224B3 Other factors that cause changes in g and need to be corrected

Note that gravity exploration is different to seismic exploration in the following way:

- In a seismic survey, the travel time depends on just the velocity of the material on a path between source and receiver.

- A gravity measurement is influenced by the gravitational attraction of local geological structure **plus** the mass distribution of the whole Earth, the moon, Sun, planets ….

B3.1 The Effect of latitude

- The acceleration of gravity at the Equator, g_E = 978,033 mgal and at the poles g_P = 983,219 mgal (Hammer, *Geophysics*, **8**, 57, 1943). This difference is 5186 mgal, which is much bigger than the anomalies we have considered, and needs to be accounted for in field measurements.

Three factors cause g to vary with latitude

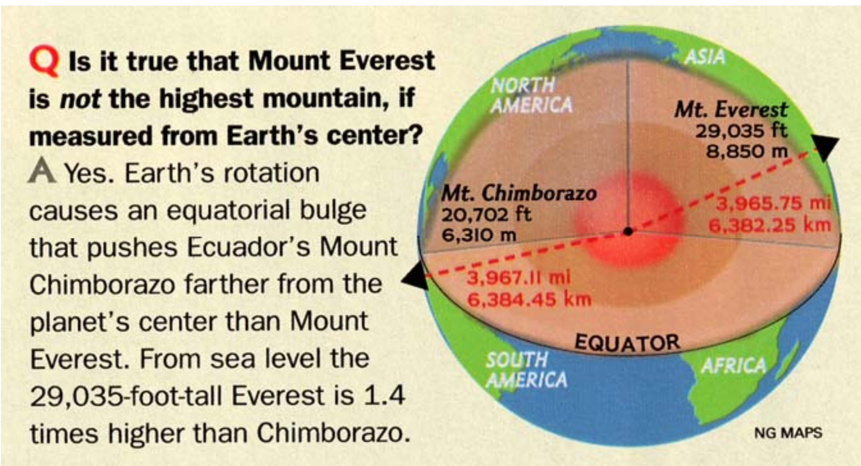
(A)The Earth is distorted by rotation

R_E = 6378 km and R_P =6357 km. The ratio of flattening is approximately 1/298. Since a point on the Equator is **further** from the centre of the Earth than the poles, gravity will be slightly weaker at the Equator.

We previously showed that for a sphere $g(r) = \dfrac{GM_E}{r^2}$

where the mass of the Earth, $M_E = 5.957\ 10^{24}$ kg.
At the North Pole, r = 6357 km and g_P = 983,219 mgal.
If we move **up** 21 km to the radius of the equator, the **decrease** in gravity will be 6467 mgal

Thus $g_E = g_P$ - 6467 mgal, which is too much to explain the observed difference between the Equator and the Poles.



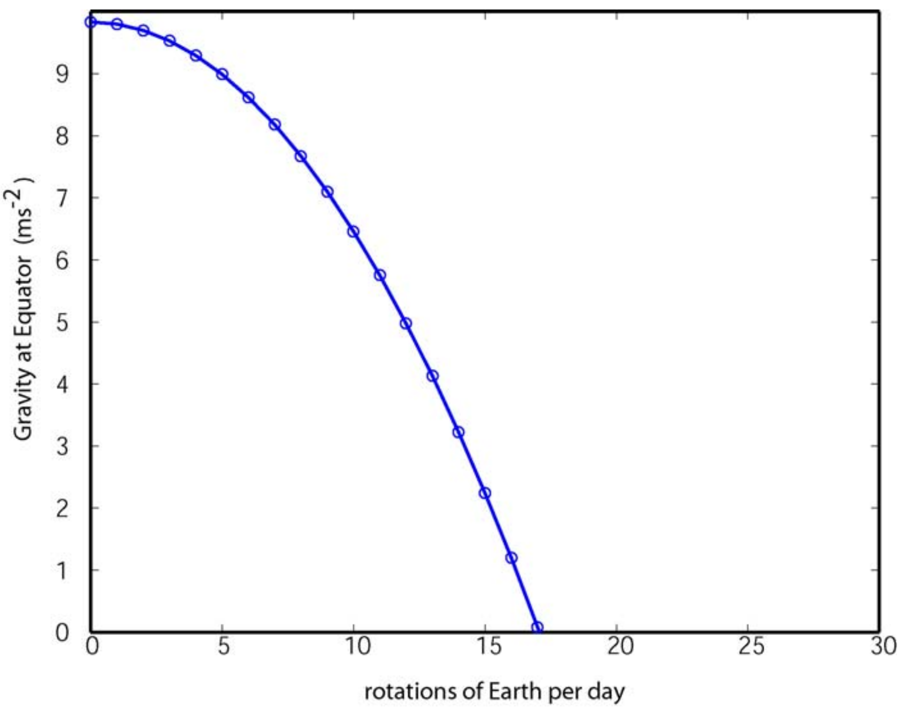
(B) Centrifugal forces vary with latitude

