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A Defense of Cagniard's Magnetotelluric Method

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FOREWORD

"A defense of Cagniard's magnetotelluric method," a report prepared for the Office of Naval Research (ONR) in 1963, was part of a study of low-frequency electromagnetic fields, earth conductivity structures, and time series analysis methods undertaken in the Massachusetts Institute of Technology Department of Geology and Geophysics under the auspices of the Electronics Division of ONR. Arnold Shostak, the director of the Electronics Division was a strong supporter of our time series analysis studies. These studies also benefited from the pioneering work of another research group in our department directed by Enders Robinson and Steve Simpson. This report applied these methods to the analysis of low-frequency electromagnetic fields on the Earth's surface, and specifically answered questions raised about the validity of single station magnetotelluric data.

The report was not submitted for publication at the time of writing due to a communication breakdown. One author was in La Jolla while the other was involved in thesis studies. The apparent resistivity phases listed will probably appear strange to those more familiar with E/H phases. The E/H phase = (apparent resistivity phase — 90)/2.

ABSTRACT

The objections to the magnetotelluric method for determining the Earth's electrical conductivity structure are examined. These objections are based on the effect the signal structure can have on the measurements, and the inconsistency of some of the observations. Theoretical solutions to study the effect of finite horizontal wavelengths were obtained using recent estimates of the Earth's conductivity as a model. These results show that the usual magnetotelluric signals should not cause difficulties in the use of the method. Actual low-frequency magnetotelluric data from Weston, Massachusetts, are presented to show how severe inconsistencies in the data can be due to a neglect of the tensor nature of the magnetotelluric relationships. The consequences of these relationships are worked out.

INTRODUCTION

In 1953, Cagniard proposed a technique for determining the electrical conductivity structure of the Earth by means of observations of the horizontal electric and magnetic field fluctuations at the surface (Cagniard, 1953). In this method, the ground impedance is determined as a function of frequency by comparing the intensity of the electric and magnetic variations, and these impedances can be used to interpret the underlying conductivity structure.

Apparent resistivity equals

$$i\, E_i^2/\omega\mu\, H_j^2, \tag{1}$$

where E_i is a horizontal electric field component at the angular frequency ω and H_j is the horizontal magnetic field component at right angles to E_i. A very attractive feature of this method is the concept that simultaneous measurements have to be made at only one station at a time, which makes the method operationally simple.

Several more recent papers have objected to this magnetotelluric method of measuring the Earth's electrical properties (Wait, 1954; Price, 1962; Quon, 1963). These papers all dealt with the errors introduced when the horizontal wavelengths were not infinite in length, as is tacitly assumed in the straightforward application of Cagniard's method. This is an important consideration because, if the horizontal wavelengths affect the results, simultaneous measurements from several stations will be necessary to determine these wavelengths, and the simplicity of the method will be lost. The papers illustrated their theses using models with poor conducting, thick substratum. Such a model accentuates the difficulties, but is not a realistic model for the Earth. The results of these calculations, however, led to doubts about the magnetotelluric method, and were used as an explanation of apparent inconsistencies in the magnetotelluric observations. We contend that when the realities of the earth conductivity structure and the source geometries are taken into account, the question of the horizontal wavelengths is not very serious. On the other hand, the strong influence of nonhorizontal conductivity structures, such as the ocean-continent boundaries or the edges of sedimentary basins, may provide an explanation of the apparent inconsistencies in the observations and therefore present the major complication in the interpretation of the measurements. This complication does not invalidate the single station concept for making magnetotelluric measurements, but the station must be moved about to sample properly the horizontal changes caused by the conductivity structures.

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EFFECTS OF FINITE HORIZONTAL WAVELENGTHS

In order to study the effect of finite horizontal wavelengths on the E/H ratios it is important to use realistic conductivity models, because the important parameter is the ratio of the horizontal wavelength to the skin depth. For a homogeneous Earth, if we define the propagation constant as k, the E/H ratio becomes

$$E/H = \frac{\omega\mu}{k_z} \tag{2}$$

where $k^2 = k_z^2 + k_y^2 = i\mu\omega\sigma$, the horizontal wavelength equals $2\pi/ky$, and the skin depth is defined as equal to $\sqrt{2}/1k1$.

