Abstracts of AHAMC 2011

Triangle Meshes for Image Representation
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In the last several years, image representations based on adaptive (i.e., irregular) sampling have been receiving an increasing amount of attention from the research community. This is largely due to the fact that, in many applications, such representations have numerous advantages over those based on traditional lattice-based sampling. Some of the many applications that can benefit from adaptive sampling include: feature detection, pattern recognition, computer vision, restoration, tomographic reconstruction, filtering, and image/video coding. Amongst the many classes of image representations based on adaptive sampling, triangle meshes (especially those based on Delaunay triangulations) have proven to be highly effective. In order to employ a triangle-mesh representation of an image, a means is needed to construct such a representation given an arbitrary lattice-sampled image. Finding effective solutions to this mesh-generation problem is a challenging task, due to the conflicting goals of obtaining the highest quality mesh (i.e., with low approximation error) at minimal computational/memory cost. In this talk, the speaker will discuss the use of triangle meshes for image representation and consider primarily the problem of mesh generation.

Multiscale Data Sampling and Function Extension
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Many kernel based methods, which are used for dimensionality reduction and data mining applications, involve an application of a SVD to a kernel matrix, whose dimensions are proportional to the size of the data. When data is accumulated over time, a method for function extension is required. We introduce a multiscale scheme for data sampling and function extension, which can be applied in any metric space, not necessarily a vector space. The scheme is based on mutual distances between datapoints. It makes use of a coarse-to-fine hierarchy of the multiscale decomposition of a Gaussian kernel. It generates a sequence of subsamples, which we refer to as adaptive grids, and a sequence of approximations to a given empirical function on the data, as well as their extensions to any newly-arrived datapoint. The subsampling is done by a special decomposition of the associated Gaussian kernel matrix in each scale in the hierarchical decomposition. In each scale, the data is sampled by an interpolative decomposition of a low-rank Gaussian kernel matrix that is defined on the data.
Reconstruction from Magnitudes of Frame Coefficients

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In this paper we present an algorithm for signal reconstruction from absolute value of frame coefficients. Then we compare its performance to the Cramer-Rao Lower Bound (CRLB) at high signal-to-noise ratio. To fix notations, assume \( \{f_i; 1 \leq i \leq m\} \) is a spanning set (hence frame) in \( \mathbb{R}^n \). Given noisy measurements \( d_i = |\langle x, f_i \rangle|^2 + \nu_i, 1 \leq i \leq m \), the problem is to recover \( x \in \mathbb{R}^n \) up to a global sign. In this paper the reconstruction algorithm solves a regularized least squares criterion of the form

\[
I(x) = \sum_{i=1}^{m} ||\langle x, f_i \rangle|^2 - d_i|^2 + \lambda \|x\|^2
\]

This criterion is modified in the following way: 1) the vector \( x \) is replaced by a \( n \times r \) matrix \( L \); 2) the criterion is augmented to allow an iterative procedure. Once the matrix \( L \) has been obtained, an estimate for \( x \) is obtained through an SDV factorization.

Constructive Digital Harmonic Analysis

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Construction: Constant Amplitude Zero Autocorrelation (CAZAC) sequences are a staple for waveform design of the type used in radar and communications theory. We construct number theoretic CAZACs whose discrete narrow-band ambiguity functions have optimal behavior to effect best possible localization.

Algorithm: Using our frame potential energy theory, a meaningful and effective classification algorithm is constructed for multi-spectral and hyper-spectral imagery data.

Theory: Linear and non-linear sampling formulas are fundamental in harmonic analysis. The linear theory, i.e., the Classical Sampling Formula, goes back to Cauchy and was applied extensively in the first half of the 20th century by the likes of Hadamard, Wiener, Shannon, et al. We extend this in the context of Beurling’s balayage theorem, which we generalize in terms of parameterization of the space of Radon measures. The non-linear approach is formulated deterministically for sparse sets of coefficients in signal representation by means of recursive quantization.