The rotation of the Earth also causes gravity to vary. Imagine you are standing at the North Pole. The rotation of the Earth will not change **g**, all that will happen is that you rotate once a day.

Now imagine you are at the Equator. If we could increase the rotation rate of the Earth enough, you would be ultimately be thrown into space (i.e. become weightless). Thus rotation makes **gravity weaker** at the equator.

With the 1 rotation per day, and $R_E = 6378$ km, can show that $g_P = g_E$ +3370 mgal

Question : How fast would the Earth need to rotate to throw you into space?



(C)Mass distribution of the Earth

The change in shape from a sphere to an ellipsoid redistributes the Earth's mass. Thus results in more mass between points on the Equator and the centre of the Earth, than between the poles and the centre of the Earth. This effect will make $g_E > g_P$ and is analogous the Bouguer correction we will discuss in section B3.2. Calculations show that $g_E \sim g_P + 4800$ mgal

Combining these three effects (A,B and C) gives

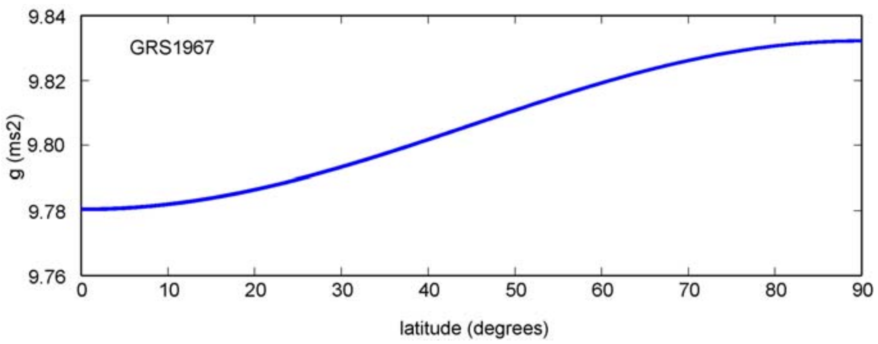
$g_P = g_E + 6467 + 3370 - 4800$ mgal $= g_E + 5037$ mgal (approximately as observed)

Equation for variation of g with latitude

The variation of g from the Equator to the pole can be written as

$$g\,(\theta) = 9.78031846\,(1 + 0.0053024 \sin^2\theta - 0.0000058 \sin^2 2\theta)$$

where θ is the latitude in degrees. This equation is called the **Geodetic Reference System** for 1967. More recent revisions are essentially the same, but with more decimal places.



Example : In Edmonton $\theta = 53^\circ\ 30'\ 25''$ N and the GRS67 equations gives

$$\begin{aligned} g &= 9.78031846\,(1 + 0.003417902 - 0.000005395)\ \text{m s}^{-2} \\ &= 9.81369388\ \text{m s}^{-2} \end{aligned}$$

The variation of g with latitude is important when a survey extends over a significant north-south distance. Differentiating the GRS67 equation with respect to θ yields

$$\frac{dg}{d\theta} = 9.78031846\,(0.0053024\ x\ 2 \sin\theta \cos\theta - 0.0000058\ x\ 4 \sin 2\theta \cos 2\theta)$$

$$= 0.049526 \quad \text{m s}^{-2} \text{ per radian}$$

$$= 0.0008655 \quad \text{m s}^{-2} \text{ per degree}$$

$$= 86.550 \quad \text{mgal per degree}$$

$$= 0.7868 \quad \text{mgal km}^{-1} \quad (\text{1 degree latitude} = 111 \text{ km})$$

All the these equations define the expected value of **theoretical gravity (or normal gravity)** at latitude θ. Differences between this value and what is actually measured are **anomalies** that we will analyse for information about subsurface density structure.