From equation (2) it is seen that when $k_y^2 \ll k^2$, the E/H ratio is insensitive to the horizontal wavelength, and thus, the conductivity value or the skin depth must be known to determine whether the horizontal wavelength is affecting the E/H ratio.

To study the influence of finite horizontal wavelengths on the E/H ratios at the Earth's surface, the electromagnetic wave equation for a wave with a horizontally polarized E field was solved using a horizontally stratified conductivity that modeled recent estimates of the Earth's conductivity structure. (McDonald, 1957; Cantwell, 1960) The numerical techniques used are reviewed in the Appendices. The model consisted of 100 layers to represent the crust and mantle, the top layer being 0.1 km thick and each successive layer 10 percent thicker than the layer above it. Two different cases were used: (1) represented an igneous area, and (2) represented a sedimentary area. Plots of these conductivity profiles are shown on a log-log scale in Figure 1. The results of the computations are given in Figures 2 and 3 for a range of horizontal wavelengths and frequencies. The lowest frequency represents a period of about one day, and the highest frequency is 100 cycles per second. The results for a horizontal wavelength of 99 000 km can be considered the infinite wavelength results. The apparent resistivity represents the resistivity a homogeneous half-space would need in order to give the same E/H ratio as the model at a particular frequency. From the results in Figures 2 and 3 we can determine the critical minimum wave length at which the estimated apparent resistivity deviates appreciably from the infinite wavelength case. These values, based on 20 percent deviation criteria, are listed in Table 1. These wavelengths are also the wavelengths at which the modes having horizontally polarized magnetic fields deviate in their E/H ratios from the modes having horizontally polarized electric fields, as shown in the Appendices.

For frequencies between 5 to 60 cycles per second most of the electromagnetic noise is due to the sferics energy which is trapped in the earth-ionosphere cavity (Balser and Wagner, 1962) These signals have wavelengths which are fractions of the Earth's circumference and far exceed 100 km. The lower frequency signals originate in the ionosphere either as magnetohydrodynamic waves or as current systems driven by some dynamo action. The wavelengths of the magnetohydrodynamic waves should be amply long, unless the waves were generated in the near vicinity of the magnetotelluric observing site. The worst cases can, therefore, be adequately represented by line sources in the ionosphere just above the observer. The maximum electron concentrations occur at heights of around 400 km and little conductivity exists below 100 km, so a reasonable lower limit for such current sources can be set at 150 km above the surface. A line source at a height Z produces a horizontal wavelength spectrum given by $e^{-\lambda z}$ where the horizontal wavelength $= 2\pi/\lambda$, so that there is an exponential cutoff of the wavelengths less than $2\pi z$. Using 150 km as the lower limit for Z, note from Table 1 that no trouble is expected with the wavelength structure of magnetotelluric signals above 10^{-3} cycles per second. Below 10^{-3} cycles per second we must consider the nature of the signal source more carefully. Fortunately, the diurnal variations and its harmonics, which represent important signals in this range, have wavelengths that are known to exceed the critical wavelengths, and also can be used for magnetotelluric studies. For frequencies lower than 10^{-5} cycles per second a spherical geometry should be used and the horizontal wavelength structure will be an important parameter in the analysis. These frequencies, however, appear to be below the practical range for magnetotelluric measurements.

EFFECT OF NONHORIZONTAL CONDUCTIVITY STRUCTURES

Despite the conclusions previously presented, often in practice magnetotelluric results do not give consistent answers, which appears to be evidence that the signals are not proper. When the Earth is horizontally stratified and

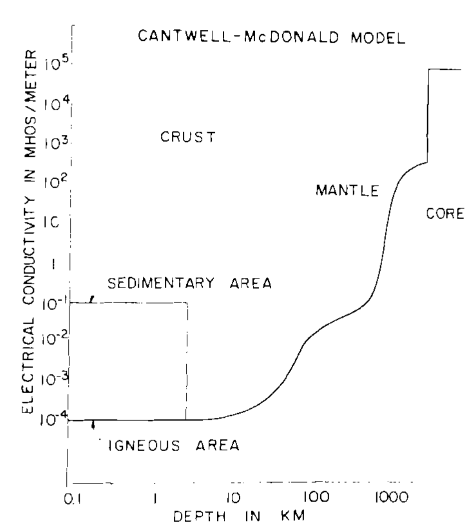

Fig. 1. Plots of Cantwell-McDonald model conductivity profiles on a log-log scale.