The Construction is a collaboration with Joseph Woodworth; the Algorithm is a collaboration with Wojciech Czaja; the Linear Theory is a collaboration with Enrico Au-Yeung; the Non-linear Theory is a reformulation of results with Onur Oktay, which themselves are inspired by work with Powell and Yilmaz.
Approximations and Fast Algorithms for Green’s Functions

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Multiresolution methods in high dimensions use separated representation of Green’s functions to obtain efficient fast algorithms designed to yield any finite accuracy requested by the user. This talk provides a brief overview of the approach with an emphasis on approximations used in the construction of convolution kernels.

We first consider separated representations of non-oscillatory free-space Green’s functions via a near optimal linear combination of decaying Gaussians. We then extend the approach to Green’s functions satisfying Dirichlet, Neumann or mixed boundary conditions on simple domains. We also briefly elucidate some delicate theoretical issues related to the construction of periodic Green’s functions for Poisson’s equation.

For oscillatory Green’s functions, we split the application of the kernel between the spatial and the Fourier domains. In the spatial domain we use a near optimal linear combination of decaying Gaussians with positive coefficients and, in the Fourier domain, a multiplication by a band-limited kernel obtained by using new quadratures appropriate for the singularity in the Fourier domain. Applying this approach to the free space and the quasi-periodic Green’s functions, as well as those with Dirichlet, Neumann or mixed boundary conditions on simple domains, we obtain fast algorithms in dimensions two and three for computing volumetric integrals.

We also briefly describe some extensions to non-convolution kernels.

Sparse Tree Approximation in High Dimensions

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In this talk, we will discuss methods of approximation in high dimensions based on adaptive partitions. The information about the function $f : X \to Y$ to be approximated comes from point cloud data $Z := \{z^i = (x^i, y^i)\}_{i=1}^N$, where $x^i \in X$ and $y^i \in Y$. We want to determine a procedure $\tilde{f}$ that for any query $x \in X$ finds an approximation $y = \tilde{f}(x)$ to $f(x)$. The analysis of the point cloud $Z$ uses a special organization of the data called sparse occupancy trees and its complexity is only mildly dependent on the dimensionality of $X$. We present results in terms of Learning Theory considering both regression and classification setups.

Improve the Results of Gaussian Integration Points and Weights with High Precision Degree of Gauss Quadrature Rules

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This paper concentrates to improve the integration rules in which both the accuracy and efficiency is assured. In order to improve the accuracy and efficiency it is found that the abscissae and corresponding weights are needed to be evaluated correctly. In this paper
Gaussian integration points are calculated from the algebraic equations generated by Legendre polynomials. Main effort is involved in this part and in the next, calculation of corresponding weights are followed. Manual calculations of weights are impossible for higher order formulae and hence a program is written and weights are calculated and tested with their property and found correct. Finally, software in FORTRAN is developed and then the formulae are demonstrated through several authors. Thus, we believe that the development of the numerical integration rules in this paper will provide the way of exact evaluation of integrals encountered in continuum mechanics problems for the analysis of real world problems accurately with less computational effort.

Affine and Quasi-affine Frames for Rational Dilations

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Quasi-affine systems were originally introduced by Ron and Shen [J. Funct. Anal. 148 (1997), 408–447] for integer, expansive dilations. In this talk we extend the definition of quasi-affine systems to the class of rational, expansive dilations. Friday, June 17, 2011 at 12:56 pm

We show that an affine system is a frame if and only if the corresponding family of quasi-affine systems are frames with uniform frame bounds. We also prove a similar equivalence result between pairs of dual affine frames and dual quasi-affine frames. Finally, we uncover some fundamental differences between the integer and rational settings by exhibiting an example of a quasi-affine frame such that its affine counterpart is not a frame. This talk is based on a joint work with Jakob Lemvig.

