325 C3 Electric current flow in a two-layer Earth

C3.1 The effect of an interface

To compute the apparent resistivity of a multi-layer Earth, we must consider what will happen when electric current crosses the interface between layers with differing resistivity values. Consider two layers with resistivity values ρ_0 and ρ_1 above and below the interface. It can shown that the following **boundary conditions** will relate the electric field above and below the interface.

(1) **Electric field parallel** to the interface (E_{par}) is **continuous**

$$E^1_{par} = E^2_{par}$$

From the definition of electric current density ($J = \sigma E$) this requires that

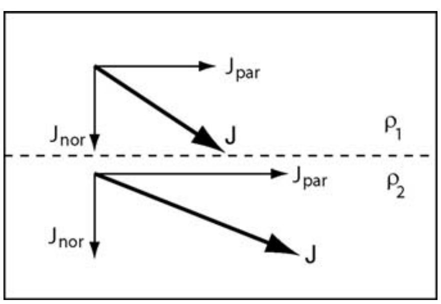
$$J^1_{par}\rho_1 = J^2_{par}\rho_2$$

(2) **Electric current normal** to the interface (J_{nor}) is **continuous**

$$J^1_{nor} = J^2_{nor}$$

This is a consequence of the conservation of electric charge.

Example 1: Resistivity decreases with depth and ρ_1 = 100 Ωm and ρ_2 = 10 Ωm

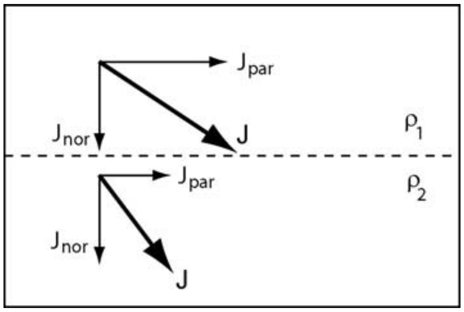


Electric current (J) approaches the interface at an oblique angle. The current, J, can be resolved into two components, normal and parallel to the interface. Using the results from above, we can show that:

$$J^1_{nor} = J^2_{nor} \quad \text{and} \quad J^1_{par} < J^2_{par}$$

Overall result: Electric current is deflected **towards** the horizontal

Example 2: Resistivity increases with depth ρ_1 = 100 Ωm and ρ_2 = 1000 Ωm



Using the same arguments as above, we can show that in this case:

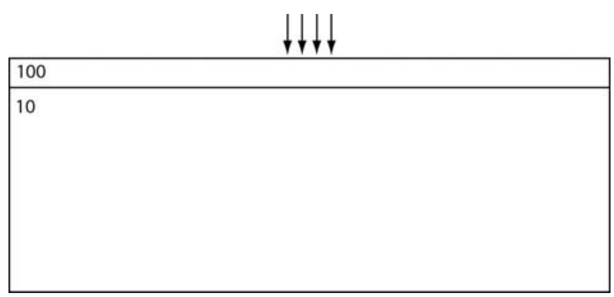
$$J^1_{nor} = J^2_{nor} \quad \text{and} \qquad J^1_{par} > J^2_{par}$$

Overall result: Electric current is deflected **away from** the horizontal

C3.2 Qualitative solution

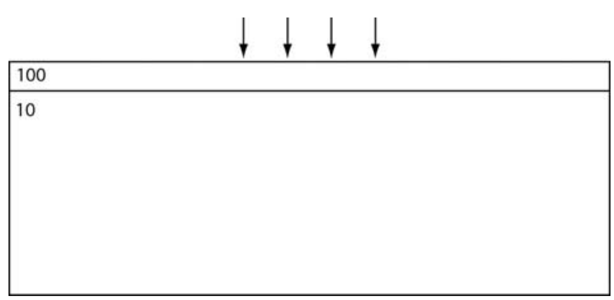
Consider the expanding Wenner array shown in the figures below. Electric current flows between the outer current electrodes, and the voltage is measured between the inner (potential) electrodes. The surface layer is 10 m thick. In each case, sketch the current flow lines.

a=1 m: Electric current is confined to the upper layer. The apparent resistivity reflects this fact and $\rho_a = \rho_1 = 100\Omega m$



a=10 m: Electric current is now flowing through both layers. The current changes direction at the interface, as described in C3.1.

