B.4 Other factors that cause changes in g and need to be corrected

Note that gravity exploration is different to seismic surveys in the following way:

- In a simple seismic survey, the travel time depends on just the velocity of the material on a path between source and receiver.

- A gravity measurement is influenced by local density changes **plus** the mass distribution of the whole Earth, the moon, Sun, planets ….

4.1 Effect of latitude

- acceleration of gravity at the Equator, g_E = 978,033 mgal and at the poles g_P = 983,219 mgal (Hammer, *Geophysics*, **8**, 57, 1943). This difference is 5186 mgal, which is much bigger than the anomalies we have considered, and needs to be accounted for in field measurements.

Three factors cause g to vary with latitude

(i) The Earth is distorted by rotation

R_E = 6378 km and R_P =6357 km. The ratio of flattening is approximately 1/298. The surface of the distorted sphere is called the **reference ellipsoid** and would be the equipotential if the Earth had a uniform structure. Density variations at depth in the mantle cause +/- 100 m differences between the reference ellipsoid and the **geoid** (the observed equipotential surface).

Since a point on the Equator is further from the centre of the Earth than the poles, gravity will be slightly weaker at the Equator.

We previously showed that for a sphere $g(r) = \frac{GM}{r^2}$ and

Now consider small changes in r, such that $r = r_0 + h$

$$g(h) = \frac{GM}{(r_o + h)^2} = \frac{GM}{r_o^2 (1 + \frac{h}{r_o})^2}$$

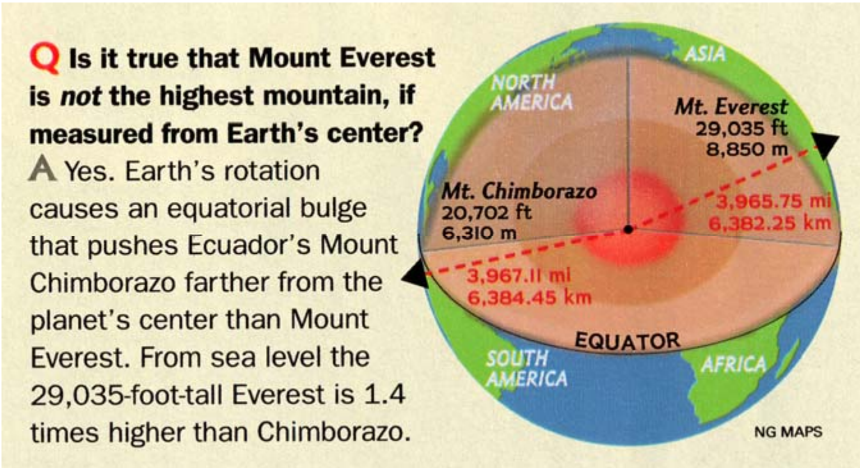
If $\frac{h}{r_o}$ is small, then this be expressed with Taylor's theorem as

$$g(h) = \frac{GM}{r_o^2}(1 - 2\frac{h}{r_o} + \ldots)$$

$$g(h) = g_o (1 - 2\frac{h}{r_o} + \ldots) \qquad \text{where } g_o = \frac{GM}{r_o^2}$$

Thus at the North Pole we have r_o = 6357 km and g_P = 983,219 mgal. When we move **up** 21 km to the radius of the equator, the **decrease** in gravity will be 6496 mgal

Thus g_E = g_P - 6496 mgal, which is too much to explain the observed difference between the Equator and the Poles.