Mapping Tensor-product Wavelet Bases and GPGPU-computing of DWTs

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An algorithm for computation of multivariate wavelet transforms (DWTs) on graphics processing units (GPUs) was proposed by some of the authors in Dechevsky, L.T., Gundersen, J., Bang, B., Computing n-Variate Orthogonal Discrete Wavelet Transforms on Graphics Processing Units, in: I. Lirkov, S. Margenov, and J. Wasniewski (Eds.) LSSC’2009, LNCS 5910, Springer-Verlag, Berlin-Heidelberg, 2010, 730–737. This algorithm was based on mapping the indices of orthonormal tensor-product wavelet bases of different number of variables and a tradeoff between the number of variables versus the resolution level, so that the resulting wavelet bases of different number of variables are with different resolution, but the overall dimension of the bases is the same. In the above-said paper, the algorithm was developed only up to mapping of the indices of blocks of wavelet basis functions. This was sufficient to prove the consistency of the algorithm, but not enough for the mapping of the individual basis functions in the bases needed for a programming implementation of the algorithm. In the present communication we upgrade this construction by passing from block-matrix index mapping on to the detailed index mapping of the individual basis functions. We consider some examples illustrating the results of the algorithm. The present work is part of the research of the R&D Group for Mathematical Modelling, Numerical Simulation and Computer Visualization at Narvik University College within two consecutive Strategic Projects of the Norwegian Research Council – ‘GPGPU (General Purpose computation on Graphics Processing Units (GPGPU): http://gpgpu.org/) – Graphics Hardware as a High-end Computational Resource’ (2004-2007) and ‘Heterogeneous Computing’ (2008-2010).
Efficient Computation of Oscillatory Integrals with Local Fourier Bases

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Computation of oscillatory integrals is required in the numerical solution of wave equations, as well as in many applications including reflection seismology, curvilinear tomography, seismic imaging. In this talk, I will discuss sparse representation and efficient computation of oscillatory kernels and oscillatory integrals using adaptive multiscale local Fourier bases.

Approximation of Functions on Unknown Manifolds defined by High-Dimensional Unstructured Data

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With the recent rapid technological advancement and significantly lower manufacturing cost in such areas as image sensor and capture, satellite and medical imaging, memory devices, computing power, convenient internet access, low-cost wireless communication, and powerful search engines, the tremendously huge amount of data information to be processed and understood is over-whelming. One of the most popular current approaches to this problem is to represent each piece of information as a point in a high-dimensional Euclidean space $\mathbb{R}^s$ and consider the collection of such points as a point-cloud $\mathcal{P}$ that lies on some unknown manifold $\mathbb{X} \subset \mathbb{R}^s$. For example, in application to photo library organization and image search engine, each point in the point-cloud in $\mathbb{R}^s$ represents a digital image thumbnail, with the dimension $s$ being the maximum resolution of the image collection. In general, when some pieces of information are only partially available or corrupted, or when the point-cloud is too large to handle, a subset $\mathcal{C} \subset \mathcal{P}$ of reliable data, called training set, is used to process $\mathcal{P}$.

Although the manifold $\mathbb{X}$ is unknown, whatever information available from the point-cloud $\mathcal{P}$ can be used to determine $\mathbb{X}$ through some symmetric positive semi-definite kernel $K$ defined by the dataset. However, it is usually not economical or even not feasible to compute the spectral decomposition of $K$ for a large point-cloud. To overcome this obstacle, we developed a class of randomized algorithms for computing the "anisotropic transformation" of the dataset to re-organize the data without the need of computing the eigenvalues directly. The transformed dataset then provides a hierarchal structure for manifold dimensionality reduction, while preserving data topology and geometry. On the other hand, to apply this manifold approach to such application areas as pattern recognition, time series event prediction, and recovery of corrupted or missing data values, certain appropriate functions of choice defined only on some desired training sets $\mathcal{C}$ must be extended to the entire unknown manifold $\mathbb{X}$. We will discuss how data geometry can be incorporated with spatial approximation to solve this extension problem. In particular, we present a point-cloud interpolation formula that provides near-optimal degree of approximation to the (unknown) target functions.

The notion of anisotropic transformation and its corresponding randomized algorithms were introduced and developed in a joint paper with Jianzhong Wang that appeared in the recently founded Springer "International Journal of Geomathematics (GEM)" in 2010; and the results on the extension of functions from training sets to the entire (unknown) manifold represent a joint work with Hrushikesh Mhaskar.
Moduli of Smoothness and Approximation on the Unit Sphere
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This is a joint work with Yuan Xu. A new modulus of smoothness based on the Euler angles is introduced on the unit sphere and is shown to satisfy all the usual characteristic properties of moduli of smoothness, including direct and inverse theorem for the best approximation by polynomials and its equivalence to a $K$-functional, defined via partial derivatives in Euler angles. Examples are given to show our new moduli of smoothness are computable.

