## Geophysics 424 A1 Final exam

## Electromagnetic and Potential field methods

Date: $\quad$ Wednesday December $15^{\text {th }} 2010,2 \mathrm{pm}-5 \mathrm{pm}$
Location CEB 442
Instructor : Dr. Martyn Unsworth
Time allowed : 3 hours
Total points $=102$

## Instructions

Attempt all questions.
Notes and books may not be used.
Calculators may be used.
Cell phones and all other electronic devices must be switched off and stored.
All questions must be directed to the invigilator.
A separate 2 page formula sheet is available.

## Question 1 : Resistivity of rocks (Total = 11 points)

(a) Electric current flows in a material that contains a single type of charge carrier.

There are $n$ charge carriers per unit volume and each has a charge $q$.
The charge carriers have a mobility $\mu$ which is defined as the drift velocity per unit electric field. $\mu=\frac{\bar{v}}{E}$

Show that the electrical conductivity of the material is given by $\sigma=n \mu q$
(b) A brine is formed by dissolving 9 g of sodium chloride in 1 litre of water.

The brine has a conductivity of $1 \mathrm{~S} / \mathrm{m}$
Estimate the mobility of the ions.
Assume that the sodium $\left(\mathrm{Na}^{+}\right)$and chloride $\left(\mathrm{Cl}^{-}\right)$ions have the same mobility

## (5 points)

## Useful data

Volume of 1 litre of water
$10^{-3} \mathrm{~m}^{3}$
Molar weight of sodium chloride
Avogadro's number
$58.4 \mathrm{~g} / \mathrm{mol}$
Charge on electron
$6.0210^{23}$
$1.610^{-19} \mathrm{C}$

## Question 2: Ground-penetrating radar (Total = 7 points)

You are studying the structure of some layered sand deposits and the survey is required to resolve layers 50 cm thick to a maximum depth of 3 m

You have the choice of using GPR (Frequency $=100 \mathrm{MHz}$ ) or seismic reflection (signal frequency $=100 \mathrm{~Hz}$ )

The sand has the following properties

- electrical resistivity of $4000 \Omega \mathrm{~m}$
- radar velocity of $0.15 \mathrm{~m} / \mathrm{ns}$
- seismic velocity of $2000 \mathrm{~m} / \mathrm{s}$.

Which geophysical method will work in this case? Explain your answer.

Hint : The thinnest layer resolved has thickness $=\mathrm{z}=\lambda / 4$, where $\lambda=$ wavelength

## Question 3: Magnetotellurics (Total = 10 points)

An MT instrument records 5 channels of data with a continuous sample rate of 8 Hz .
Each data sample takes 16 bytes of disk space.
The instrument has an 16 MB memory card.
(a) What will be the highest frequency recorded?
(b) How long can data be recorded before the memory is full?
(c) What will be the lowest frequency recorded?
(d) Give two reasons for burying the magnetometer in an MT survey.

## Question 4: Magnetotellurics (Total = 21 points)

(a) The figure below shows a section through the continental lithosphere. It is not drawn to scale.


Label approximate depths for each interface.
Label typical resistivity values (nearest factor of 10).
What is the physical cause of the observed resistivity values in each layer?

## (15 points)

(b) Describe two applications of magnetotellurics in hydrocarbon exploration.

For each

- describe the contrasts in resistivity that make MT applicable.
- State why seismic exploration fails in each case


## Question 5: Maxwell's Equations (Total $=20$ points)

An electromagnetic (EM) wave is vertically incident on the Earth's surface.
The EM fields vary with time as $\mathrm{e}^{\mathrm{i} \omega \mathrm{t}}$ with angular frequency $\omega$
At this location, the Earth has a uniform conductivity $\sigma$
The electric field is polarized in the $x$-direction and $E_{x}$ does not vary in the $x$-direction.


