## Geophysics 524A1-424 A1 - Final exam - Fall term

Electromagnetic and Potential field methods

| Date : | Friday December 18th 2015, 2-5 pm |
| :--- | :--- |
| Location | CCIS L1-029 |
| Instructor : | Dr. Martyn Unsworth |
| Time allowed | 3 hours |
| Total points | $\mathbf{1 1 8}$ |

## Instructions

Attempt all questions.
Notes and books may not be used.
Calculators may be used.
Cell phones and all other electronic devices must be switched off and stored.
All questions must be directed to the invigilator.
A separate formula sheet will be distributed

## Question 1 : Resistivity of rocks (Total = 18 points)

An ore deposit consists of crystalline rock with sulphide minerals in aligned cracks.
The volume fraction of sulphides in the rock is $V$
The sulphides have a resistivity of $\rho_{s}$ and the rock matrix has a resistivity of $\rho_{\mathrm{r}}$
Consider a cube that is 1 mx 1 mx 1 m
(1a) Derive an equation for the resistance of the cube when electric current flows parallel to the cracks. Consider $0<V<1$
(1b) Derive an equation for the resistance of the cube when electric current flows normal to the cracks. Consider $0<V<1$
(3 points)
(1c) Sketch these resistance values on a graph for the range $0<V<1$
Assume that $\rho_{\mathrm{s}}=0.01 \Omega \mathrm{~m}$ and $\rho_{\mathrm{r}}=1000 \Omega \mathrm{~m}$
Give numerical values of the resistance for $V=0,0.5$ and 1
(4 points)
(1d) Calculate the resistance predicted by Archie's Law for the range $0<\mathrm{V}<1$.
Plot the resistance on the graph.
Assume that $\rho_{\mathrm{s}}=0.01 \Omega \mathrm{~m}$ and $\rho_{\mathrm{r}}=1000 \Omega \mathrm{~m}$
Give numerical values of the resistance for $V=0,0.5$ and 1
(5 points)
(1e) Comment on how Archie's Law compares to the curves you plotted in (1c)
(3 points)

## Question 2 : Magnetotellurics (Total = 18 points)

Broadband MT data (1000-0.001 Hz) are being used to image a coastal aquifer in the Canadian Arctic. Site A is located on rock, while B is located on sea ice
(2a) Sketch the apparent resistivity and phase data at sites A and B (12 points)
(2b) To avoid the problem of working on sea ice, the researchers returned when the sea ice had melted and placed a seafloor MT instrument at site C. Sketch the apparent resistivity that would be measured at C ( 6 points)

- You can approximate the structure at each location is 1-D.
- Be quantitative where possible. Ignore the resistivity of the sea ice
- Assume that EM signals can be detected after travelling 1 skin depth in seawater
B

| A | B |  |
| :---: | :---: | :---: |
|  | Seawater $0.3 \Omega \mathrm{~m} \quad \mathrm{C}$ |  |
| Rock $1000 \Omega \mathrm{~m}$ | 0.3 km |  |
| Aquifer $10 \Omega \mathrm{~m}$ | 1 km |  |
| Rock $1000 \Omega \mathrm{~m}$ | 2 km |  |

Station $A$ and $B$





Page 3 of 10

## Question 3 : Maxwell's Equations (Total = 21 points)

An MT survey is being carried out at a location, where the Earth has a 2-D conductivity structure that is invariant in the $x$-direction.

The EM fields have an angular frequency $\omega$, and time variation $\mathrm{e}^{\mathrm{i} \omega \mathrm{t}}$

(3a) Expand Maxwell's equations for the six components of the electromagnetic field ( $E_{x}, E_{y}$, $E_{z}, B_{x}, B_{y}$ and $B_{z}$ ) in the frequency domain.
(3b) These six equations simplify over a 2-D Earth structure. Derive a differential equation for the TM mode in terms of $\mathrm{B}_{\mathrm{x}}(\mathrm{y}, \mathrm{z})$. Explain any assumptions you have made ( 7 points)
(3c) Consider a point on the surface of the Earth at $\mathrm{z}=0$
The vertical electric field above and below the surface are $E_{z}^{1}$ and $E_{z}^{2}$ The electrical conductivity above and below the surface are $\sigma_{1}$ and $\sigma_{2}$ Derive a relationship between $E_{z}^{1}$ and $E_{z}^{2}$
(4 points)
(3d) Consider the case when $\sigma_{1}=0 \mathrm{~S} / \mathrm{m}$ and $\sigma_{2}=0.001 \mathrm{~S} / \mathrm{m}$
Show that for the TM mode, $B_{x}(\mathrm{y})$ at $\mathrm{z}=0$ is constant for all values of $y$
(4 points)