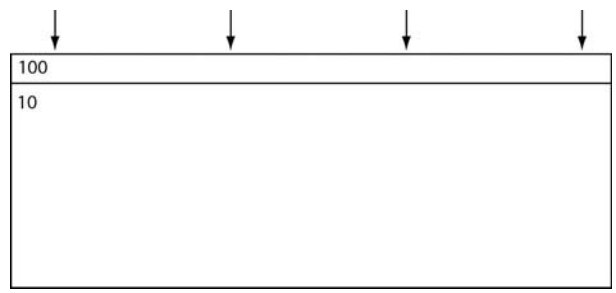
$$\rho_2 < \rho_a < \rho_1$$

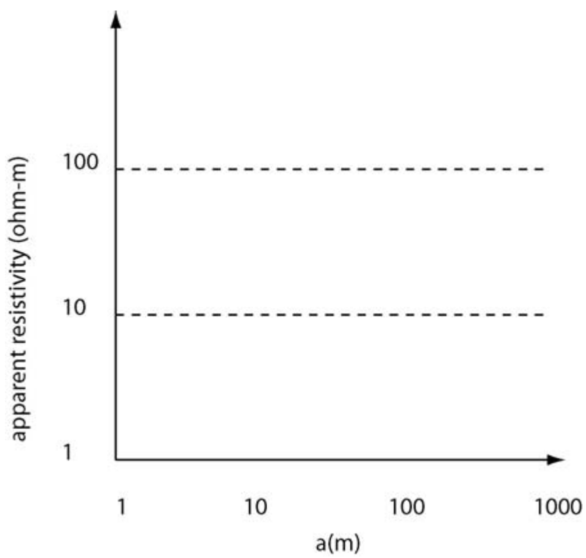


a=100 m: Most of the electric current flows in the lower layer because of geometric arguments. This is enhanced by the fact that the lower layer is a route of low resistivity, essentially a short circuit, and electric current flows preferentially in this layer.

$$\rho_2 \sim \rho_a < \rho_1$$

As the electrode spacing continues to increase, $\rho_a \rightarrow \rho_2$





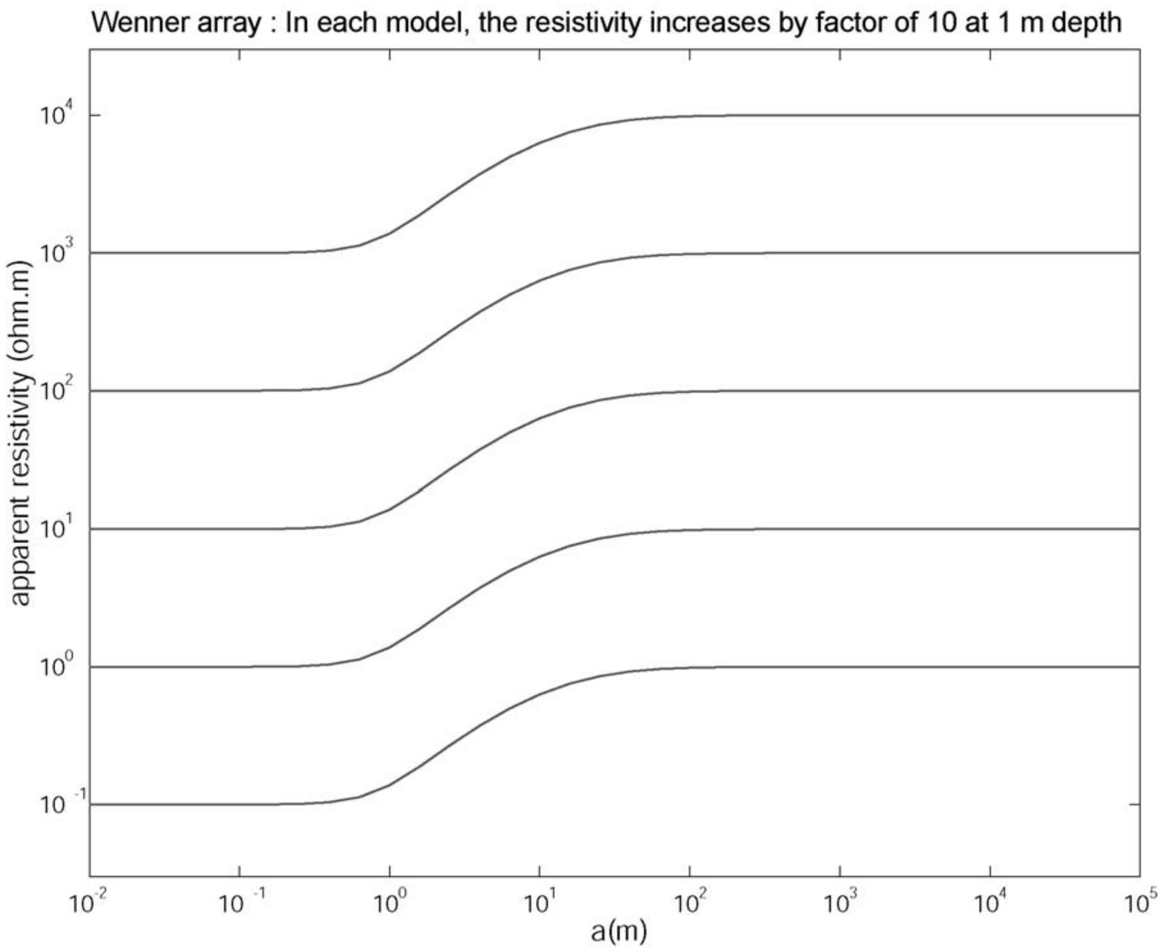
The results can be summarized in a plot of apparent resistivity as a function of the a-spacing. Since the electric current goes deeper as the *a*-spacing increases, this curve gives an impression of how electrical resistivity varies with depth.

