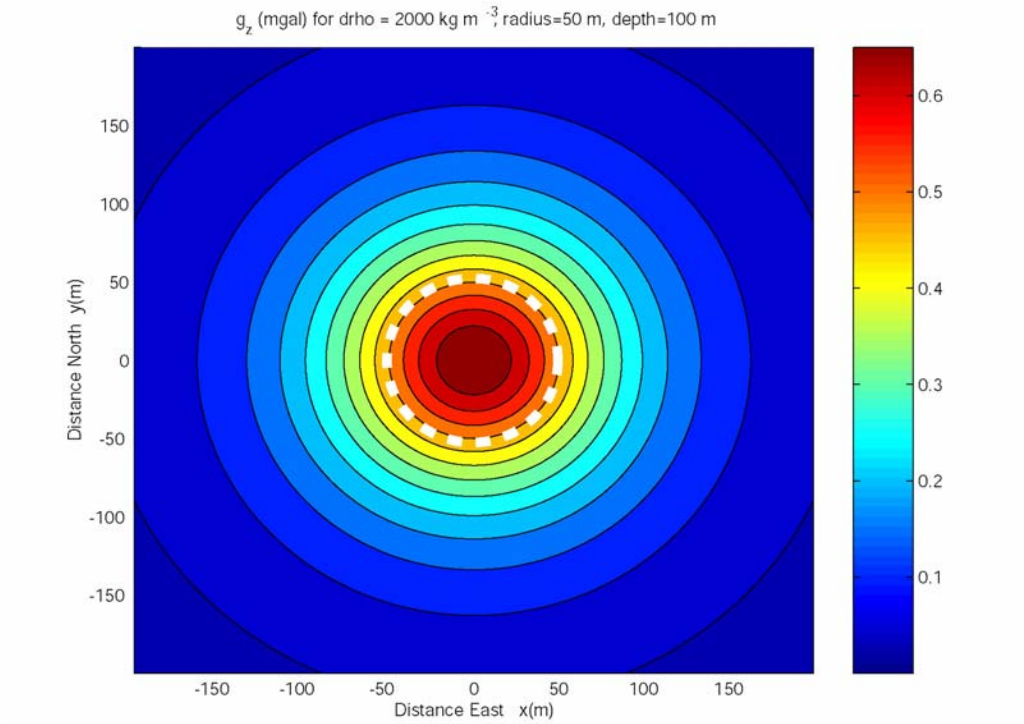
$$g_z^{max} = \frac{2G\pi a^2\Delta\rho}{z}$$

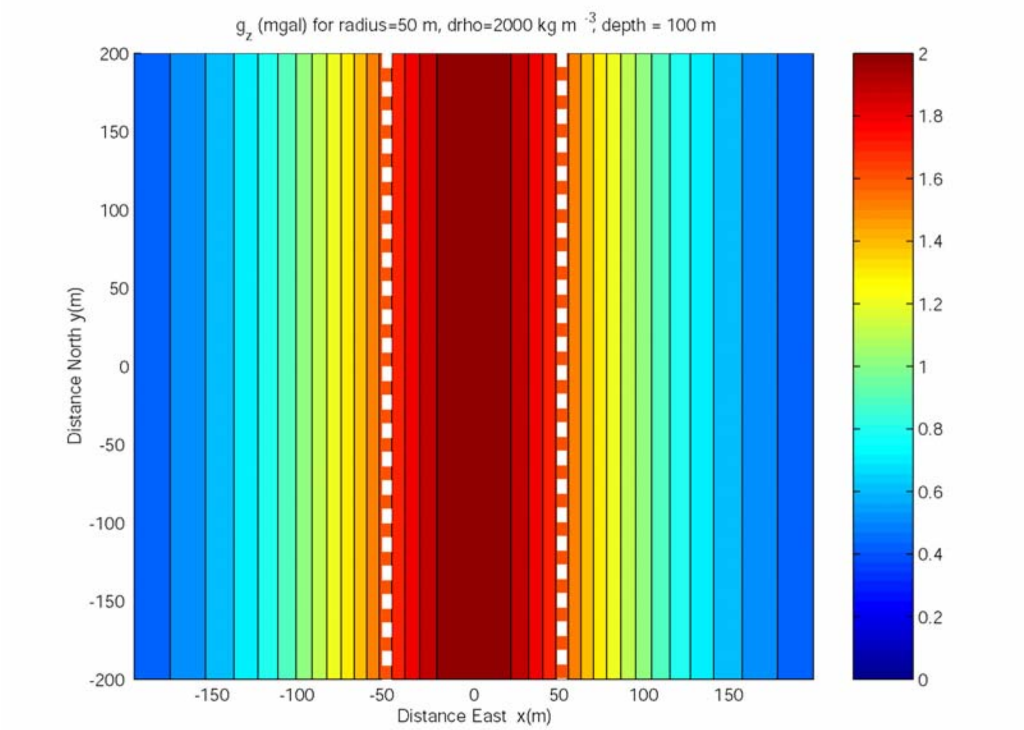
- From the plots, we can see that this value is larger than g_z^{max} for a sphere? Why?