(ii) Centrifugal forces vary with latitude

$r = \text{R} \cos\theta$

Centripetal acceleration = $-\text{r}\,\omega^2$

Component towards the centre of the earth $= -\text{r}\,\omega^2 \cos\theta$

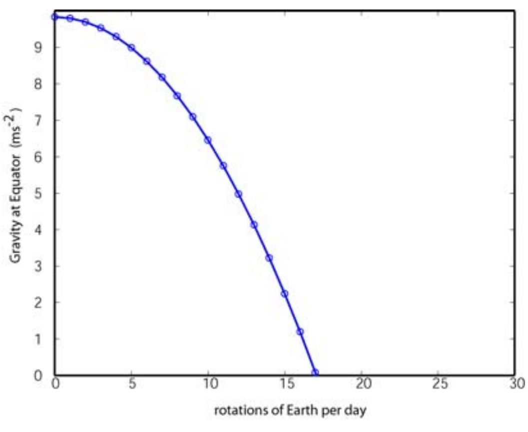
$= -\text{R}\,\omega^2 \cos^2\theta$

Using $\text{R} = \text{R}_P$ = 6378 km and $\omega = 2\pi$ per day = $7.27\ 10^{-5}$ rad s^{-1} gives

$\text{R}\omega^2$ = 3370 mgal

Thus $g_P = g_E$ +3370 mgal

How fast would the Earth need to rotate to throw objects at the Equator into space?



(iii) Mass distribution of the Earth

The change in shape from a sphere to an ellipsoid redistributes the Earth's mass. Thus results in more mass between points on the Equator and the centre of the Earth, than between the poles and the centre of the Earth. This effect will make $g_E > g_P$ and is analogous the Bouguer correction we will discuss in section B4.2. Calculations show that $g_E \sim g_P + 4800$ mgal

Combining these three effects gives

$g_P = g_E + 6496 + 3370 - 4800$ mgal $= g_E + 5056$ mgal (approximately as observed)

These effects can be quantified analytically and it can be shown that $g(\theta)$ can be written either as a truncated power series

$$g(\theta) = g_E\,(1 + \alpha \sin^2\theta + \beta \sin^2 2\theta + \ldots)$$

or exactly as Somigliana equation $$g(\theta) = g_E \frac{1 + k\sin^2\theta}{\sqrt{1 - e^2\sin^2\theta}}$$

where α, β, k and e are all constants (Blakely, pages 129-136)

Over the past century, these constants have been used by the International Association of Geodesy to define standard gravity in a series of formula with ever increasing accuracy.

1930 International Gravity Formula

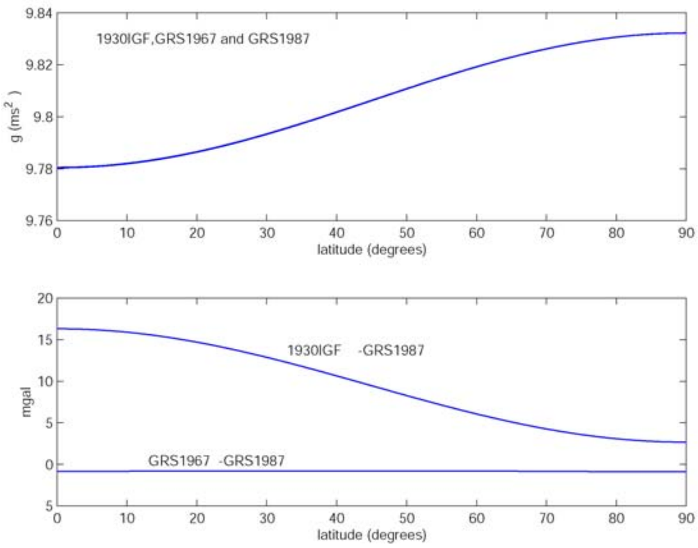
$$g(\theta) = 9.78049 \quad (1 + 0.0052884\sin^2\theta - 0.0000059\sin^2 2\theta)$$

Geodetic reference system 1967

$$g(\theta) = 9.78031846\,(1 + 0.0053024\sin^2\theta - 0.0000058\sin^2 2\theta)$$

Geodetic reference system 1987

$$g(\theta) = 9.7803267714\,\frac{1 + 0.00193185138639\sin^2\theta}{\sqrt{1 - 0.0066943799013\sin^2\theta}}$$