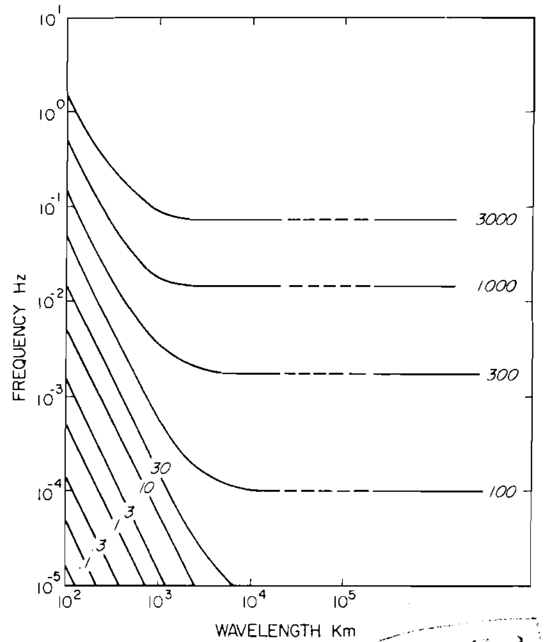

Figure 2

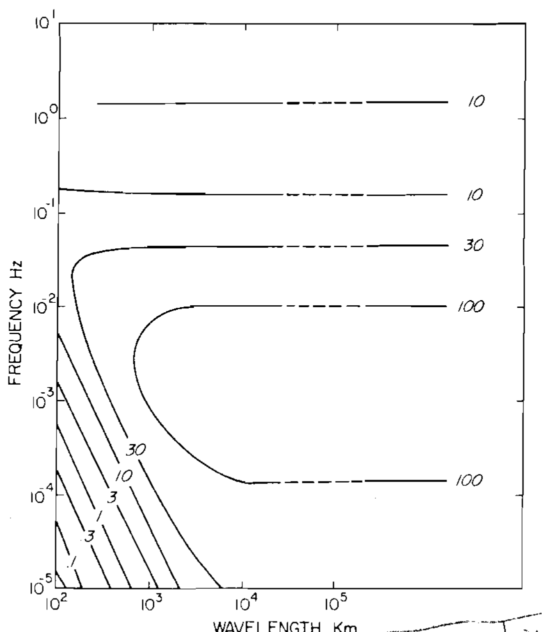

Figure 3

electrically isotropic, the ground impedance can be studied from measurements of one horizontal magnetic component and the electric component perpendicular to it. When the magnetotelluric signals are of sufficiently long wavelengths, the electric-magnetic relationships should be consistent, irrespective of the signal fluctuations. When the Earth's conductivity structure is no longer horizontally stratified, or if a strong electrical anisotropy exists, the electric field in one direction may reflect magnetic variations parallel to, as well as perpendicular to, its direction. The ground impedance in such a case must be represented by a second-order tensor (Cantwell, 1960; Bostick and Smith, 1962), and the neglect of this fact can produce apparent inconsistencies.

A good example of such a case is provided by some low-frequency magnetotelluric data collected in Weston, Massachusetts. This site is close enough to the continental margin that the electric currents in the Earth at frequencies below 10^{-3} cycles per second are flowing at right angles to the continental margin which is trending north-east. Magnetic variations in any direction will cause a NW-SE electric field, and, therefore, the electric field measured in some direction can be due to a magnetic variation in any direction. Under these conditions an electric field component is not expected to be consistently related to the magnetic field component at right angles to it. In fact, this particular data had little coherency between the electric north and the magnetic east components. In such a situation the E/H ratios observed have a strong dependence on the orientation of the magnetic fluctuation field, but when the tensor nature of the E/H relationships is taken into account, a consistent picture begins to emerge. The electric north data referred to above is actually over 90 percent predictable from the magnetic data, but both components must be considered.

