## Geophysics 424 A1 Final exam <br> Electromagnetic and Potential field methods

Date : $\quad$ Monday April $23^{\text {rd }} 2012,9 \mathrm{am}$ - noon
Location CCIS L1-029
Instructor : Dr. Martyn Unsworth
Time allowed : 3 hours
Total points $=127$

## Instructions

Attempt all questions.
Notes and books may not be used.
Calculators may be used.
Cell phones and all other electronic devices must be switched off and stored.
All questions must be directed to the invigilator.
A separate 2 page formula sheet will be distributed

## Question 1 : Resistivity of rocks (Total $=16$ points)

A crystalline rock contains graphite in aligned cracks.
The volume fraction of graphite is $V$
The graphite has a resistivity of $\rho_{\mathrm{g}}$ and the rock matrix has a resistivity of $\rho_{\mathrm{r}}$

Consider a cube that is $1 \mathrm{mx} 1 \mathrm{~m} \times 1 \mathrm{~m}$.
(1a) Derive an equation for the resistance of the cube when electric current flows parallel to the cracks. Consider $0<V<1$
(3 points)
(1b) Derive an equation for the resistance of the cube when electric current flows normal to the cracks. Consider $0<V<1$
(1c) Sketch these resistance values on a graph for the range $0<V<1$
Assume that $\rho_{\mathrm{g}}=0.1 \Omega \mathrm{~m}$ and $\rho_{\mathrm{r}}=1000 \Omega \mathrm{~m}$
Give numerical values for $V=0,0.5$ and 1
(1d) Calculate the resistance predicted by Archie's Law. Plot on the graph.
Assume that $\rho_{\mathrm{g}}=0.1 \Omega \mathrm{~m}$ and $\rho_{\mathrm{r}}=1000 \Omega \mathrm{~m}$
Give numerical values for $V=0,0.5$ and 1
(4 points)
(1e) Comment on how Archie's Law compares to the curves you plotted in (1c)
( 2 points)

## Question 2: Ground-penetrating radar (Total = 12 points)

GPR is being used to study the subsurface structure shown below.
A metal pipe is buried at a depth $d=4 \mathrm{~m}$ and horizontal location $x=0 \mathrm{~m}$
Velocity in the soil $=\mathrm{v}_{1}=0.1 \mathrm{~m} / \mathrm{ns}$
Radar frequency is 200 MHz and TX-RX distance is 1 m
(2a) On the graph below, sketch the travel times for the air wave and ground wave at the RX
(2b) The TX-RX are located at a distance $x$ from the pipe. Derive an equation for the travel time of a signal diffracted from the pipe. Assume that the TX and RX are coincident.
(2c) Add the travel time for the diffracted signal to the graph
(2d) Calculate the moisture content of the soil $\left(\theta_{\mathrm{v}}\right)$


TX---RX


## Question 3: Magnetotellurics (Total = 11 points)

An MT instrument records 5 channels of data with a sample rate of 320 Hz .
Each data sample takes 16 bytes of disk space and 36 hours of data are recorded.
A 24-bit analogue to digital converter is used with a full-scale range of 4 V
Explain you answer to each of the following.
(3a) What will be the highest frequency that can be obtained from these data?
(2 points)
(3b) What will be the lowest frequency that can be obtained from these data?
(2 points)
(3c) What is the weakest electric field that can be detected?
(3 points)
(3d) What is an anti-alias filter? Describe the filter that would be needed in this example?
(4 points)

## Question 4 : Marine CSEM methods (Total = 10 points)

(4a) How does frequency domain marine controlled source EM exploration measure variations of resistivity with depth?
(2 points)
(4b) Explain the geometry used in broadside and inline configurations in marine CSEM. Include a diagram in your answer.
(2 points)
(4c) How are these configurations used to detect hydrocarbon reservoirs? Describe the physics and include a diagram of measured data in your answer. (6 points)

