

2D Magnetic Modeling: Implementation of a MATLAB based .exe software package

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Summary

This presentation will focus on the implementation of a newly developed MATLAB based .exe software package that is used for modeling 2D magnetic anomalies. This software will allow a user to model arbritray shaped 2D magnetic objects and contrast the calculated anomaly to that obtained from a magnetic profile.

Introduction

In order to properly interpret magnetic anomalies within a magnetic survey a clear understanding of the potential size and shape of their source(s) is required. While simple geometric shapes have relatively simple equations associated with them, modeling arbitrarily shaped magnetic sources efficiently requires extensive computation. Additional complications arise when numerous magnetic objects, each with its own unique magnetization, exist within a magnetic profile. To deal with the complications of real world data we have developed a new software tool that is based on re-derived formulas from Talwani (1964). Within this software package the user must define each magnetic object by its induced magnetization, remnant magnetization, and magnetic susceptibility. The formulas from Talwani (1964), slightly modified, allow for the proper calculation of the magnetic anomaly associated with any number of arbitrarily shaped objects.

Theory and/or Method

Consider that there exists a elemental volume contained within an irregularly shaped body. This elemental volume extends from negative to positive infinity in the y-direction. Bodies of irregular shapes can be approximated by a polygon, which can be reduced to solving semi-infinite two dimensional polygons (Talwani, 1964). Now consider a small volume element with dimensions dx, dy, dz (Figure 1A) and its properties.

The magnetic potential, Ω , at the origin is given by:

$$\Omega = \frac{\vec{m} \cdot \vec{R}}{4\pi R^3} \tag{1}$$

where m is the magnetic moment of the volume element and R is the distance from the origin (Fig. 1A).

The magnetic moment in terms of Cartesian coordinates x, y, z, can be written as:

$$\Omega = \frac{J_x x + J_y y + J_z z}{4\pi (x^2 + y^2 + z^2)^{1.5}} dxdydz$$
 (2)

Using the assumption that the body extends from negative infinity to positive infinity in the y-direction and integrate equation 2 with respect to y, the magnetic potential has the form:

$$\Omega = \int_{-\infty}^{\infty} \frac{J_x x + J_y y + J_z z}{4\pi (x^2 + y^2 + z^2)^{1.5}} dy = \frac{(J_x x + J_z z)}{2\pi (x^2 + z^2)} dx dz$$
 (3)

The vertical (V) and horizontal (H) components of the magnetic strength can be derived by differentiating equation (3) with respect to z and x respectively. If we assume the body extends to positive infinity we produce the following equations:

$$V = \int_{x}^{\infty} \frac{2J_{x}xz - J_{z}(x^{2} - z^{2})}{2\pi(x^{2} + z^{2})^{2}} dxdz = \frac{J_{x}z - J_{z}x}{2\pi(x^{2} + z^{2})} dz$$
 (4)

$$H = \int_{x}^{\infty} \frac{2J_{z}xz + J_{x}(x^{2} - z^{2})}{2\pi(x^{2} + z^{2})^{2}} dxdz = \frac{J_{x}x - J_{z}z}{2\pi(x^{2} + z^{2})} dz$$
 (5)

Equations (4) and (5) are the components produced by the rod KLMNK in Figure 1B. Integrating these equation from z_1 to z_2 , the magnetic field strength for the prism AFGBA in Figure 1B becomes:

$$V = \frac{1}{2\pi} \left(J_x Q - J_z P \right) \tag{6}$$

$$H = \frac{1}{2\pi} \left(J_z Q + J_x P \right) \tag{7}$$

where,

$$Q = \gamma_z^2 \ln \left(\frac{r_2}{r_1}\right) - \delta \gamma_z \gamma_x (\alpha_2 - \alpha_1), P = \gamma_z \gamma_x \ln \left(\frac{r_2}{r_1}\right) + \delta \gamma_z^2 (\alpha_2 - \alpha_1)$$

$$\gamma_z = \frac{z_{21}}{\sqrt{x_{21}^2 + z_{21}^2}}, \gamma_x = \frac{x_{21}}{\sqrt{x_{21}^2 + z_{21}^2}}, r_1 = \sqrt{x_1^2 + z_1^2}, r_{12} = \sqrt{x_2^2 + z_2^2}$$