Example : In Edmonton $\theta = 53^\circ\ 30'\ 25''$ N, the GRS67 equations gives

$$g = 9.78031846\,(1+0.003417902-0.000005395) \text{ m s}^{-2}$$

$$= 9.81369388 \text{ m s}^{-2}$$

The variation of g with latitude is also important when a survey extends over a significant north-south extent. Differentiating the GRS67 equation with respect to θ yields

$$\frac{dg}{d\theta} = 9.78031846\ (0.0053024\ x\ 2 \sin\theta \cos\theta - 0.0000058\ x\ 4\sin 2\theta \cos 2\theta)$$

$$= 0.049526 \quad \text{m s}^{-2} \text{ per radian}$$

$$= 0.0008655 \quad \text{m s}^{-2} \text{ per degree}$$

$$= 86.550 \quad \text{mgal per degree}$$

$$= 0.7868 \quad \text{mgal km}^{-1} \quad (\text{1 degree latitude} = 111 \text{ km})$$

All the these equations define the expected value of **theoretical gravity (or normal gravity)** on the reference ellipsoid at latitude θ. Differences between this value and what is actually measured are **anomalies** that we will analyse for information about density contrast below the survey area.

325 B.4.2 Elevation of the measurement location

(i) The Free air correction

We previously showed that as we move to higher elevations, the acceleration of gravity decreases.

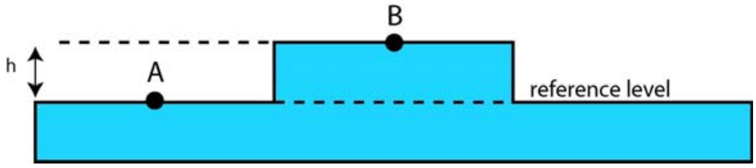
$$g(h) = \frac{GM}{r_o^2}\left(1 - 2\frac{h}{r_o} + \ldots\right)$$

Thus $\dfrac{dg}{dh} = -\dfrac{GM}{r_o^2} \text{ x } \dfrac{2}{r_o} = -g_o \text{ x } \dfrac{2}{r_o}$

With $g_o = 9.81 \text{ ms}^{-2}$ at a radius $r_o = 6357$ km, this gives

$$\frac{dg}{dh} = -3.077 \times 10^{-6} \ \text{m s}^{-2} \text{ per m of elevation gain}$$

$$= -0.3077 \text{ mgal m}^{-1}$$



Gravity measurements are made at points ‘A’ and ‘B’. Before we can consider if changes in **g** are due to subsurface density variations, we must consider the effect of elevation on these measurements.

We will “*correct*” the data at ‘B’ for the effect of elevation. This will give the value that would be observed at ‘B’, if it were at the **reference level**.

Thus to remove the effect of elevation on gravity measurements at a ‘B’, we must **add**

$$\Delta g_{FA} = 0.3077\, h \text{ mgal} = \textbf{free air correction}.$$

to the observed gravity measurement.

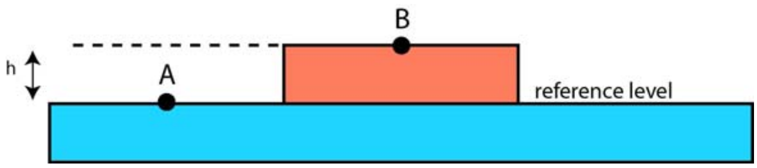
In general, any reference level could be chosen, but sea level is commonly chosen.

Point ‘A’ is already at the reference level, so no correction is needed.

Question

To keep gravity data accurate to within 0.1 mgal, how accurately must we know the elevation?

(ii) The Bouguer correction



The previous discussion ignores the fact that between the observation point ‘B’ and the reference level, there is (**red**) material that will increase **g** at the observation point ‘B’ compared to ‘A’.