## Question 5: Magnetotellurics (Total = 19 points)




Figure above shows the crustal resistivity structure of a coastal area.
Stations A and B are onshore
Station C is on the seafloor where the seawater is 3 km deep.
(5a) Sketch the apparent resistivity and phase curves at stations A and B on the left hand graph. Be quantitative where possible. (11 points)
(5b) Sketch the apparent resistivity and phase curves at station C on the right hand graph. Be quantitative where possible. (8 points)

Note : Assume that an EM signal will not be detectable after travelling 3 skin depths

## Question 6: Maxwell's Equations (Total = 23 points)

An MT survey is being carried out at a location, where the Earth has a 2-D conductivity structure that is invariant in the $x$-direction.

The EM fields have an angular frequency $\omega$, and time variation $\mathrm{e}^{\mathrm{i} \omega t}$

(6a) Expand Maxwell's equations for the six components of the electromagnetic field ( $E_{x}, E_{y}, E_{z}, B_{x}, B_{y}$ and $B_{z}$ ) in the frequency domain.
(6b) These six equations simplify over a 2-D Earth structure. Derive a differential equation for the TM mode in terms of $\mathrm{B}_{\mathrm{x}}(\mathrm{y}, \mathrm{z})$. Explain any assumptions you have made
(7 points)
(6c) Consider a point on the surface of the Earth at $\mathrm{z}=0$
The vertical electric fields above and below the surface are $E_{z}^{1}$ and $E_{z}^{2}$
The electrical conductivity above and below the surface are $\sigma_{1}$ and $\sigma_{2}$
Derive a relationship between $E_{z}^{1}$ and $E_{z}^{2}$
(4 points)
(6d) Consider the case when $\sigma_{1}=0 \mathrm{~S} / \mathrm{m}$ and $\sigma_{2}=0.01 \mathrm{~S} / \mathrm{m}$
Show that for the TM mode, $B_{x}(\mathrm{y})$ at $\mathrm{z}=0$ is constant for all values of $y$
(4 points)

## Question 7 : Frequency domain EM methods (Total = $\mathbf{1 3}$ points)

A


Am EM34 survey takes place where a clay layer overlies bedrock.
The basement outcrops on the left and has a resistivity of $800 \Omega \mathrm{~m}$.
The clay layer is very thick on the right and has a resistivity of $20 \Omega \mathrm{~m}$
TX-RX separation was 10 m and frequency was 6.4 KHz .
TX and RX placed on ground to make measurements.
(7a) Prove that for a $20 \Omega \mathrm{~m}$ halfspace, the near-field approximation is valid ( $\mathbf{3}$ points)
(7b) At 'A', the instrument measures an apparent resistivity of $240 \Omega \mathrm{~m}$ in the vertical dipole mode. How thick is the clay layer?
(5 points)
(7c) At 'A', calculate the apparent resistivity that would be measured in horizontal dipole mode. Comment on your answer.

## Question 8 : Time domain EM methods (Total = 22 points)

A time domain survey is using a sawtooth transmitter waveform.
The primary magnetic field is shown on the next page in figure (A)
The ore body ( a conductor) behaves as an inductor and resistor in series.
(8a) Define the mutual inductance between two electric circuits
(8b) The secondary voltage induced in the conductor is shown in B .
Explain how the secondary voltage is related to the primary magnetic field.
(8c) Sketch the time variation of the secondary current.
Show both GOOD and BAD conductors on same graph.
(6 points)
(8d) Sketch the time variation of the secondary magnetic field at the RX.
Show both GOOD and BAD conductors on same graph.
(8e) Sketch the time variation of the total magnetic field at the RX Show both GOOD and BAD conductors on same graph.

Your answer will be qualitative in (8c) - (8e).
Briefly explain how you obtained your answer in each case.
(A) Primary magnetic field at conductor

(B) Secondary voltage induced in conductor

(C) Secondary current in conductor

(D) Secondary magnetic field at RX

(E) Total magnetic field at RX