- For a cylinder, we can show that the half-width $x_{½} = z$

Question: Compare the profiles across the sphere and a cylinder. Would this information allow you to decide if the buried object was a sphere or a cylinder?

Question: If g_z is measured on a grid of points, what will the resulting map look like? Would this be a better way to distinguish between a sphere and cylinder?





Forward and inverse problems in geophysics

B2.1 and B2.2 illustrate the gravity anomaly that we would expect to observe above a known geological target. This is called a **forward problem** in geophysics, and is a useful exercise in understanding if measurements would be able to detect a particular structure.

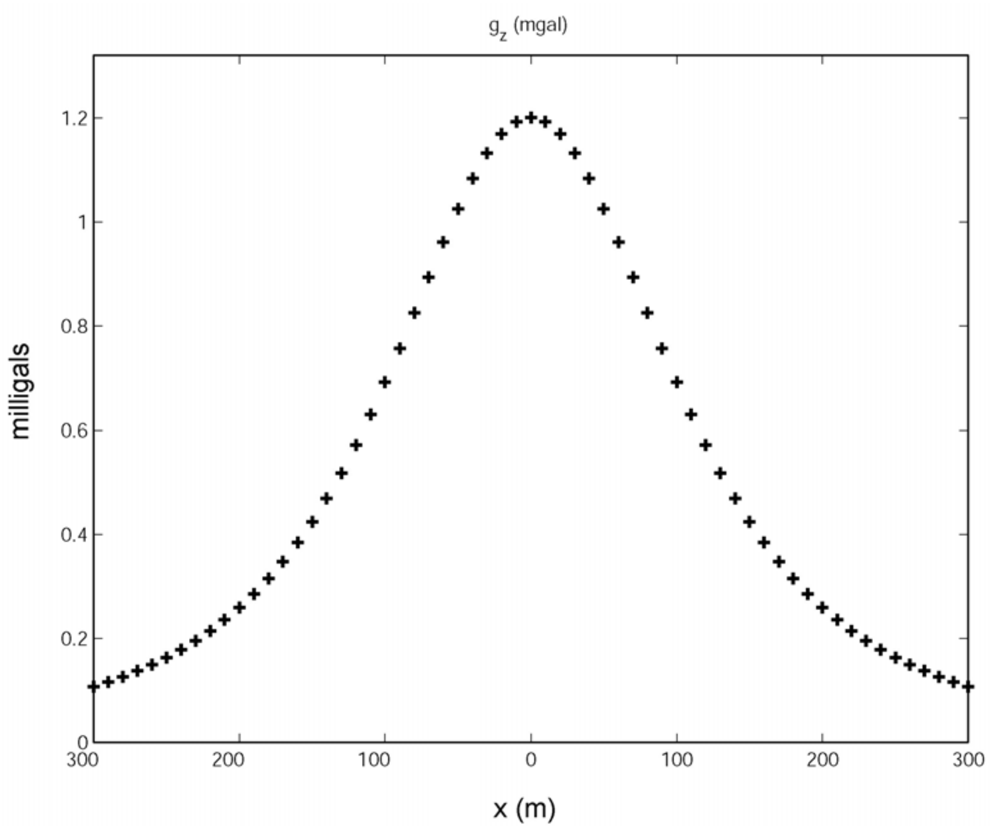
Forward problem: *Density model of Earth > Predicted gravity data(anomaly)*

However, we are usually more interested in solving the opposite problem. When gravity data has been collected in a field survey, we want to find out the depth and size of the target. This is called an **inverse problem** in geophysics.

Inverse problem: *Measured gravity data > Density model of Earth*

Example : Gravity data interpretation example

Consider some gravity data collected on a profile crossing a spherical iron ore body.



