

**Geophysics 424 A1 - Final exam - Fall term**  
**Electromagnetic and Potential field methods**

**Date :** Tuesday December 16th 2014, 2-5 pm  
**Location** CCIS L1-029  
**Instructor :** Dr. Martyn Unsworth  
**Time allowed** 3 hours  
**Total points** **104**

**Instructions**

*Attempt all questions.*

*Notes and books may **not** be used.*

*Calculators may be used.*

*Cell phones and all other electronic devices must be switched off and stored.*

*All questions must be directed to the invigilator.*

*A separate formula sheet will be distributed*

**Question 1 : Resistivity of rocks (Total = 10 points)**

(1a) Electric current flows in a material that contains a single type of charge carrier.

There are  $n$  charge carriers per unit volume and each has a charge  $q$ .

The charge carriers have a mobility  $\mu$  which is defined as the drift velocity per unit

electric field.  $\mu = \frac{\bar{v}}{E}$

Show that the electrical conductivity of the material is given by  $\sigma = n\mu q$

**(5 points)**

(1b) A brine is formed by dissolving 30 g of sodium chloride in 1 litre of water.

The brine has a conductivity of 3 S/m

Estimate the mobility of the sodium ions.

Assume that the sodium ( $\text{Na}^+$ ) and chloride ( $\text{Cl}^-$ ) ions have the same mobility

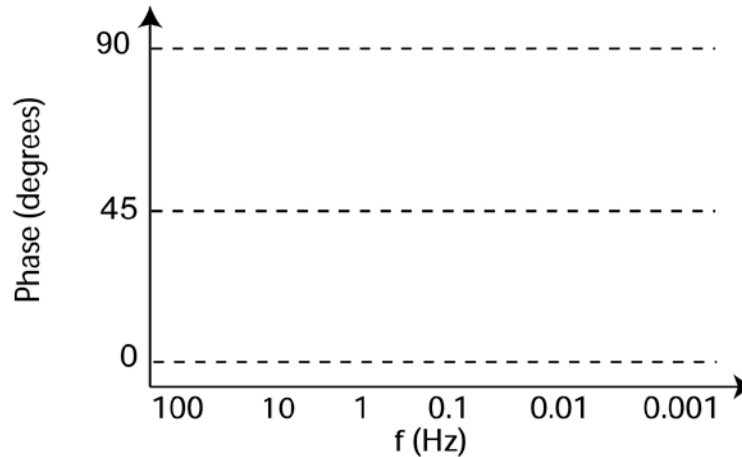
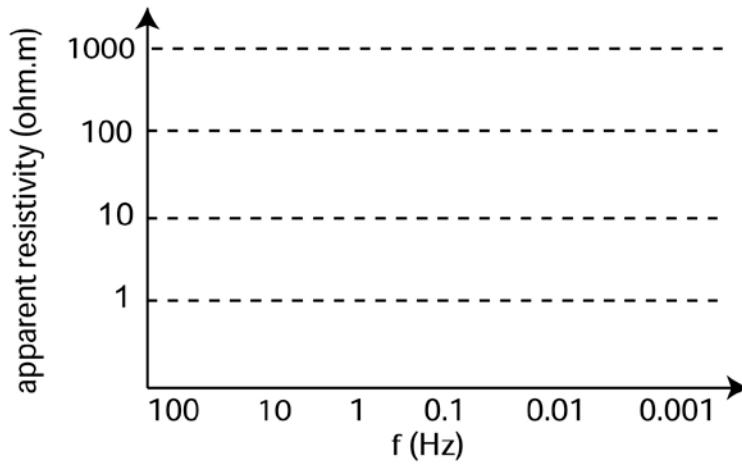
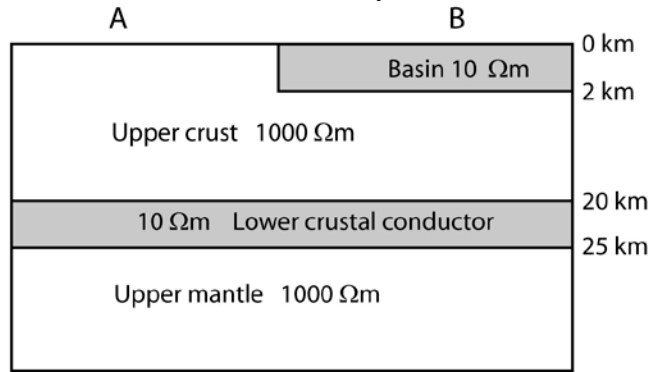
**(5 points)**