In any given frequency band we must, in general, write for the horizontal components,

$$E_i = A_{ij} H_j . \tag{3}$$

Designating the cross-power spectral matrix as $(<E_i\bar{H}_j>)$ we have

$$(<E_i\bar{H}_j>) = A_{ik} (<H_k\bar{H}_j>) . \tag{4}$$

$$\therefore A_{ik} = (<E_i\bar{H}_j>)(<H_k\bar{H}_j>)^{-1} . \tag{5}$$

This can be expanded and rewritten in terms of coherencies and magnitudes where

$$\text{coh } E_iH_j = \frac{<E_i\bar{H}_j>}{(<E_i\bar{E}_i><H_j\bar{H}_j>)^{1/2}} , \tag{6}$$

and

$$|E_i| = (<E_i\bar{E}_i>)^{1/2} , \tag{7}$$

giving

$$A_{i1} = \frac{|E_i|}{|H_1|}\left(\frac{\text{coh } E_iH_1 - \text{coh } E_iH_2 \text{ coh } H_2H_1}{1 - |\text{coh } H_1H_2|^2}\right), \tag{8}$$

TABLE 1
Minimum wave length for Cagniard's magnetotelluric method

Freq. in cps	Cantwell-McDonald Model	Cantwell-McDonald Model with 3km sedimentary layer
10^{-5}	7 000 km	7 000 km
10^{-4}	3 000 km	3 000 km
10^{-3}	1 500 km	1 000 km
10^{-2}	1 000 km	500 km
10^{-1}	750 km	< 100 km
1	300 km	< 100 km
10	< 100 km	< 100 km
100	< 100 km	< 100 km

TABLE 2
Apparent resistivity values of Weston, Massachusetts low frequency magnetotelluric data

Freq. Hz	Based on raw E_i/H_j ratios E_N/H_E (Ω•m)	E_E/H_N (Ω•m)	Based on A_{ij} values $E_N/H_{E\ only}$ (Ω•m)	$E_E/H_{N\ only}$ (Ω•m)
.000 104	3 200	4 400	36	3 600
.000 122	4 100	4 200	50	3 600
.000 139	3 500	3 900	66	3 200
.000 156	3 700	4 800	110	3 900
.000 174	4 700	4 400	100	3 800
.000 191	4 000	4 000	98	3 500
.000 208	4 800	3 600	120	3 100
.000 226	4 400	3 400	280	2 800
.000 243	2 500	4 000	200	3 100
.000 260	3 200	4 700	79	3 200
.000 278	3 400	4 000	280	2 900
.000 295	2 900	3 500	110	2 500
.000 312	2 700	4 400	130	3 000
.000 330	2 600	5 400	28	3 300
.000 347	4 400	5 600	120	3 100
.000 365	4 300	5 700	2	3 400
.000 382	3 400	5 400	45	2 900
.000 400	2 600	4 600	150	2 500
.000 417	1 800	3 600	170	2 200
.000 434	1 000	3 500	64	2 400

and

$$A_{i2} = \frac{|E_i|}{|H_2|}\left(\frac{\text{coh } E_iH_2 - \text{coh } E_iH_1 \text{ coh } H_1H_2}{1 - |\text{coh } H_1H_2|^2}\right), \quad (9)$$

the A_{ij}'s are invariant to the changes in the orientation and polarization of the observed magnetotelluric fields, provided the horizontal wavelengths are adequate and provided the recording system is not introducing spurious noise. From this tensor the E/H ratio for any given magnetic field geometry is easily calculated. For instance, the E_1/H_2 ratio is equal to A_{12} when the magnetic field fluctuations are linearly polarized and in the H_2 direction. Table 2 shows the apparent resistivities computed from the observed E_i/H_j ratios and from the A_{ij} values. The last two columns are based on the predicted E/H ratios for a linearly polarized magnetic field.

The apparent resistivities associated with the E north-H east are very different for the two computations, the typical variation being a factor of 20:1, and yet both values can, in principle, be obtained from raw E_i/H_j ratios if the appropriate magnetic field orientation is occurring. The higher scatter in the A_{NE} values is to be expected, as the A_{NE} values are so small their determination is sensitive to the errors in the data.