● Where is the centre of the ore body? x = _________metres

● What is the maximum value of g_z? g_z^{max} = _________mgal

● At what distance ($x_{½}$) has g_z fallen to half this value? $x_{½}$ = _________metres

● The **depth** of the sphere can be derived using the equation $x_{½} = 0.766\, z$

Rearranging this gives $z = 1.306\, x_{½}$

z = ______metres

● To determine the **excess mass**, we use the equation $g_z^{max} = \dfrac{GM_S}{z^2}$

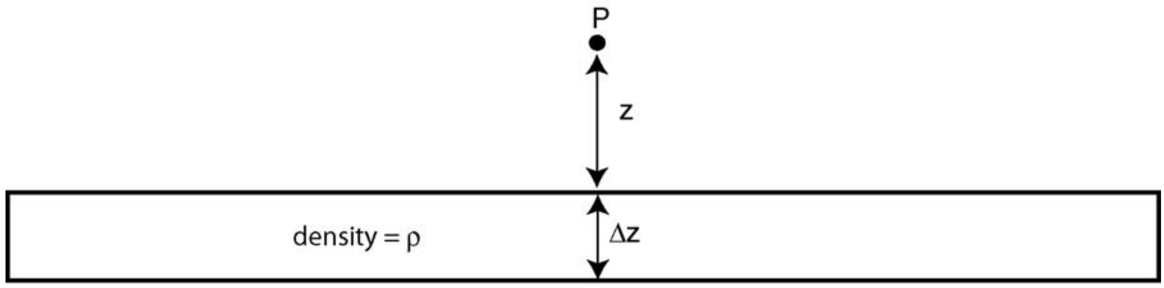
We know z and have measured g_z^{max} so we need to rearrange this equation to find M_s

This gives $$M_s = \frac{g_z^{max} z^2}{G}$$

Remember to convert g_z^{max} from milligals to ms^{-2}! M_s = _________kg

● Can the radius (a) and density contrast ($\Delta\rho$) of the sphere be determined?

B2.3 Uniform layer

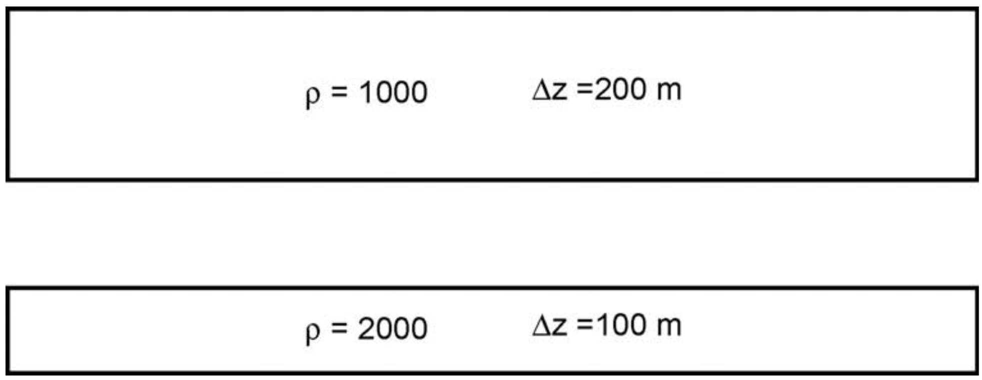


A layer has a infinite extent, thickness Δz and a density ρ. The gravitational attraction of this slab at P is:

$$g_z = 2\pi G\rho\Delta z$$

Note that g_z does **not depend** on the distance from the layer to the point P. Why?

Consider the two density models shown below.



What can we say about the gravitational acceleration (g_z) of the two models?

This is an example of **non-uniqueness** in geophysics, and occurs when more than one Earth model can explain the same set of geophysical data.