**Useful data**

Volume of 1 litre of water	$10^{-3} \text{ m}^3$
Molar weight of sodium chloride	58.4 g / mol
Avogadro's number	$6.02 \cdot 10^{23}$
Charge on electron	$1.6 \cdot 10^{-19} \text{ C}$

**Question 2 : Magnetotellurics (Total = 21 points)**

MT data (100-0.001 Hz) are being used to image a lower crustal conductor. Site A is located on exposed basement rocks, while B is in a sedimentary basin.



(2a) Sketch the MT apparent resistivity and phase data at sites 'A' and 'B'. You can approximate the structure at each location is 1-D. Be quantitative where possible.

**(14 points)**

(2b) One theory suggests that the lower crust is conductive because of **interconnected** graphite films. The conductivity of pure graphite is 1000 S/m. What **volume fraction** of graphite is required to account for a bulk resistivity of 10  $\Omega\text{m}$ ?

**(3 points)**

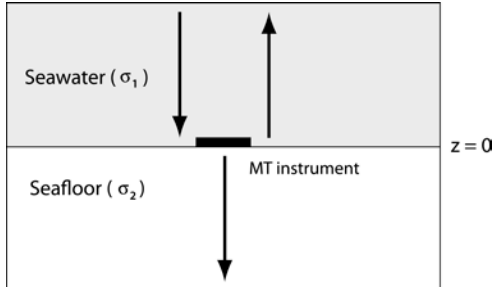
(2c) The other theory is that saline fluids cause the low resistivity in the lower crust. The fluid distribution is not known. Assuming a fluid conductivity of 10 S/m what **range of porosities** is required to account for a bulk resistivity of 10  $\Omega\text{m}$ ?

**(4 points)**

**Question 3 : Maxwell's Equations (Total = 22 points)**

A seafloor MT survey is being used to measure the resistivity of the seafloor.

A **plane** EM wave has an angular frequency,  $\hat{E}$ , and travels **vertically** downwards in seawater in the  $z$ -direction. The electric field is **polarized** in the  $x$ -direction.



Down going signal in seawater

$$E_x(z, t) = Ae^{-k_1 z} e^{-i\omega t}$$

Up going signal in seawater

$$E_x(z, t) = Be^{k_1 z} e^{-i\omega t}$$

Down going signal in seafloor

$$E_x(z, t) = Ce^{-k_2 z} e^{-i\omega t}$$

$$k_1 = \sqrt{-i\omega\mu_0\sigma_1} \text{ and } k_2 = \sqrt{-i\omega\mu_0\sigma_2}$$

(3a) Use Maxwell's equations to prove that the **horizontal electric field** is continuous at the seafloor ( $z = 0$ )

**(6 points)**

(3b) Use Maxwell's equations to show that for a plane, polarized EM signal

$$H_y(z) = \frac{1}{i\omega\mu} \frac{\partial E_x(z)}{\partial z} \quad \text{(5 points)}$$

(3c) Derive an expression for  $\frac{C}{A}$  at the seafloor in terms of  $\sigma_1$  and  $\sigma_2$ . State any boundary conditions that you use.

**(6 points)**

(3d) The MT instrument measures electric and magnetic fields at a frequency  $\hat{E}$ .

At  $z = 0$ , the measured electric and magnetic fields is  $E_x^m$  and  $H_y^m$

Derive an equation for the seafloor conductivity ( $\sigma_2$ ) in terms of  $E_x^m$  and  $H_y^m$

**(5 points)**

#### **Question 4 : Marine EM methods (Total = 17 points)**

- (4a) How does frequency domain marine controlled source EM exploration measure variations of resistivity with depth? **(2 points)**
- (4b) In a study area, the seafloor has a uniform porosity of 10% and the pore space is poorly connected. The seawater conductivity is 3 S/m. It is required to transmit EM signals over a distance of 5 km through the seafloor. What transmission frequency is needed? **(5 points)**
- (4c) Draw a map of **broadside** and **inline** configurations of receivers in marine CSEM when the transmitter is a horizontal electric dipole. **(4 points)**
- (4d) How are these configurations used to detect **hydrocarbon reservoirs**? Explain the physics and include a diagram in your answer. **(6 points)**