However, there is not an exact correlation between the *a*-spacing and depth being sampled by the electric currents. Modeling and interpretation is needed to convert the *a*-spacing into a true depth.

C3.3 Quantitative solution

A quantitative solution can be derived through the method of images, or other more complex calculations. Details can be found in the textbook. A simple MATLAB program was used to compute the apparent resistivity as a function of *a*-spacing.

Example 1: Two layer models. Resistivity increases by a factor of 10 at depth of 1m in all the models.

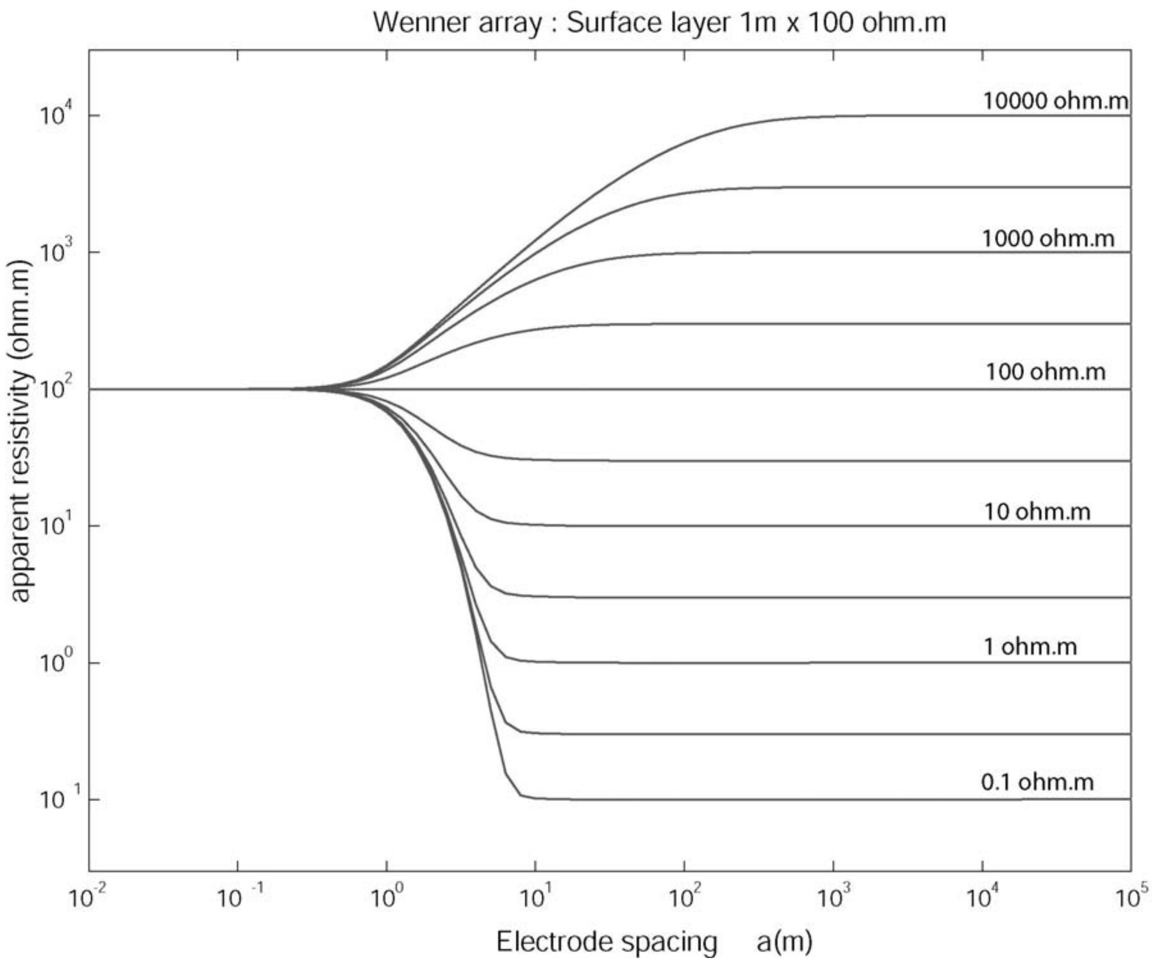


Note that:

(1) The increase in apparent resistivity occurs when the *a*-spacing of the Wenner array is approximately equal to the layer thickness.

(2) If the vertical axis was plotted as $\frac{\rho_a}{\rho_1}$ then all the curves would be identical. This will be useful when **master curves** are considered later in this section.

Example 2: In all models, the surface layer has a resistivity of 100 Ωm and is 1 m thick. The second layer has a range of resistivities, as shown in the figure.



Note that:

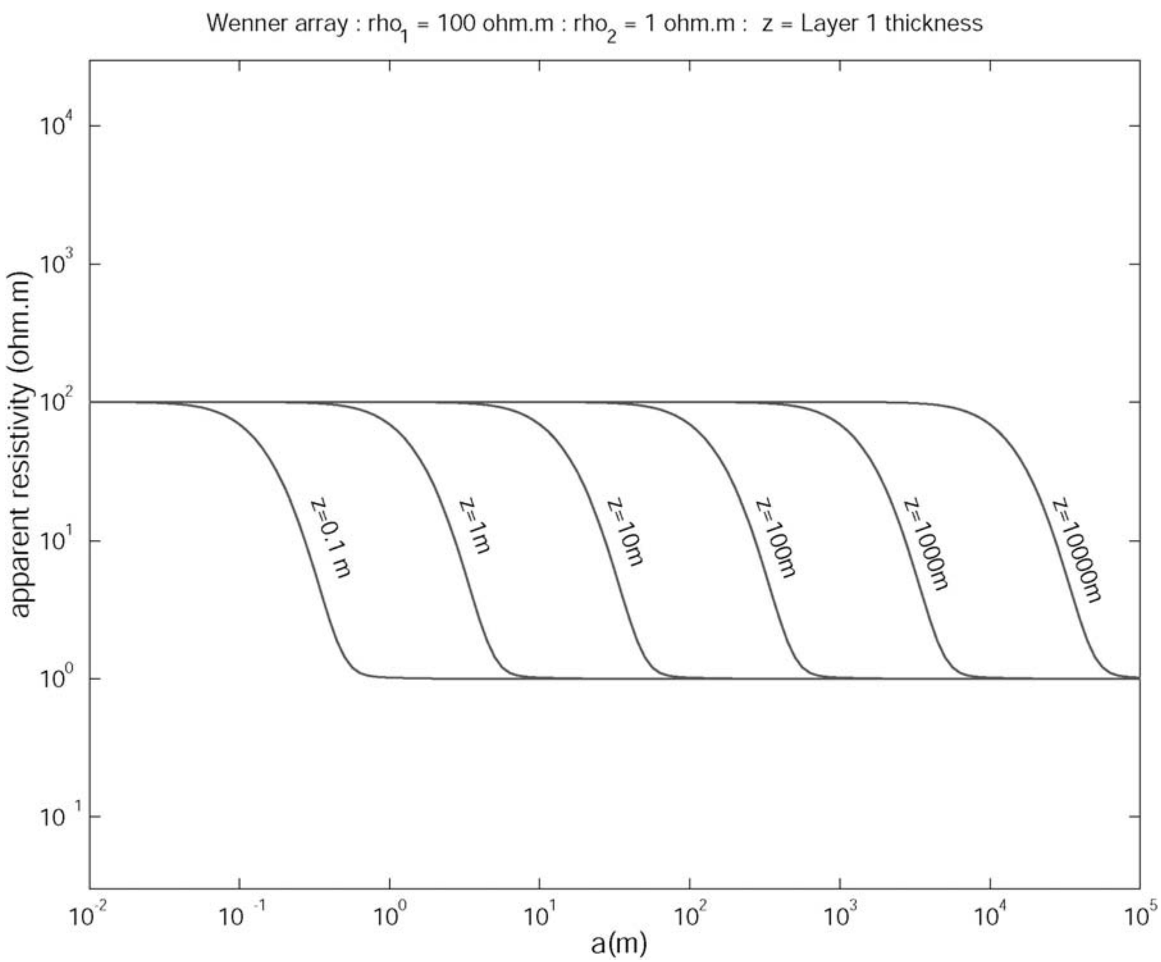
(1) The effect of the lower layer is only observed when the *a*-spacing is greater than the layer thickness.