Smoothness and Best Approximation on the Sphere
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The rate of best approximation of functions on the unit sphere in $\mathbb{R}^d$ by spherical harmonic polynomials of degree $n$ using different norms or quasinorms is related to various measures of smoothness. Recent results comparing classical and new moduli of smoothness on the sphere are described.

Shearlet Ginzburg-Landau Energy, Anisotropic Analogues, Associated Operators and Applications
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The authors prove the $\Gamma$-convergence of a shearlet-adapted Ginzburg-Landau(-type) functional to a multiple of the $TV$ seminorm. The design of the functional was inspired by the diffuse-interface wavelet Ginzburg-Landau(GL) energy introduced by J.D. in collaboration with A.Bertozzi. The shearlet GL energy provides the isotropy that the wavelet GL lacks. It opens a new perspective on the implementation and utilization of the TV-related methods.

The generalized notions of the direction-adaptive functionals (primarily related to variational image processing) based on the energies of a similar class are considered. The problem of recovering a weighted-TV-like functional corresponding to any given Wulff shape is addressed. The discussion is enriched with particular examples of modeling such energies using shearlets and wavelets. Basic examples of the above applications are illustrated with numerical experiment data; current/future research directions are outlined.
Palindromic Matrices of Order Two and Three-Point Subdivision Schemes

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We introduce a family of three-point subdivision schemes related to palindromic pairs of matrices of order 2. We apply the Mößner Theorem on palindromic matrices to the $C^0$ convergence of these subdivision schemes. We study the Hölder regularity of their limit functions. The Hölder exponent which is found in the regular case is optimal for most limit functions. In the singular case, the modulus of continuity of the limit functions is of order $\delta \log \delta$. These results can be used for studying the $C^1$ convergence of the Merrien family of Hermite subdivision schemes.

Convolution Inequalities Associated with Irregular Gabor and Wavelet Systems

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We consider certain convolution inequalities for translation-bounded, positive measures on $\mathbb{R}^n$ and the $a x + b$ group. We show how such inequalities are related to the notion of Beurling density in Euclidean spaces and how they can be used to define a corresponding notion of density for the $a x + b$ group. Applications of these results to irregular Gabor and wavelet systems will be given.

Interpolatory Subdivision Schemes with Fractal Limit Curves

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We consider a one-parameter class of interpolatory subdivision schemes, and provide parameter intervals on which subdivision convergence is guaranteed. It is then shown that, for parameter values inside certain sub-intervals of the convergence intervals, fractal subdivision curves are obtained. Graphical illustration are provided.
Ambiguity Functions, Sampling and Uncertainty Principles in Parameter Estimation

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The standard approach for joint estimation of time delay and Doppler shift of a signal is to estimate the point at which the cross ambiguity function of the original and modified signals attains its maximum modulus. We shall present a fast and accurate method on band-limited signals for this parameter estimation problem. The method acts on approximated signals obtained from discrete samples and given by truncated Shannon series. It uses Newton’s method to estimate the time delay and Doppler shift by calculating a point at which the cross ambiguity function of the approximated signals attains its maximum modulus. Numerical experiments demonstrated that the method generally outperformed other methods in estimating both time delay and Doppler shift. We shall also discuss extension of the ideas and setup to a Hilbert space context, and introduce the notion of a generalized cross ambiguity function. This extension unifies various problems of interest, including the joint estimation of time delay and time scale in wideband signal processing. Under the abstract generalization, error bounds for estimating the parameters are derived, and we will reveal a connection between these bounds and a new type of uncertainty principle. The estimation error is shown to lie in an ellipsoidal region and the uncertainty principle gives an upper bound to the size of the region. This is based on joint work with Tim N. T. Goodman and Fuchun Shang.