Using the results from section B3.2 we can calculate that the magnitude of this attraction is approximately:

$g_z = 2\pi G \Delta\rho h$

Thus, the slab **raises** the value of **g** observed at 'B' is by an amount $2\pi G \Delta\rho$ h, compared to measurements at 'A'

Thus to remove this effect we need to **subtract** $\Delta g_B = 2\pi G \Delta\rho$ h from the gravity measurement.

This is called the **Bouguer correction** it is simple to show that

$$\Delta g_B = -0.00004193 \Delta\rho \text{ h} \quad \text{mgal}$$

To apply the correction we need to estimate $\Delta\rho$, the density that lies between 'B' and the reference plane. Using the value $\Delta\rho = 2670$ g m^{-3} this gives

$$\Delta g_B = -0.1119\text{h} \quad \text{mgal}$$

This value of $\Delta\rho$ represents an **average density** for crustal rocks. Other information can be used to give a better estimate of the density ($\Delta\rho$). These include:

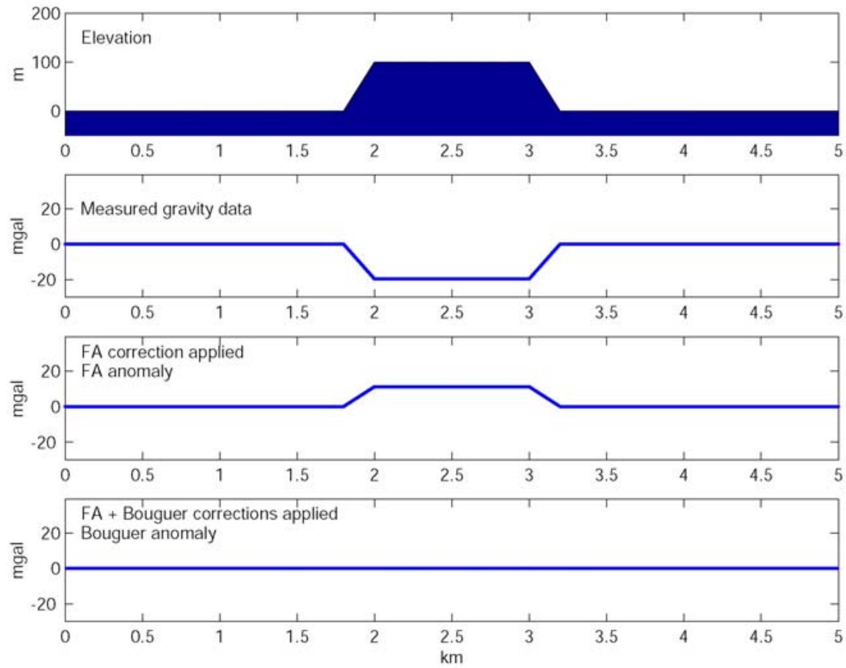
- borehole gravity data
- direct density measurements of rock samples from the field area
- Nettleton's method (see Geophysics 437 when field school data are analysed)


Pierre Bouguer

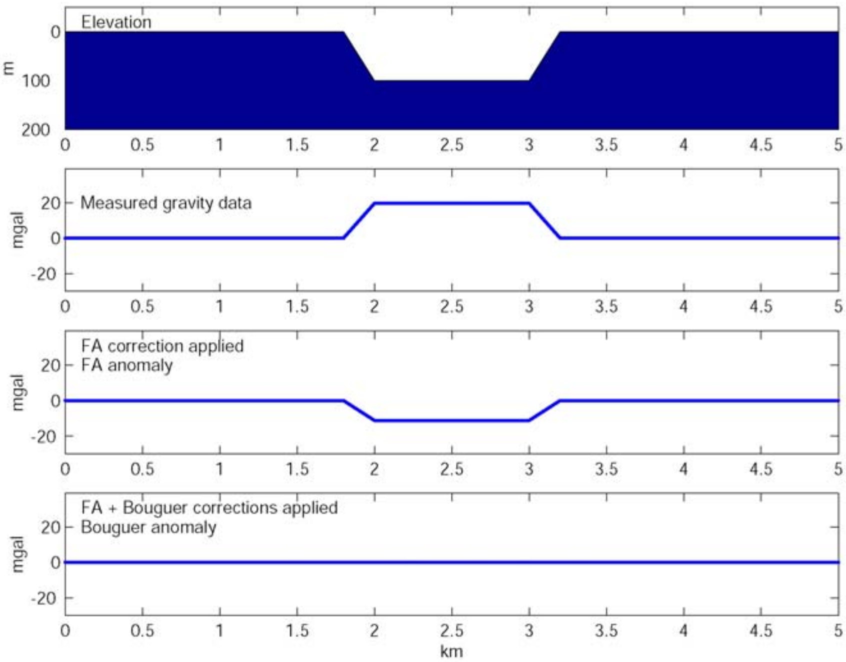
Note that the Bouguer correction is approximate in two ways:

- The density is not known exactly
- In general, a mountain is not an infinite slab.

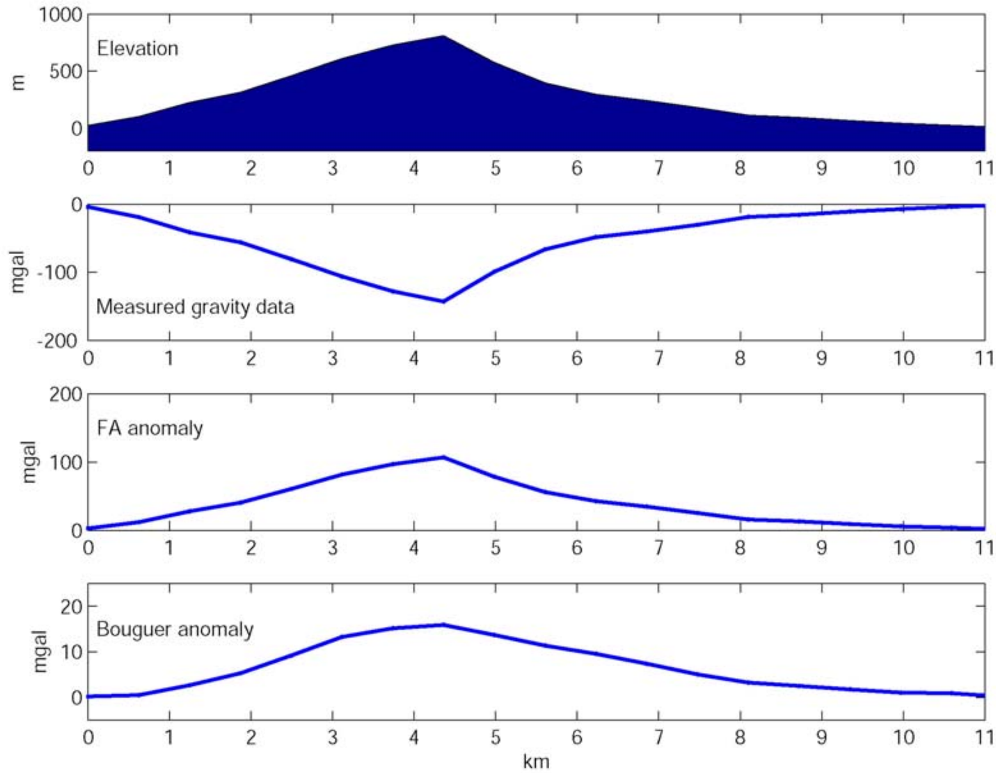
Example 1 : Gravity survey across Table Mountain (synthetic data)



Example 2 : Gravity survey across a River Valley (synthetic data)



Example 3 : Gravity survey for geothermal exploration across Mt. Cabalian, Southern Leyte in the Philippines. Gravity data was provided by Philippine National Oil Company.



What does the positive Bouguer anomaly tell about sub-surface density?

What could explain this density feature beneath a recently active volcano?

How could you estimate the depth of the body that causes the gravity anomaly