The high apparent resistivities associated with the E east: H north are due to the high concentration of current flowing from the ocean which has not yet distributed itself to depths appropriate for the skin depths in the continental regions.

The low apparent resistivity values associated with A_{NE} are only slightly lower than the values predicted from the Cantwell-McDonald conductivity model as given in Table 1. We cannot use the Weston values to study the Earth's conductivity structure until a detailed solution appropriate to the ocean-continent geometry is worked out. The continental margin is far enough removed from the site for the ground impedance at higher frequencies to behave more nearly isotropically. At frequencies above 10^{-3} cycles per second the ocean effect rapidly disappeared.

It is interesting to note from the denominator of equations (8) and (9) that the value of A_{ij} is indeterminate if the signals are too highly coherent. This is in direct contrast to the natural inclination to accept as more valid those data where the consistency of the observed field leads to high coherencies and consistent ratios for the raw E_i/H_j signals. It does serve to emphasize, however, the importance of considering the tensor nature of these E/H relationships.

CONCLUSIONS

The magnetotelluric method suggested by Cagniard (1953) has not proved to be as useful as it first appeared, because results appear inconsistent. These inconsistencies could be due to source geometry or to nonhorizontal earth conductivity structure.

The source geometry is important, but it must be considered along with a realistic view of the Earth's vertical variation in electrical conductivity. When this is done, we show that the effects of source geometry, which would be evident in finite horizontal wavelengths, do not invalidate Cagniard's simple assumptions.

Nonhorizontal earth conductivity structures are evident on the Earth's surface, and can cause large variations in the observed E/H ratios recorded at different times from the same station. However, a simple tensor treatment can be used to find the time-invariant tensor properties which describe the E/H relationships. An example of such data from Weston, Massachusetts is studied, and the strong influence of the oceanic-continental slope boundary is clearly seen in the marked anisotropy of this tensor. In order to use this data to study conductivity structures it is necessary to consider magnetotelluric solutions for nonhorizontally layered structures, and to obtain adequate geographic coverage with the magnetotelluric measurements.

The Cagniard assumption of infinite horizontal wavelength is accurate enough to make single station data valid. A further conclusion is that nonhorizontal conductivity structures cause many of the apparent magnetotelluric inconsistencies and necessitate a tensor treatment of the data. It is recommended that methods of handling such earth conductivity models be undertaken to develop further the magnetotelluric method.

ACKNOWLEDGMENTS

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We also thank A. M. Dow of Holliston, A. Litchfield of South Acton, and J. Murray of Northboro, who made it possible for us to set up electrical ground probes on their properties.

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APPENDIX A
MAGNETOTELLURIC SOLUTIONS IN HORIZONTALLY LAYERED MEDIA

The magnetotelluric problem is one of a class of problems where the desired solution involves the ratio of two quantities which are interrelated by coupled first-order partial differential equations (Friedman, 1965; Eckhardt, 1961). From these coupled equations a new first-order differential equation can be derived to describe the spatial dependence of the desired ratio. Such an approach was used by Eckhardt in treating the magnetic induction problem for a spherical earth (Eckhardt, 1961). In the magnetotelluric problem, the ratio involves E/H, and the coupled first order partial differential equations are Maxwell's equations. A straightforward way of deriving the desired equation is to consider the solution in a homogeneous medium. For a horizontally polarized E field we can write

$$E = E_x\,(Ae^{ik_z z} + Be^{-ik_z z})\,e^{ik_y y}\,e^{-i\omega t}, \quad \text{(A-1)}$$

and

$$H = \frac{\nabla \times E}{1\mu\omega} \quad \therefore\ Hy = \frac{-k_z}{\omega\mu} (Ae^{ik_z z} - Be^{ik_z z})\, e^{ik_y y} e^{-i\omega t}, \quad \text{(A-2)}$$

defining the apparent resistivity as is done in the usual magnetotelluric method

$$\varrho_a = iE_x^2/\omega\mu H_y^2 = \frac{i\omega\mu\,(Ae^{ik_z z} + Be^{-ik_z z})^2}{k_z^2\,(Ae^{ik_z z} - Be^{-ik_z z})}. \quad \text{(A-3)}$$