Sparse Evolution of Functions Using Diffusion Wavelets

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Its almost been 20 years since we are using wavelet theory for numerical solutions of PDEs. It is well known fact that many nonlinear PDEs arising in real world have solutions containing local phenomena (e.g. formation of shocks) and interaction between several scales (e.g. Turbulence). Such solutions can often be well represented in wavelet spaces because of the nice properties of wavelets like compact support, vanishing moments etc. Traditionally, wavelet construction is in unbounded or periodic domains but its application for solving partial differential equations (PDEs) on general manifold is still in infancy stage. Diffusion wavelets (invented by Coifman et al. at Yale university) introduced the multiresolution geometric construction for the efficient computation of high powers of local diffusion operators, which have high powers with low numerical rank. Classes of operators satisfying these conditions include discretization of differential operators, in any dimension, on manifolds, and in non-homogeneous media. The construction yields multiresolution scaling functions and wavelets on domains, manifolds, graphs and other general classes of metric spaces. The aim of this paper is to obtain sparse representations of functions which is essential for compression and is a first step towards solving PDEs using diffusion wavelet.
An Alternative Approach to Designing Clinical Trials: Budgeted Learning of Effective Classifiers

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Researchers often use clinical trials to collect the data needed to evaluate some hypothesis, or produce a classifier. During this process, they have to pay the cost of performing each test. Many studies will run a comprehensive battery of tests on each subject, for as many subjects as their budget will allow — ie, "round robin" (RR). We consider a more general model, where the researcher can sequentially decide which single test to perform on which specific individual; again subject to spending only the available funds. Our goal here is to use these funds most effectively, to collect the data that allows us to learn the most accurate classifier.

We first explore the simplified "em coins version" of this task. After observing that this is NP-hard, we consider a range of heuristic algorithms, both standard and novel, and observe that our "biased robin" approach is both efficient and much more effective than most other approaches, including the standard RR approach. We then apply these ideas to learning a naive-bayes classifier, and see similar behavior. Finally, we consider the most realistic model, where both the researcher gathering data to build the classifier, and the user (eg, physician) applying this classifier to an instance (patient) must pay for the features used — eg, the researcher has $10,000 to acquire the feature values needed to produce an optimal $30/patient classifier. Again, we see that our novel approaches are almost always much more effective that the standard RR model.

See http://webdocs.cs.ualberta.ca/ greiner/RESEARCH/BudgetedLearning/. This is joint work with Aloak Kapoor, Dan Lizotte and Omid Madani.

An Empirical Feature-based Learning Algorithm Producing Sparse Approximations

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A learning algorithm for regression is studied. It is a modified kernel projection machine in the form of a least square regularization scheme with $\ell^1$-regularizer in a data dependent hypothesis space based on empirical features (constructed by a reproducing kernel and the learning data). The algorithm has three advantages. First, it does not involve any optimization process. Second, it produces sparse representations with respect to empirical features under a mild condition, without assuming sparsity in terms of any basis or system. Third, the output function converges to the regression function in the reproducing kernel Hilbert space at a satisfactory rate. Our error analysis does not require any sparsity assumption about the underlying regression function.
High-Performance Time-Warped Multiscale Signal Reconstruction

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Multiscale methods are becoming increasingly interesting as computational scientists look to model interesting physical phenomenon that span vast ranges of spatiotemporal scales. Often, these technique model inherently-connected phenomenon at separate scales in order to achieve a computationally more efficient solution than solving the full fine scale problem. In other cases such as turbulent flows, self-similarity across scales is exploited to produce more computationally-efficient models.

At the core of these methods is some notion of scale which facilitates representation of ranges of spatiotemporal scales. Typical notions of scale used in simulations are based on the assumption of periodic functions or spatial homogeneity of solutions, represented in terms of signal-independent basis functions (Fourier, wavelets, etc.). In contrast, time-warped signal processing uses the signal itself in order to guide basis construction. This allows for a signal-dependent notion of scale that has nice properties with regards to nonstationary and aperiodic signals.

