# Geophysics 424 A1 / 524 A1 Final exam 

Electromagnetic and Potential field methods
Date: December 202017 2- 5 pm
Location CCIS L1-029
Instructor : Dr. Martyn Unsworth
Time allowed : 3 hours
Total points $=115$

## Instructions

Attempt all questions
Notes and books may not be used
Non-programmable calculators may be used
Cell phones and all other electronic devices must be switched off and stored
All questions must be directed to the invigilator
A separate formula sheet will be distributed

## Name :

## Question 1 : Maxwell's Equations (Total = 28 pts)

- A plane EM wave is travelling vertically downwards in the Earth in the $z$-direction.
- The wave has an angular frequency, $\omega$, and time dependence $\mathrm{e}^{\mathrm{i} \omega t}$
- The electric field is polarized in the $x$-direction.
- At this location, the Earth has the following properties

$$
\begin{array}{lll}
\text { Magnetic permeability } & =\mu=\mu_{0} & =4 \pi \times 10^{-7} \mathrm{H} / \mathrm{m} \\
\text { Dielectric permittivity } & =\varepsilon=\varepsilon_{0} & =8.85 \times 10^{-12} \mathrm{~F} / \mathrm{m} \\
\text { Electrical conductivity } & =\sigma & =0.01 \mathrm{~S} / \mathrm{m} \\
\text { Speed of light } & =\mathrm{c} & =3 \times 10^{8} \mathrm{~m} / \mathrm{s}
\end{array}
$$

(1a) Expand Faraday's and Ampere's Law to give equations for the six components of the electric and magnetic fields. Simplify these equations for the case when the electric field is polarized in the x -direction.
( 6 points)
(1b) Show that Maxwell's equations reduce to an ordinary differential equation for $E_{x}$

$$
\begin{equation*}
\frac{d^{2} E_{x}(z)}{d z^{2}}=i \omega \sigma \mu E_{x}(z)-\omega^{2} \mu \varepsilon E_{x}(z) \tag{3points}
\end{equation*}
$$

(1c) Indicate the type of electric current that is represented by each term on the right hand side of this equation. At a frequency $f=10 \mathrm{~Hz}$, which term is larger? Simplify this equation by discarding the smaller term.
(5 points)
(1d) Find a solution to the equation in (1b) of the form $E_{x}=A e^{k z}$ where z increases positively into the Earth. Boundary conditions require that $E_{x}=E_{o}$ at $z=0 \mathrm{~m}$

Derive values for $A$ and $k$.
(6 points)
(1e) The skin depth $(\delta)$ is defined as the depth at which $\left|E_{x}\right|$ has fallen to $1 / \mathrm{e}$ its value at the surface. Derive an equation for the skin depth in terms of $\sigma$ and $\omega$
(4 points)
(1f) A radio station transmits with wavelength of 740 m . You are listening to the radio in your car when you enter a tunnel. The depth of the tunnel below the surface of the Earth slowly increases.

At what depth will you no longer be able to hear the radio? The ground has a resistivity of $100 \Omega \mathrm{~m}$. Reception ceases when the signal is reduced to $0.1 \%$ of the surface value.
(4 points)

## Question 2: Magnetotellurics (Total = 15 points)

An MT survey takes place at a mine where a mineralized zone is located at a depth of 1 km . The MT system used records data in the frequency band $10000-1 \mathrm{~Hz}$
(2a) Sketch the apparent resistivity curve that would be measured on the surface at point 'A'
(4 points)
(2b) The instrument was then deployed in a mine tunnel at a depth of 900 m .
Sketch the apparent resistivity curve that will be measured at 'B'.
(5 points)
(2c) What type of sensor is needed to measure the magnetic field in this MT survey?
(2 points)
(2d) What sample rate and recording time would be needed to collect MT data in this frequency range?
(4 points)



## Question 3: Magnetotellurics (Total = 10 points)

- An MT survey takes place in a location where the resistivity structure is 2-D
- A crustal conductor is buried at a depth of 4 km close to the coast.
- A profile of MT stations is shown by the black dots.

(3a) What is an induction vector? (2 points)
(3b) On the MAPS below, draw the direction and relative magnitude of the induction vectors at frequencies of 100 Hz and 1 Hz . ( 6 points)
(3c) State the convention used to plot the induction vectors ( 2 points)
MAP VIEW : INDUCTION VECTORS $\mathrm{f}=100 \mathrm{~Hz}$


MAP VIEW : INDUCTION VECTORS $\mathrm{f}=1 \mathrm{~Hz}$


## Question 4 : Frequency domain EM methods (Total = 12 pts)

A frequency domain EM survey is being used to investigate the near-surface conductivity structure at a construction site.