From the continuity conditions, Ex/Hy is constant passing from one horizontal layer to the next. The parameter ϱ_a, therefore, from equation (A-3) must also be continuous across the boundary if μ is assumed constant. ϱ_a will vary, however, in passing through a layer, but if the layer is homogeneous, the differentiation of equations (A-3) is very simple and gives

$$\left(\frac{d\varrho_a}{d_z}\right)_{k_{\text{const.}}} = -\frac{2\omega\mu}{k_z} \left\{ \frac{(Ae^{ik_z z} + BE^{-ik_z z})\,(Ae^{ik_z z} + Be^{-ik_z z})^3}{(Ae^{ik_z z} - BE^{-ik_z z})\,(Ae^{ik_z z} - Be^{-ik_z z})^3} \right\}. \quad \text{(A-4)}$$

Using (A-3) this can be rewritten as

$$\left(\frac{d\varrho_a}{dZ}\right)_{k_{\text{const.}}} = -2\sqrt{\frac{\omega\mu}{i}}\,\varrho_a^{\,1/2}\left\{1 - \frac{k_z^2}{i\omega\mu}\,\varrho_a\right\} = -2\,\frac{\omega\mu}{i}\,\varrho_a^{\,1/2}\left\{1 - \frac{k^2 - k_y^2}{i\omega\mu}\,\varrho_a\right\}. \quad \text{(A-5)}$$

Since ϱ_a is continuous across the boundaries, equation (A-5) holds throughout the entire region, and in the limit of infinitely thin layers, the restriction k = constant can be forgotten.

For a horizontally polarized H field the solution is slightly different

$$H = Hy = (Ae^{ik_z z} + Be^{-ik_z z})\,e^{ik_x x}\,e^{-i\omega t}, \quad \text{(A-6)}$$

ignoring displacement currents,

$$E = \frac{1}{\sigma}\,\nabla \times H,\ E_x = \frac{ikz}{\sigma} (Ae^{ik_z z} - Be^{-ik_z z})\,e^{ik_x x}\,e^{-i\omega t}, \quad \text{(A-7)}$$

$$\varrho_a = \frac{ik_z^2}{\omega\mu\sigma^2}\,\frac{(Ae^{ik_z z} - Be^{-ik_z z})^2}{(Ae^{ik_z z} + Be^{-ik_z z})^2}, \quad \text{(A-8)}$$

and

$$\left(\frac{d\varrho_a}{dZ}\right)_{k_{\text{const.}}} = \frac{2k_z^2}{\omega\mu\sigma^2} \left\{ \frac{(Ae^{ik_z z} - Be^{-ik_z z})}{(Ae^{ik_z z} + Be^{-ik_z z})} - \frac{(Ae^{ik_z z} - Be^{-ik_z z})^3}{(Ae^{ik_z z} + Be^{-ik_z z})^3} \right\}. \quad \text{(A-9)}$$

From equations (A-8) and (A-9) we have

$$\left(\frac{d\varrho_a}{dZ}\right) = \frac{2k_z^2}{\sigma}\sqrt{\frac{1}{\omega\mu}}\,\varrho_a^{\,1/2}\left\{1 - \frac{i\omega\mu\sigma^2}{k_z^2}\,\varrho_a\right\}. \quad \text{(A-10)}$$

When the horizontal wavelengths are long compared to the skin depth $k_z^2 \cong k^2$ and both equations (A-10) and (A-5) reduce to

$$\left(\frac{d\varrho_a}{dZ}\right) \cong -2\sqrt{\frac{\omega\mu}{i}}\,\varrho_a^{\,1/2}\left\{1 - \sigma\varrho_a\right\} \quad \text{(A-11)}$$

and similar magnetotelluric results will be obtained for horizontal H fields as for horizontal E fields. It appears from the previous discussion that the usual magnetotelluric signals do allow this approximation.

In order to start the integration of equation (A-5), it is usual to assume an infinitely thick layer at some depth. In such a layer $\varrho_a = \frac{i\omega\mu}{k_z^2}$ = resistivity of the layer if $k_z^2 \cong k^2$.