In this talk, we explore applying the time-warping idea to the Fourier basis and experiment with performance and quality concerns with regards to multiscale reconstruction. We show that signals can be reconstructed using time-warped convolution techniques with higher quality than classical convolution reconstruction, with a only a relatively small computational penalty. Moreover, we explore how standard Fourier-based signal tools can be interpreted and computed in the time-warped context and analyze the high-performance computing properties. Potential applications of this framework include multiscale visualization and simulation as well as various computational photography and image processing problems.

Frequency-based Directional Framelets

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In this talk, we discuss some recent theoretical developments on wavelets and framelets. First, we introduce the notion of frequency-based basic framelets. Next, we provide a complete characterization for frequency-based basic framelets and discuss their connections to wavelet/framelet filter banks. We shall see that there is a one-one correspondence between a wavelet/framelet filter bank and a frequency-based basic framelet. We provide examples of bivariate directional framelets. Some connections to shearlets and curvelets will be discussed. Related papers are available at http://www.ualberta.ca/~bhan
Wavelets Centered on a Knot Sequence
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We develop a general notion of orthogonal wavelets ‘centered’ on an irregular knot sequence and present two families of orthogonal wavelets that are continuous and piecewise polynomial. As an application, we construct piecewise quadratic, orthogonal wavelet bases on the quasicrystal lattice consisting of the $\tau$-integers where $\tau$ is the golden-mean.

Numerical Techniques for Approximating Fourier Integral Operators
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Here we discuss recent and well known methods for approximating Fourier Integral Operators and a subset thereof called Pseudo-Differential Operators. Recent theoretical and computational work has resulted in methods which are accurate, robust, and fast. Gabor Multipliers can exactly represent linear operators, and these computational methods can be used to approximate them. The butterfly algorithm has been adapted to be massively parallel and to handle general amplitude and phase functions. A brief discussion of these methods is given, followed by applications to medical and seismic imaging, and solving linear partial differential equations.

Modulation Spaces, BMO, BLT, and Rectangular Partial Sums
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The modulation spaces quantify the time-frequency concentration of functions and distributions. We relate the modulation spaces, the space BMO of functions with bounded mean oscillation, and the Balian–Low Theorem. The embedding of modulation spaces into VMO (the space of functions of vanishing mean oscillation) is seen to lie behind the essential limitation of the time-frequency localization of Gabor systems that form Riesz bases. We also prove that a type of Balian–Low Theorem holds for Gabor Schauder bases, which raises interesting questions about the convergence of rectangular partial sums of Fourier series in higher dimensions.

This talk is based on joint work with Alex Powell (Vanderbilt University) and Ramazan Tinaztepe (Alabama A&M University).
Wavelet packets and wavelet frame packets on local fields
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Using a prime element of a local field $K$ of positive characteristic $p$, the concepts of
multiresolution analysis (MRA) and wavelet can be generalized to such a field. We prove a
version of the splitting lemma for this setup and using this lemma we have constructed the
wavelet packets associated with such MRAs. We show that these wavelet packets generate
an orthonormal basis by translations only. We also prove an analogue of splitting lemma for
frames and construct the wavelet frame packets in this setting.

Non-negative Subdivision and Non-homogeneous Markov Chains
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Recent work by X. L. Zhou, see [4] and the references there, has settled a long-standing
question of characterizing convergence of non-negative, univariate subdivision schemes. We
relate some of these results to methods used in the analysis of non-homogeneous Markov
chains. In particular, the convergence result in [1] is a strong and so far less known basic
theorem, from which convergence of non-negative subdivision can be characterized.

We will develop the main ideas and proofs following this approach through properties of
stochastic matrices, and of products of families of such matrices. In particular, the notion of
SIA matrices (coined by Wolfowitz [3]) is useful and applicable to multivariate non-negative
subdivision when the mask is finitely supported. In this case, an ’ergodic coefficient’ as
introduced by Hajnal [2] leads to a contraction argument for families of SIA matrices as they
appear in subdivision when looking at the corresponding submasks of the subdivision scheme.
This is joint work with my PhD student Xianjun Li.