$$\alpha_1 = \tan^{-1} \left(\frac{\delta(z_1 + gx_1)}{x_1 - gz_1} \right), \alpha_2 = \tan^{-1} \left(\frac{\delta(z_2 + gx_{z1})}{x_2 - gz_z} \right), g = \frac{x_2 - x_1}{z_2 - z_1} = \frac{x_{21}}{z_{21}}$$

$$\delta = 1$$
 if $x_1 > gz_1$, $\delta = -1$ if $x_1 < gz_1$

For a arbitrarily shaped polygon a point x_i , z_i represents a corner of the polygon and a point x_{i+1} , z_{i+1} to be the next nearest corner of the polygon. Equations 6 and 7 represent the magnetic strength of the rectangular region AFGBA for only one side of the polygon. For a polygon with n-sides there are a n number of prisms of the same form as AFGBA. By choosing the proper sign for each prism that comprise the polygon and summing their contribution of the magnetic field strength at the origin we can produce the magnetic anomaly for the entire polygon (AFGBA), at that point.

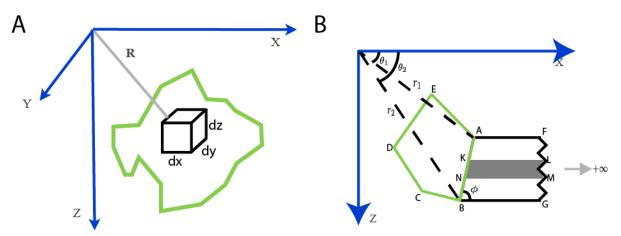


Figure 1: Modified from Talwani (1964) A) Shows a volume element for arbitrary shaped body b) KLMNK is a rod that extends to positive infinity. AFGBA is a semi infinite rectangular prism. ABCDEA is an arbitrary polygon.

Example

A simple example of how the program is implemented is shown below. Consider that we have a magnetic profile and we decide to model a transect shown by the black line in Figure 2A. The user will then choose appropriate magnetic properties for both the background (Background Tab) and magnetic object (Object Tab, Figure 2B). The user then draws a shape they think may be causing the anomaly (blue line). The user then clicks "Calculate" which produces the associated magnetic anomaly for the drawn shape (green line). The user then can compare the magnetic profile data (red line) with that of the user created anomaly. If the user is not satisfied with the result they can modify the shape or create additional shapes. Once the user is satisfied with the result all data and figures can be saved in a variety of formats (e.g. .xls, .png, .tiff, ect...).

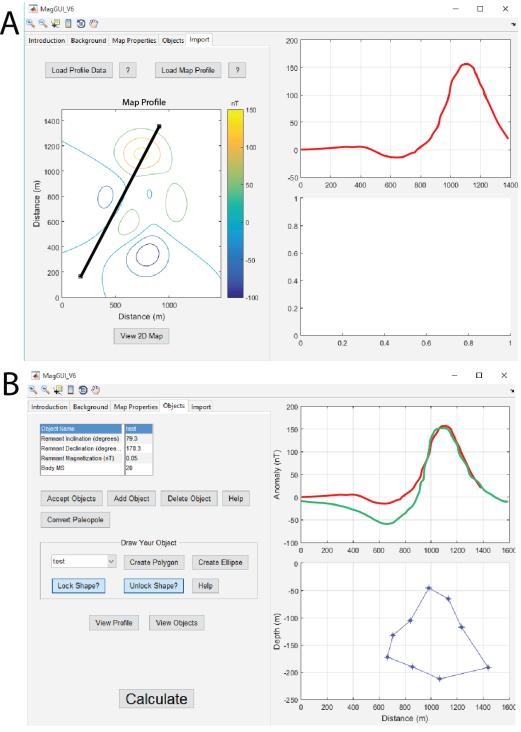


Figure 2: A) Import Profile Tab - User selects a magnetic profile which is then plotted in red B) Object Properties Tab - User creates an arbitrary shaped polygon (blue) and calculates its associated magnetic anomaly (green)

References

Talwani, M., Heirtzler, J. R., 1964. Computation of magnetic anomalies caused by two dimensional structures of arbitrary shape. *Computers in the mineral industries*, part 1: Stanford University publications, Geol. Sciences 9, 464-480.