For an accurate numerical integration of equation (A-5) from the starting depth to the surface we use $\triangle Z$ steps small enough so that $\triangle\varrho_a \ll \varrho_a$. This criterion involves two inequalities

$$k\triangle Z \ll 1, \quad \text{(A-12)}$$

and

$$\left(\frac{\varrho_a}{\varrho}\right)^{1/2} k\triangle Z \ll 1. \quad \text{(A-13)}$$

The second inequality is bothersome, because it is difficult to preset the $\triangle Z$ steps as ϱ_a is not known a priori. When σ increases with depth, however, $(\varrho_a/\varrho) \leq 1$ and only the first inequality need be considered.

An alternate approach to this problem is to consider the exact solution for $\triangle\varrho_a$ across a layer of constant ϱ. This is the essence of the multilayered approach to such boundary-value problems. By incorporating small $k\triangle Z$ values, however, it is possible to use expansions of the functional terms involved and reduce the problem to numerical integration again even though $(\varrho_a/\varrho)^{-1/2}k\triangle Z$ is not $\ll 1$.

The multilayered solutions are derived from the transfer properties of a single layer. For the horizontal E fields we have

$$\begin{bmatrix} E_{\text{top}} \\ H_{\text{top}} \end{bmatrix} = \begin{bmatrix} \cos(k_z\triangle z) & -\frac{i\mu\omega}{k_z}\sin(k_z\triangle_z) \\ -\frac{ik_z}{\mu\omega}\sin(k_z\triangle z) & \cos(k_z\triangle z) \end{bmatrix} \begin{bmatrix} E_{\text{bottom}} \\ H_{\text{bottom}} \end{bmatrix}. \quad \text{(A-14)}$$

Let

$$\frac{E_{\text{bottom}}}{H_{\text{bottom}}} \frac{k_z}{\mu\omega} = a,$$

and

$$\frac{E_{\text{top}}}{H_{\text{top}}} = \left(\frac{E_{\text{bottom}}}{H_{\text{bottom}}}\right)\left(\frac{\cos(k_z\triangle z) - \dfrac{i\sin(k_z\triangle z)}{a}}{\cos(k_z\triangle z) - ia\sin(k_z\triangle z)}\right). \quad \text{(A-15)}$$

For $k_z\triangle Z \ll 1$ we can expand the sine and cosine values giving

$$\triangle\left(\frac{E}{H}\right) \cong \left(\frac{E}{H}\right)\left(\frac{a - \dfrac{1}{a}}{1 - ik_z a\triangle z}\right) ik_z\triangle z. \quad \text{(A-16)}$$

With the use of digital computers there is no difficulty in using either approach, and the numerical results discussed previously were actually computed from equation (A-15).

APPENDIX B
LOW FREQUENCY TELLURIC STATION

The data used to study the tensor properties of the *E*-*H* relationship consisted of the magnetic recordings of the Weston College Geophysical Observatory, and electric recordings made by the M.I.T. Geophysics Laboratory. The electric recordings were made from measurements of the voltage fluctuations between three electrodes at South Acton, Northboro, and Holliston, Massachusetts. The electrodes were lead electrodes buried 5 ft below the surface to escape the temperature variations, and the voltage levels were fed directly to the recording site by all-metal telephone line connections. These signals were filtered with a .002 cycles per second low-pass two-term Butterworth filter and recorded on Rustrak recorders. Northboro was used as the reference point, so that the signals represented the pickup of 15-mile dipoles, one running northeast and one running southeast from Northboro, which is about 27 miles west of Boston. Weston, itself, could not be used as an electrode site as the Boston public transit system has a streetcar terminal nearby and sizable low-frequency signals were produced during the rush hours.

The data were digitized from the strip chart records by the Research Calculations Company.

APPENDIX C
RESULTS OF MACHINE COMPUTATIONS

The following computed results are included.

1. Figures C-1 thru C-4 are power density spectra plots for E_{north}, E_{east}, H_{north}, H_{east} in the frequency range. 174×10^{-4} to $.851 \times 10^{-3}$ Hz. (Note that "south" should read "east" in all titles.)
2. Figures C-5 thru C-8 are apparent resistivities and phases for the infinite wavelength case of the two Cantwell-McDonald models of Figure 1.
3. Figures C-9 thru C-14 are power density spectra and coherencies for *E* and *H*.