

Geophysics 699 - Magnetotelluric data processing

Assignment 2 : 2-D magnetotellurics

(1) Papers to read

d'Erceville, I., G. Kunetz, The effect of a fault on the Earth's electromagnetic field, *Geophysics*, **27**, 651-665, 1962.

Rankin, D., The magnetotelluric effect on a dike, *Geophysics*, 666-676, 1962.

(2) Finite difference approximation in 2-D

In this question we will develop the equations for a 2-D finite difference solution for the TE mode. You will solve :

$$\frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} + i\omega\mu\sigma(y,z)E_x = 0 \dots (1)$$

for $E_x(y,z)$. To keep things simple, we will start by considering a single frequency, and an Earth model with uniform conductivity (halfspace).

Set up a rectangular mesh with constant spacing in the y-direction (Δy) and z-direction (Δz).

Let the grid have n_y points in the y-direction and n_z in the z-direction.

Total number of nodes, $N = n_y \times n_z$

Number the nodes from top left to bottom right ($i = 1$ to N). Note that this is different to the way we did this in class with 2 indices.

Write a finite difference form of equation (1) for the electric field (E_x^i) at the i^{th} node. This will include the electric fields at the surrounding 4 nodes.

Will need to add a special case for nodes on each boundary.

Assemble the matrix A , where

$$\mathbf{A} \cdot \mathbf{E} = \mathbf{B}$$

and \mathbf{E} is a vector of the electric fields at each node.

Plot the matrix in MATLAB using the **pcolor** function. It should have a banded form. Can you explain the width of the band?

We will continue with this numerical solution in later assignments.

(3) Computation

In this question, we will use a 2D forward modelling code to investigate the accuracy of the quarter space model of d'Erceville and Kunetz (1962).

- (a) Design the mesh in Winglink. Use skin depth arguments to select size of smallest elements and overall mesh size. Consider both the highest and lowest frequencies.
- (b) Export the inversion files from Winglink. Perform a forward calculation using the external NLCG6 inversion (request 0 iterations).
- (c) Compare your apparent resistivity and phase values with those computed by d'Erceville and Kunetz (1962), Table 2 and which are listed on the next page. Note that these numbers are for two quarter spaces i.e. the contact between ρ_1 and ρ_2 extends to infinite depth.

In tables below, quarter space resistivities are ρ_1 and ρ_2 and $R = \frac{\rho_2}{\rho_1}$

$$d_1 = \sqrt{\frac{\rho_1}{2\pi\omega}} \text{ (skin depth in medium } \rho_1\text{); } \quad x = \text{distance from fault}$$

rho_ratio_4.mat

$\frac{x}{d_1\sqrt{2}}$	ρ_a	Φ
-2.0	0.50483	0.81004
-1.0	0.48314	0.85925
-0.6	0.45301	0.8839
-0.4	0.42683	0.89145
-0.2	0.3866	0.88425
0.0	0.3075	0.785
0.0	1.2299	0.785
0.1	1.182	0.76155
0.2	1.1531	0.75131
0.4	1.1127	0.74154
0.6	1.0847	0.73839
1.0	1.0483	0.74034

rho_ratio_9.mat

$\frac{x}{d_1\sqrt{2}}$	ρ_a	Φ
-2.0	0.33814	0.82926
-1.0	0.31367	0.90847
-0.6	0.28226	0.94939
-0.4	0.25573	0.96439
-0.2	0.21563	0.95836
0.0	0.14361	0.785
0.0	1.2545	0.785
0.1	1.2134	0.76288
0.2	1.1858	0.75259
0.4	1.146	0.74141
0.6	1.1173	0.73629
1.0	1.0777	0.73423
2.0	1.0277	0.74438

rho_ratio_39.mat

$\frac{x}{d_1\sqrt{2}}$	ρ_a	Φ
-2.0	0.16312	0.85944
-1.0	0.14462	0.97873
-0.6	0.12301	1.0449
-0.4	0.1054	1.0767
-0.2	0.07959	1.0917
0.0	0.03103	0.783
0.0	1.21234	0.785
0.2	1.1672	0.76345
0.4	1.1416	0.75513
0.6	1.1222	0.75041
1.0	1.094	0.74611

rho_ratio_100.mat

$\frac{x}{d_1\sqrt{2}}$	ρ_a	Φ
-2.0	0.102	0.8728
-1.0	0.08879	1.0085
-0.6	0.07387	1.0867
-0.4	0.06187	1.1286
-0.2	0.04441	1.1619
0.0	0.01656	0.795
0.0	1.1615	0.785
0.1	1.1467	0.77454
0.2	1.1355	0.76989
0.4	1.1183	0.764
0.6	1.1051	0.76044
1.0	1.0854	0.75674
2.0	1.0553	0.75512

(4) Optional reading for the enthusiasts

Rankin, D., G. D. Garland, K. Vozoff, An Analog Model for the Magnetotelluric Effect, *JGR*, **70**, 1939-1945, 1965.

Jones, F.W., A. Price, The perturbations of alternating geomagnetic fields by conductivity anomalies, *Geophys. J.R.A.S.*, **20**, 317-334, 1970.

Jones, F.W., A. Price, Geomagnetic effects of sloping and shelving discontinuities of Earth conductivity, *Geophysics*, **36**, 58-66, 1971.

Also see the discussion of the above paper in *Geophysics* **37**, 541, 1972.

L.R. Lines and F.W. Jones, The perturbation of Alternating geomagnetic fields by an island near a coastline, *Can. J. Earth Sci.*, 10, 510-518, 1973