(2) At large values of *a*-spacing, the apparent resistivity asymptotically approaches the true resistivity of the lower layer.

(3) When the lower layer is more resistive, the apparent resistivity rises slowly as the a-spacing increases. This is because the electric current preferentially flows in the lower resistivity (upper) layer, and apparent resistivity is the average resistivity of the region in which current is actually flowing.

(4) When the lower layer has the lowest resistivity, the apparent resistivity falls quickly, as the *a*-spacing increases. Explanation is the converse of that in (3).

Example 3: In all models, the surface layer has a resistivity of 100 Ωm and the thickness is variable ranging from 0.1 to 10000 m. The second layer has a range of resistivity of 1 Ωm.



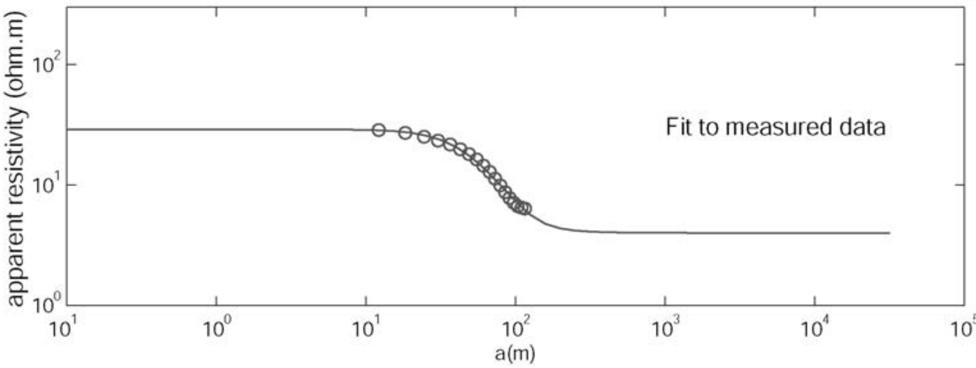
Note that:

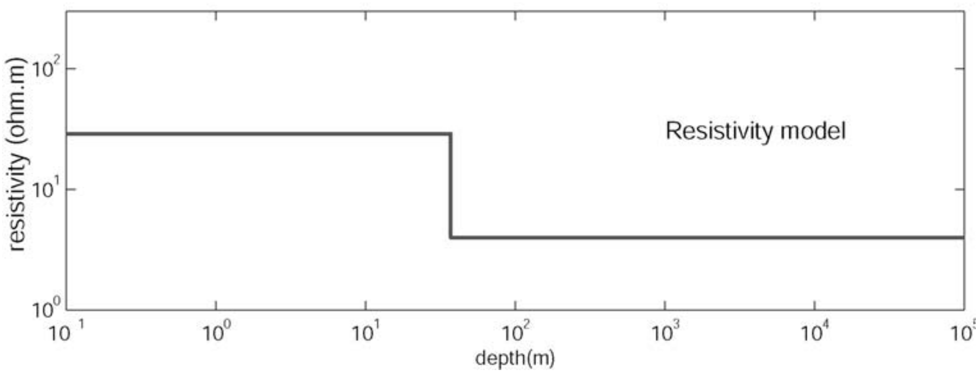
(1) The effect of the lower layer is only observed when the *a*-spacing is greater than the layer thickness.