B2.4 General polygon – simple computer modelling

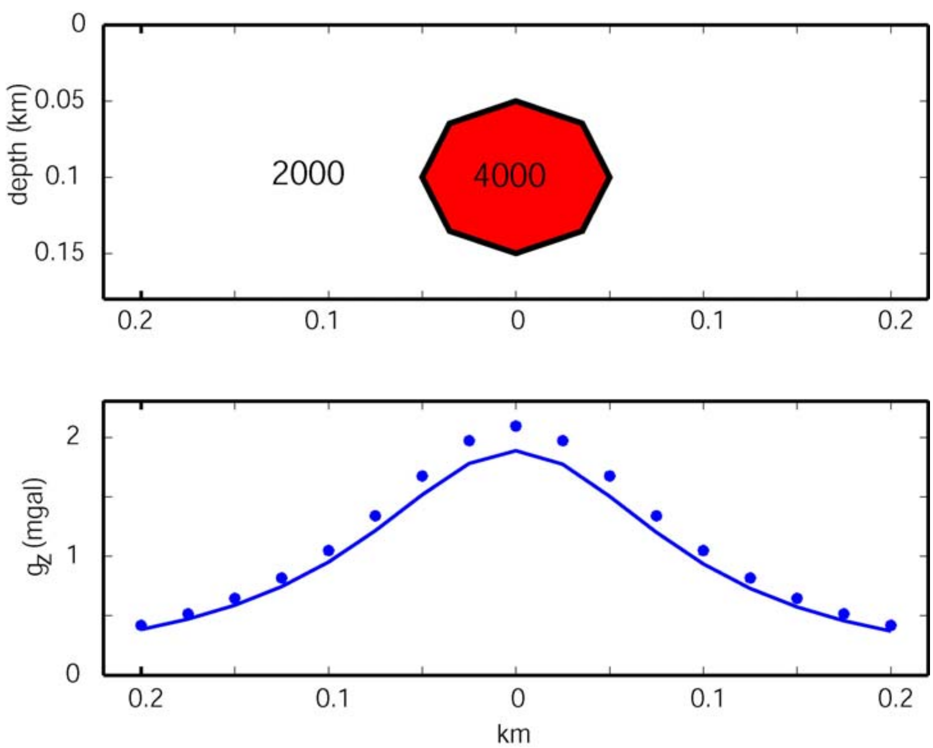
Simple density models such as a sphere and a cylinder can help us get an idea of what gravity anomaly (g_z) is produced by simple density models. However, to represent the real Earth, more complicated model geometries are clearly needed. The development of powerful, portable computers has made this possible.

In the lectures and labs we will use a very simple MATLAB program that computes g_z for a N-sided prism. This is called 2-D modelling, since the polygon is assumed to be infinite in length in the strike direction (out of the plane of the page). Fully 3-D gravity modelling is also widely used, but will not be discussed in detail in Geophysics 224.

B2.4 Example 1 – verification

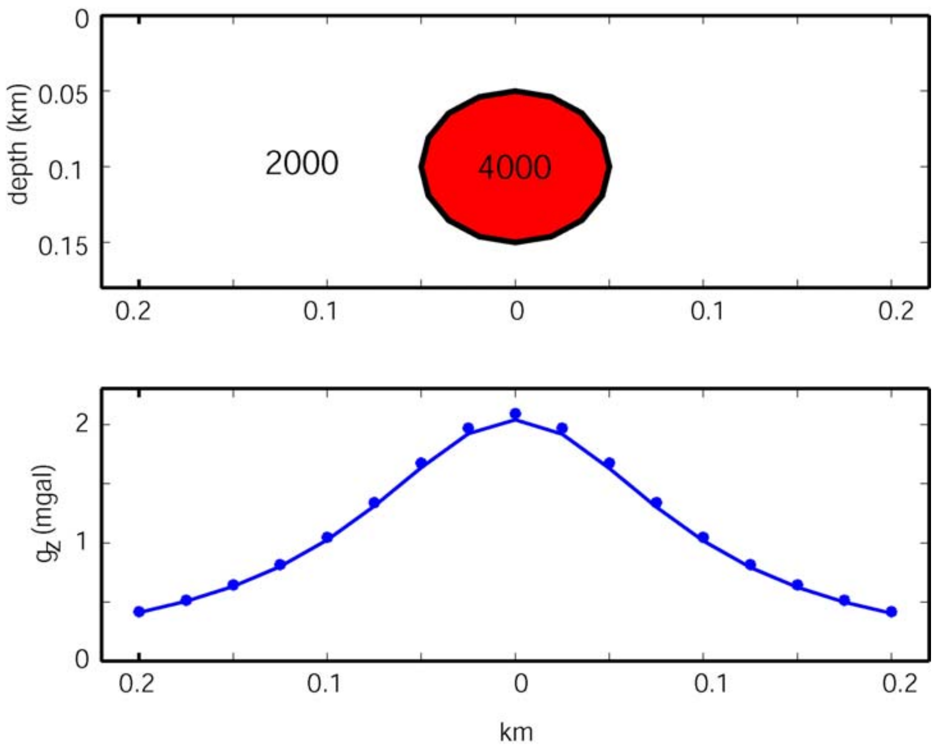
When using a computer modelling program, it is important to be **suspicious** about the answer. The first time the program is used, you should try and use a simple model for which the answer is already known. If you can get the same answer by two methods, then **maybe** you can trust the program.