## Question 4 : Marine EM methods (Total = 8 points)



In a marine CSEM survey, the EM signals travel diffusively from transmitter (TX) to receiver (RX) by the two routes shown above. In the CSEM survey shown above the seawater depth was 400 m and the transmission frequency was 4 Hz
(4a) Estimate the TX-RX offset at which the two signals have the same strength?
(6 points)
Hint : Ignore geometric spreading, consider only attenuation.
(4b) Name two commonly used types of transmitter in seafloor CSEM
(2 points)

## Question 5 : Frequency domain EM methods (Total = 24 points)

(5a) A good conductor generates a secondary magnetic field that is in-phase with the primary magnetic field. Explain the physics of this statement in detail.

Your answer should include a phase diagram.
(6 points)
(5b) An EM survey uses horizontal co-planar transmitter and receiver loops as shown below. Consider the situation when the transmitter (TX) and receiver (RX) are located above a conductive dike.

Sketch the primary magnetic field $\left(\mathrm{H}^{\mathrm{P}}\right)$ and in-phase secondary magnetic field $\left(\mathrm{H}^{\mathrm{S}}\right)$ lines on the diagram below.
What will be the sign of $\frac{H^{S}}{H^{P}}$ at the receiver?

(5c) The figure on the next page shows three basement conductors.
Calculate the value of the in-phase and quadrature responses when the TX-RX system is located above each conductor. System operates at 3800 Hz ( 6 points)
(5d) Sketch the variation of the in-phase and quadrature responses along the profile on the diagram below ( 6 points)

$$
\begin{array}{lll}
\text { Response parameter, } & \mathrm{p}=\mu \omega \sigma \mathrm{WL} & \text { (constant on solid lines) } \\
\text { Depth parameter, } & \mathrm{D}=\mathrm{z} / \mathrm{L} & \text { (constant on dashed lines) }
\end{array}
$$

$\mathrm{W}=$ width of conductor
$\mathrm{L}=\mathrm{TX}-\mathrm{RX}$ separation, $\omega=\mathrm{TX}$ frequency $(\mathrm{rad} / \mathrm{s})$
$\sigma=$ conductivity of conductor
$z=$ depth to top of conductor





Page 7 of $\mathbf{1 0}$

## Question 6 : Time domain EM methods (Total = 12 points)

(6a) A GEOTEM survey is being conducted at a location where the Earth's magnetic field is horizontal and $\mathrm{B}_{\mathrm{E}}=30000 \mathrm{nT}$.

The $z$-axis receiver is in a towed bird that oscillates with an amplitude of $1^{\circ}$ at a frequency of 0.1 Hz . This receiver is a horizontal loop.
Calculate the amplitude of noise in $\frac{d B_{z}}{d t}$ caused by the oscillation (6 points)
(6b) The transmitter has a current $I=1000 \mathrm{amps}$, area $A=100 \mathrm{~m}^{2}$ and 5 turns.
The noise level is that computed in part (a). The Earth has a resistivity of $1000 \Omega \mathrm{~m}$. What is the latest time at which the transient can be observed? (4 points)
(6c) What depth of exploration does this represent?
(2 points)

## Question 7: Ground-penetrating radar (Total = 17 points)

GPR is being used to study the subsurface structure shown below.
A reflector is located at a depth $d$
Velocity in the Earth is $\mathrm{v}_{1}$ above the reflector and $\mathrm{v}_{2}$ below
Radar frequency is 200 MHz and TX-RX distance is x m The transmitter is fixed and the receiver is moved.

(7a) Derive an equation for the travel time of the air wave when the transmitter (TX) and receiver (RX) are separated by a distance $x$.
(7b) Derive an equation for the travel time of the direct wave travelling in the ground when the transmitter (TX) and receiver (RX) are separated by a distance $x$.
(7c) Derive an equation for the travel time for a reflected wave when the transmitter (TX) and receiver (RX) are separated by a distance $x$.
(3 points)
(7d) Sketch these two travel time curves on the figure above.
(3 points)
(7e) GPR travel times for the reflection are listed in the table below. Calculate $v_{l}$ and $d$
(5 points)

| $x(\mathrm{~m})$ | $\mathrm{T}(\mathrm{ns})$ |
| :--- | :--- |
| 1 | 34.36 |
| 2 | 37.27 |
| 3 | 41.67 |
| 4 | 47.14 |
| 5 | 53.36 |

(7f) Calculate the moisture content of layer $1\left(\theta_{\mathrm{v}}\right)$
(2 points)