224 B3.2 How does elevation effect gravity measurements?

(i) The Free air correction

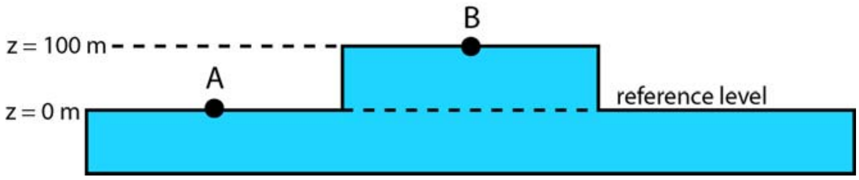
Newton's Theory of Gravitation states that at a distance, r, from the centre of the Earth

$$g(r) = \frac{GM}{r^2}$$

This means that as you move away from the centre of the Earth, the acceleration of gravity (g) decreases. In Edmonton, g = 9.81 ms^{-2} and if you move **up** a distance, Δh, the acceleration of gravity will **decrease** by

$$\Delta g = 3.086 \ \Delta h \times 10^{-6} \quad m\,s^{-2}$$
$$= 0.3086 \ \Delta h \quad \text{mgal}$$

Consider the exciting topography of a flat topped mountain:



Gravity measurements are made at points A and B. The difference in elevation means that g_B will be less than g_B by an amount

$$\Delta g = 0.3086 \times 100 = 30.86 \text{ mgal}$$

When collecting gravity data, our real interest is to determine the density of the rocks below ground. The change in elevation from 'A' to 'B' will thus contaminate the data. The **Free Air correction** is a mathematical way of undoing the effect of elevation. It allows us to correct the data collected at 'B' in order to make it equivalent to data collected at the same elevation as 'A'.

In gravity surveys, we always define a **reference level** for the survey. Free Air corrections are made relative to this level. In general, any reference level could be chosen, but sea level is commonly chosen in coastal areas. In Alberta, the average level of the prairies would be a good choice.

If a gravity measurement was made Δh **above** the reference level, we must **add**

$$C_{FA} = 0.3086 \ \Delta h \quad mgal$$

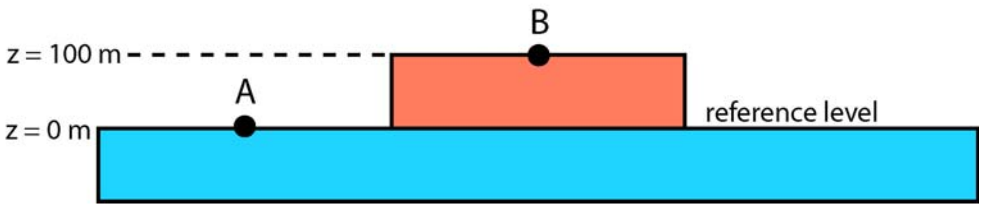
C_{FA} is called the Free Air correction for a given gravity measurement.

Similarly, if a gravity measurement was made Δh **below** the reference level, we must **subtract**

$$C_{FA} = 0.3086 \ \Delta h \quad mgal$$

Question : to keep data accurate to 0.1 mgal, how accurately must we know the elevation?

(ii) The Bouguer correction



Unfortunately, this is not the end of story! Compare the gravity measurements at 'A' and 'B'. At point A, the gravity measurement is solely due to structure below the reference level (blue). At 'B' the gravity measurement is due to structure below the reference level, **plus** the gravitational pull of the 100 metres of mountain (red). The net result is that $g_B > g_A$

From section B2.3 the magnitude of this extra gravitational attraction is approximately

$$g_B - g_A = 2\pi G \rho \Delta h$$

where ρ is the density of the mountain.

Thus to remove this effect we need to **subtract** $C_B = 2\pi G$ from the observed gravity measurement at 'B'. This is called the **Bouguer correction** and

$$C_B = 0.00004193 \ \rho \Delta h \quad \text{mgal}$$

Note that to apply the Bouguer correction we need to estimate ρ, the density that lies between 'B' and the reference level. Using the value $\rho = 2670$ kg m $^{-3}$ this gives

$$C_B = -0.1119 \ \Delta h \quad \text{mgal}$$

This value represents an average density for crustal rocks. Other information (*e.g* borehole gravity data or Nettleton's method) may be used to give a better estimate of the density.


Pierre Bouguer

Summary

Measurement **above** reference level	**Add** Free Air correction	**Subtract** Bouguer correction
Measurement **below** reference level	**Subtract** Free Air correction	**Add** Bouguer correction

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Free Air and Bouguer corrections