Magnetic permeability of ground Dielectric permittivity of ground Electrical conductivity of ground
Frequency

$$
\begin{array}{ll}
=\mu=\mu_{0} & =4 \pi \times 10^{-7} \mathrm{H} / \mathrm{m} \\
=\varepsilon=\varepsilon_{0} & \\
=8.85 \times 10^{-12} \mathrm{~F} / \mathrm{m} \\
=\sigma & \\
=0.01 \mathrm{~S} / \mathrm{m} \\
=\mathrm{f} &
\end{array}
$$

(a) Use Maxwell's equations to show that $E_{x}$ satisfies the following partial differential equation

$$
\begin{equation*}
\frac{\partial^{2} E_{x}}{\partial y^{2}}+\frac{\partial^{2} E_{x}}{\partial z^{2}}=i w \mu \sigma E_{x} \tag{9points}
\end{equation*}
$$

State any assumptions made in your derivation.
(b) Consider a non-plane wave whose amplitude varies as a function of y as :

$$
E_{x}(y, z)=E_{x}(z) \sin \left(\frac{2 \pi y}{\lambda}\right)
$$

Show that in this case, the partial differential equation in (a) simplifies to an ordinary differential equation

$$
\frac{d^{2} E_{x}}{d z^{2}}+k_{a}^{2} E_{x}=0 \quad \text { and derive a value for } \mathrm{k}_{\mathrm{a}}
$$

## (6 points)

(c) Using the numerical values listed above, estimate the highest frequency at which the non-planar nature of the wave will be noticed when $\lambda=100 \mathrm{~km}$.

## Question 6: Frequency domain EM methods (Total = 20 points)



A frequency domain electromagnetic system with co-axial transmitter (TX) and receiver ( RX ) is flown beneath a helicopter. The system parameters are:

- primary field frequency, $\mathrm{f}=20,000 \mathrm{~Hz}$
- transmitter-receiver offset, $\mathrm{L}=10 \mathrm{~m}$.
- ground clearance of TX-RX bird, $\mathrm{h}=30 \mathrm{~m}$
- noise level 10 ppm (both in phase and quadrature)
(a) The EM system was flown across a vertical conductor and the following response $\left(\mathrm{H}_{\mathrm{S}} / \mathrm{H}_{\mathrm{P}}\right)$ was measured.


Explain the sign of the in-phase response with a diagram showing magnetic field lines for primary and secondary magnetic fields.
(b) Determine as much as possible about the target.

List all the assumptions you make.
(10 points)
(c) A vertical conductor has a conductance of 4 S . What is the maximum depth at which it can be detected by this system?
(5 points)

## Characteristic curves for co-axial TX-RX and a vertical plate



Response parameter, $\mathrm{Q}=\sigma \mathrm{Wf} \quad$ Depth parameter, $\mathrm{D}=\mathrm{H} / \mathrm{L}$

$$
\begin{array}{ll}
\mathrm{W} & =\text { width of conductor } \\
\sigma & =\text { conductivity of conductor } \\
\mathrm{H} & =\text { depth to top of conductor below TX-RX bird } \\
\mathrm{f} & =\text { TX frequency }(\mathrm{Hz}) \\
\mathrm{L} & =\text { TX-RX separation }
\end{array}
$$

## Question 7 : Time domain EM methods (Total = $\mathbf{1 3}$ points)



A time-domain EM system is being used to measure the resistivity during a ground water survey. The system parameters are:

| Transmitter geometry | $10 \mathrm{~m} \times 10 \mathrm{~m}$ square loop |
| :--- | :--- |
| Transmitter current | $\mathrm{I}=100 \mathrm{amps}$ |
| Number of turns on transmitter | $\mathrm{N}=10$ |
| TX-RX separation | $\mathrm{r}=100 \mathrm{~m}$ |

(a) The transient below was collected at 'A'.

Selected data values are listed in the table on the next page.


| Time $(\mathrm{s})$ | $-(\mathrm{nT} / \mathrm{s})$ |
| :--- | :--- |
| 0.0001 | $1.100 \mathrm{e}+004$ |
| 0.001 | $5.599 \mathrm{e}+001$ |
| 0.01 | $1.852 \mathrm{e}-001$ |
| 0.1 | $5.883 \mathrm{e}-004$ |

Calculate the conductivity of the gravel. Justify any assumptions you make.

## (7 points)

(b) A second transient was recorded at ' $B$ ' where a high conductivity clay layer was present. Sketch the transient at ' $B$ ' on the graph above. Be quantitative where possible.
(6 points)