(2) The shape of the curve is the same in each case.

(3) If the horizontal axis was (*a/z*) then all the curves would be identical. This is the physical basis of the **master curves** discussed later on in the notes.

3.4 Fitting Wenner array data with a two-layer resistivity model





Data were collected at Malagash, Nova Scotia and are listed in Telford. In this area, the subsurface is saturated with brine, quite close to the surface.

These data will be interpreted in class by trial-and-error modeling with the MATLAB script **wenner2lay_fit.m**

a(ft)	rho (ohm-m)
40.	28.5
60.	27.1
80.	25.3
100.	23.5
120.	21.7
140.	19.8
160.	18.0
180.	16.3
200.	14.5
220.	12.9
240.	11.3
260.	9.9
280.	8.7
300.	7.8
320.	7.1
340.	6.7
360.	6.5
380.	6.4

3.5 Curve matching (a history lesson)

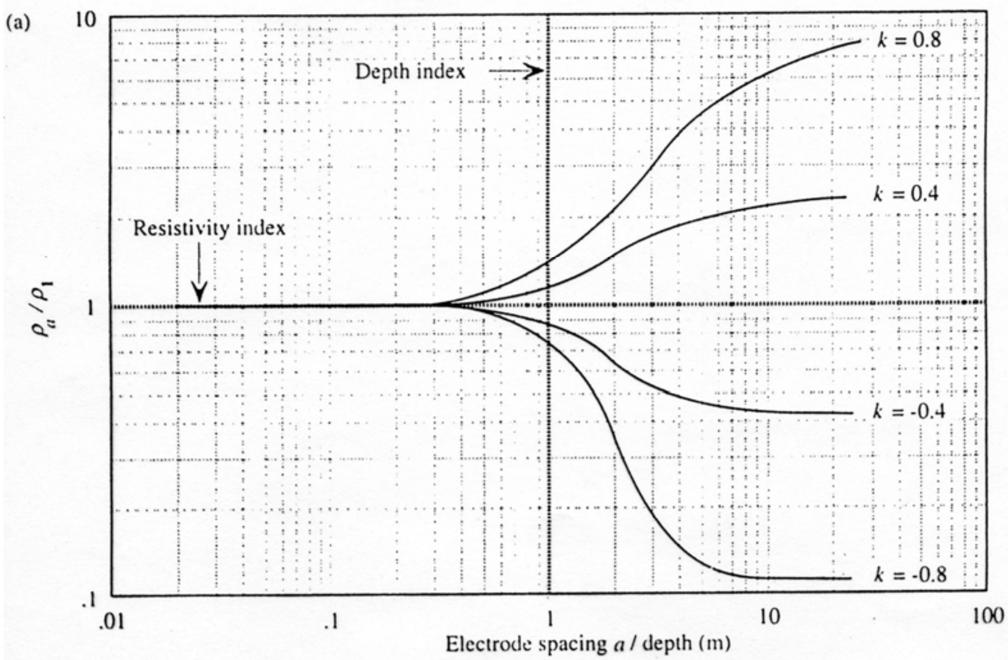


Figure 5-33 Basic procedures for curve matching. (a) Master curves for several possible *k*-values. (b) Field data plotted on the same scale as (a). (c) Answer obtained by superimposing field-data curve on master curves.

This set of curves summarizes **all possible combinations** of upper layer resistivity (ρ_1), upper layer resistivity (ρ_2) and layer thickness (d).

This is achieved by plotting apparent resistivity normalized by ρ_1 on the vertical axis.

Similarly, the horizontal scale is normalized by plotting apparent resistivity divided by layer thickness (*a/d*). This reflects the fact that the lower layer will be detected when the electrode spacing is approximately equal to the layer thickness (when *a~d* or *a/d =1*).

The individual apparent resistivity curves represent a range of lower layer resistivity values (ρ_2) and $k = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$. Positive values of *k* correspond to an increase in resistivity with depth, while negative values correspond to a decrease in resistivity with depth.

Note that on Assignment 3, the master curves are labeled with the ratio (ρ_2 / ρ_1)

These master curves are used by overlaying a plot of the actual data points on a set of master curves and sliding the overlay **up-down** and **left-right** until a good fit is found to one of the curves. More details will be found on Assignment 3.

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