- The ground conductivity is the range $0.001-0.1 \mathrm{~S} / \mathrm{m}$
- The instrument TX-RX separation is 4 m
- Operating frequency is 1000 Hz
(4a) Is this instrument working in the near field or far field?
(3 points)
(4b) In the first location, the Earth has a uniform conductivity.


The instrument is carried 1 m above the surface. In vertical dipole mode, the instrument reads $0.05 \mathrm{~S} / \mathrm{m}$. Calculate the conductivity of the ground $\left(\sigma_{1}\right)$
(4c) In a second location, the Earth has a two layer structure, with an interface at a depth of 2 m . The instrument is placed on the ground


The instrument measures $0.0736 \mathrm{~S} / \mathrm{m}$ in vertical dipole mode and $0.0473 \mathrm{~S} / \mathrm{m}$ in the horizontal dipole mode. Calculate the conductivity of the two layers ( $\sigma_{1}$ and $\sigma_{2}$ )
(6 points)

## Question 5 : Marine CSEM methods (Total = 14 points)

A marine CSEM survey is using a horizontal electric dipole (HED) as a transmitter.

The seawater is very deep in the study area.
(5a) How does frequency domain marine controlled source EM exploration measure variations of resistivity with depth?
(3 points)
(5b) In a study area, the seafloor has a uniform porosity of $15 \%$ and the pore space is poorly connected. The seawater conductivity is $3 \mathrm{~S} / \mathrm{m}$. It is required to transmit EM signals from a seafloor transmitter to a seafloor receiver at a distance of 5 km . What transmission frequency is needed?
(5 points)
(5c) Explain the geometry of transmitters and receivers in the broadside and inline configurations. How are they used to detect hydrocarbon reservoirs?

Your answer should include diagrams of

- Map of transmitter and receivers on the seafloor
- the EM fields in the seafloor and
- the data measured at the seafloor
(6 points)


## Question 6: Time-domain EM (Total = 22 points)

A time-domain EM system is being used to measure the resistivity during a ground water survey. The system parameters are:

- Transmitter geometry
- Transmitter current
- Number of turns on transmitter
- TX-RX separation
- Noise level

Circular loop with area $100 \mathrm{~m}^{2}$
$\mathrm{I}=100 \mathrm{amps}$
$\mathrm{N}=10$
100 m
$0.01 \mathrm{nT} / \mathrm{s}$

The transient T1 was recorded with this system and is plotted on page 8. Selected data values are listed in the table.
(6a) Calculate the conductivity of the Earth. Explain any assumptions you make.
(4 points)
(6b) What is the maximum depth of investigation with this system? (3 points)
(6c) On the diagram on page 9 , draw the distribution of secondary electric current and secondary magnetic field at the three times listed in the figure
(9 points)
(6d) The transmitter current is increased to 1000 amps and another transient is recorded. Sketch this transient on the graph on page 8 and label it T2
(3 points)
(6e) The transmitter current is returned to 100 amps . The TX-RX separation is increased to 200 m . Sketch this transient on the graph on page 8 and label it T3
(3 points)


| Time $(\mathrm{s})$ | $\frac{d B_{Z}}{d t}(\mathrm{nT} / \mathrm{s})$ |
| :--- | :--- |
| 0.000001 | 716197 |
| 0.00001 | 629912 |
| 0.0001 | 16510 |
| 0.001 | 129.9 |
| 0.01 | 0.4456 |
| 0.1 | 0.001420 |

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## Question 7 : Ground-penetrating radar (Total = 14 points)



- GPR is being used to study the glacier shown above
- The radar system has a closely spaced transmitter and receiver system.
- Radar frequency is 200 MHz
- TX-RX distance is very small compared to the thickness of the ice.
- A rock is located at the base of the ice at $\mathrm{x}=0 \mathrm{~m}$ and acts as a diffractor
(7a) Derive an equation for the two-way travel time of the reflection from the bed of the glacier
( 2 points)
(7b) Derive an equation for the travel time of a signal diffracted from the rock. Assume that the TX and RX are coincident.
(3 points)
(7c) The figure below shows radar data collected on the glacier. Numerical values are also given in a table. Calculate $v_{l}$ and $d$
(7d) Estimate the velocity in the bedrock $\left(v_{2}\right)$
(3 points)


| $\mathrm{x}(\mathrm{m})$ | arrival 1(ns) | arrival 2(ns) |
| :---: | :---: | :---: |
| -300.000 | 2000 | 5385 |
| -200.000 | 2000 | 3887 |
| -100.000 | 2000 | 2603 |
| 0.000 | 2000 | 2000 |
| 100.000 | 2000 | 2603 |
| 200.000 | 2000 | 3887 |
| 300.000 | 2000 | 5385 |