#### **Question 5 : Frequency domain EM (Total = 14 points)**

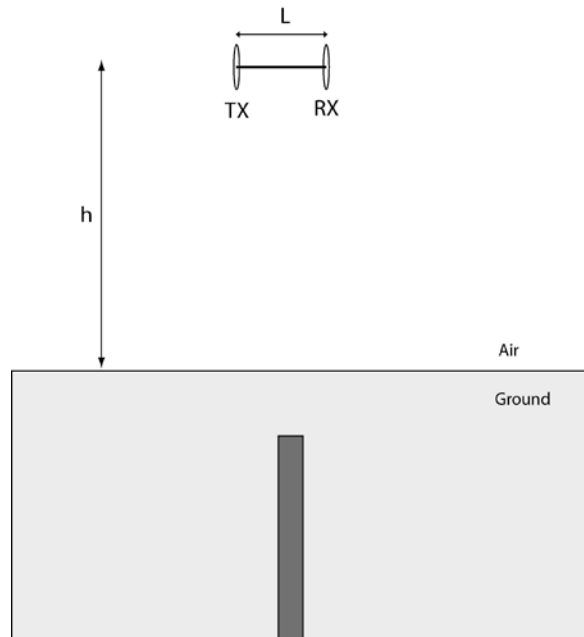
Frequency domain EM exploration can be modelled by considering the transmitter (TX), receiver (RX) and ore body as electrical circuits. An EM system generates a primary field with a frequency of 900 Hz.

- (5a) Define the **mutual inductance** between the TX and RX. Include a diagram **(3 points)**
- (5b) Why is relative motion between the TX and RX a problem in this type of system? **(4 points)**
- (5c) The primary magnetic field has a magnitude of 100 nT at the receiver. The secondary magnetic field has in-phase and quadrature components at the receiver of 10 and 2 nT, respectively.

Draw a **phase diagram** of the primary, secondary and total magnetic fields at the RX **(3 points)**

- (5d) Calculate the **magnitude of** the total magnetic fields. **(2 points)**
- (5e) Calculate the difference in **phase angle** between the total and primary magnetic fields. **(2 points)**

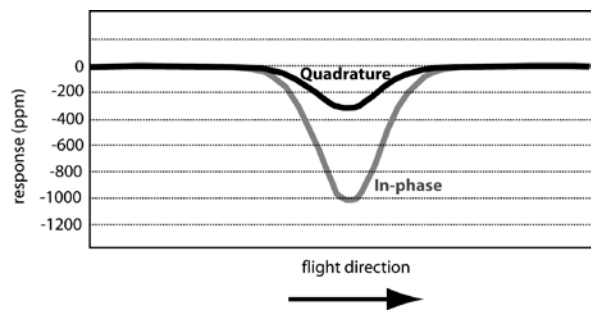
**Question 6 : Frequency domain EM methods (Total = 20 pts)**



A frequency domain electromagnetic system with **co-axial** transmitter (TX) and receiver (RX) is flown beneath a helicopter. The system parameters are:

- primary field frequency,  $f = 10,000$  Hz
- transmitter-receiver offset,  $L = 20$  m.
- ground clearance of TX-RX bird,  $h = 30$  m
- noise level 10 ppm (both in phase and quadrature)

(6a) The EM system was flown across a **conductor** and the following response ( $H_S / H_P$ ) was measured.



Explain the **sign** of the **in-phase** response with a **diagram** showing magnetic field lines for primary and secondary magnetic fields.