Consider using the program to compute the gravity anomaly of a cylinder. The cylinder can be approximated by an 8-sided polygon, as shown below. The dots show the exact (known) answer for a cylinder.



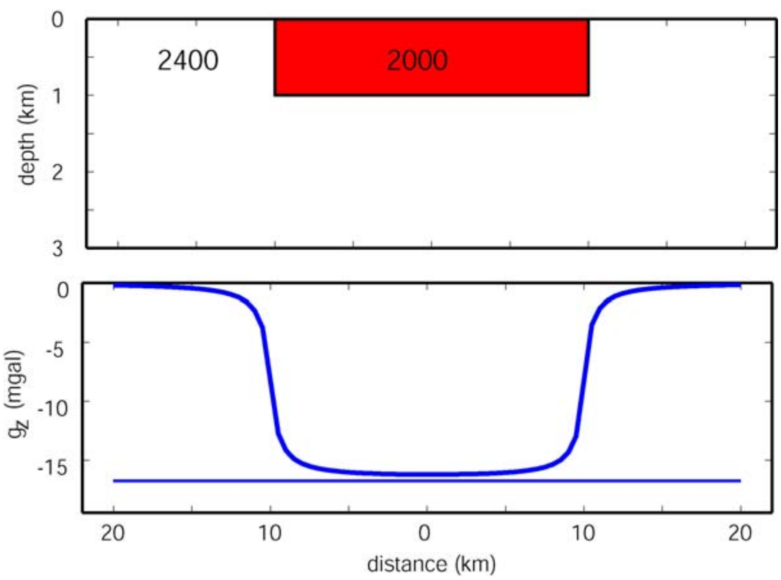
Question : Why is g_z for the polygon less than for a cylinder?

With a 16-sided polygon, the agreement is significantly better.

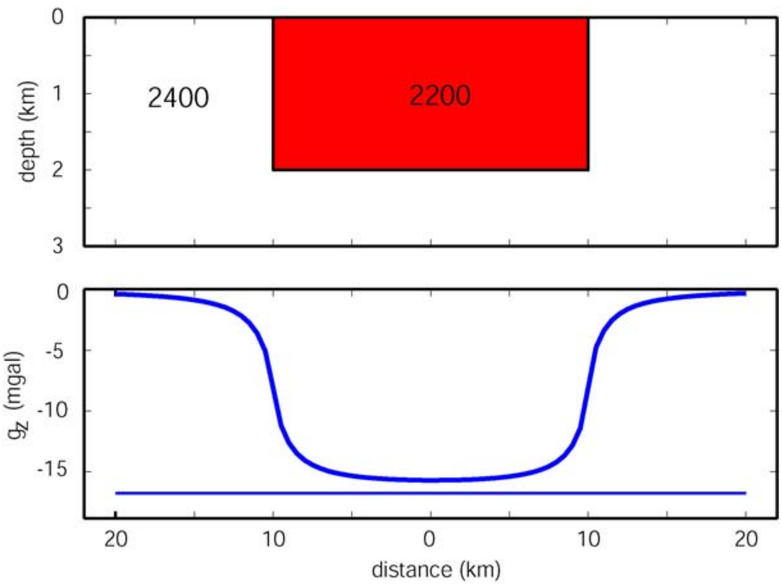


B2.4 Example 2 – sedimentary basin

An over simplified sedimentary "basin" is shown below sheet. Note that the "basin" is less dense than the basement rocks, so the pull of gravity is slightly weaker over the basin. The horizontal line shows the expected value of g_z for an infinite layer with the same thickness and density as the basin.



Now consider a similar basin, that is twice as deep and with a lower density contrast.



Note the following two features:

● The minimum value of g_z is the same as for the first basin. Why?

● The edge effects extend further into the basin than in the first case.

In the lab we will use this program to analyse some real gravity data collected across a sedimentary basin in Washington and an impact crater in Mexico.

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