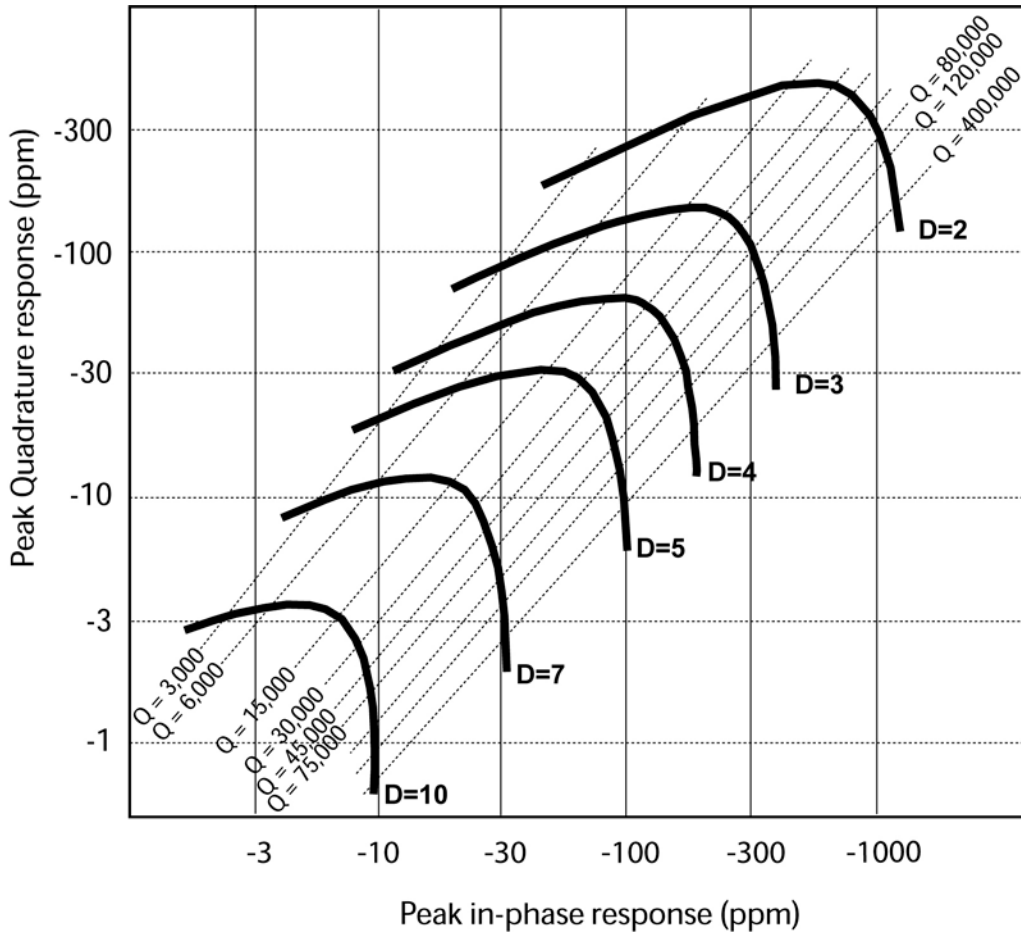
**(5 points)**

(6b) Determine as much as possible about the conductor.  
List all the assumptions you make.

**(10 points)**

- (6c) A vertical conductor has a conductance of 4 S.  
 What is the maximum depth at which it can be detected by this system?  
**(5 points)**

**Characteristic curves for co-axial TX-RX and a vertical plate**



Response parameter,  $Q = \tilde{A}Wf$     Depth parameter,  $D = H / L$

- W = width of conductor
- $\tilde{A}$  = conductivity of conductor
- H = depth to top of conductor below TX-RX bird
- f = TX frequency (Hz)
- L = TX-RX separation



**Question 7 : Ground-penetrating radar (Total = 13 points)**

GPR is being used to study the subsurface structure shown below.

A metal pipe is buried at a depth  $d = 4$  m and horizontal location  $x = 0$  m

Velocity in the soil =  $v_1 = 0.15$  m / ns

Radar frequency is 200 MHz and TX-RX distance is 1 m

(7a) On the graph below, sketch the **travel times** for the air wave and ground wave at the RX **(4 points)**

(7b) The TX-RX are located at a distance  $x$  from the pipe. Derive an equation for the travel time of a signal **diffracted** from the pipe. Assume that the TX and RX are coincident. **(3 points)**

(7c) Add the travel time curve for the diffracted signal **(3 points)**

(7d) Calculate the moisture content of the soil ( $\theta_v$ ) **(3 points)**