References:
14 (1958), 733–737.
[4] X.-L. Zhou, Positivity of refinable functions defined by nonnegative masks, Appl. Com-
put. Harmonic Analysis 27 (2009), 133–156.
Explicit Iteration Schemes of the Uzawa Algorithm for Minimization Problems Arising from Image Processing

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Recently the Uzawa algorithm has been widely used in image processing and compressive sampling. In this talk we indicate that many apparently different algorithms can be viewed as variations of the Uzawa algorithm. We discuss convergence of the Uzawa algorithm. In particular, we propose explicit iteration schemes based on matrix splitting. When the matrix splitting is done by the symmetric Gauss-Seidel method, we establish convergence of the scheme with no restriction on the step size of the iteration.

Bi-frames with 6-fold Axial Symmetry for Hexagonal Data and Triangle Surface Multiresolution Processing

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In this talk we will discuss the construction of highly symmetric affine bi-frames for hexagonal data/image and triangle surface multiresolution processing. Compared with the conventionally used square lattice, the hexagonal lattice has several advantages, including that it has higher symmetry. It is desirable that the filter banks for hexagonal data also have high symmetry which is pertinent to the symmetric structure of the hexagonal lattice. While in the field of CAGD, when the filter banks are used for surface multiresolution processing, it is required that the corresponding decomposition and reconstruction algorithms for regular vertices have high symmetry so that these algorithms could be used to process surfaces with extraordinary vertices.

In this talk, we will show that the 6-fold axial symmetry is the desired symmetry which the filter banks and bi-frames should possess when they are used for hexagonal data and triangle surface multiresolution processing. We will also discuss the construction of bi-frames with such a symmetry. The multiresolution algorithms of the constructed b-frames are given by templates (stencils) so that it is easy to implement them for surface processing applications such as surface progressive transmission, sparse representation and surface shrinkage.

Multidimensional A/D Conversion and Directional Bias

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Digital Filters for $d$-dimensional signals are generated as the fourier transforms of square-integrable $\mathbb{Z}^d$-periodic functions. Depending on the variability of the decay rate of the filter’s Fourier transform, which we call directional bias of the filter, the reconstruction errors of an input signal $f$ may vary. This variation also depends on the rotations of the input image. We study the effects on the truncation error $E_N(f) = \inf_I \{ \| f - \sigma_I(f) \| : I \subset \mathbb{Z}^d, |I| \leq N \}$ of the directional distribution of the decay of $\hat{\varphi}$, where $\sigma_I(f) = \sum_{n \in I} \langle f, \tau_n \varphi \rangle \tau_n \varphi$, with $\tau_n g(x) = g(x - n)$. 

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We sketch how ideas are from engineering of signals (e.g., frequency bands) recently having seen a renascence in computational harmonic analysis. They involve such mathematical tools as operator theory, representation theory, infinite-dimensional groups, spectral analysis, and approximation theory.

We sketch the use of recursive input-output models, and filters, adapted in mathematics where this has not been common fare until recently. One of our motivations is the desire to extend and refine these methods as they are used in wavelet algorithms, or more generally in multi-scale resolutions.

In the simplest cases, this takes the form of operator valued functions (generating function) of a complex variable. And in many applications, it is possible to encode data as vectors in a Hilbert space $H$, and to do this in such a way that a finite selection of bands will then correspond to a cascading system of closed subspaces in $H$.

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**An Introduction to Synchrosqueezed Transforms**

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This talk reviews some recent developments in empirical harmonic analysis (Daubechies et al. 2011) which propose a “synchrosqueezed” approach for sharpening the resolution of a time-frequency representation $R(t,f)$ by considering the behavior of its local gradient. We then examine variations on this approach and show results which demonstrate its effectiveness at resolving temporally co-located heart valve closure sounds collected by electronic stethoscope.

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**Numerical Approximations and Stability Analysis of 1D Heat Equation**

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Abstract: The present investigation deals with the solution of the 1D diffusion equations. We choose a thin, laterally insulated bar in which heat is constrained to flow along the bars axis and no heat can enter or leave through the lateral surface. A solution of the initial-boundary value problem is obtained against along the x-axis on the entire surface of the bar $[0, L]$. Following methods are used to solve the equation such as (i) Analytically (ii) Crank-Nicholson Implicit Method (C-N) (iii) Explicit Forward-Difference Method (FTCS) and (iv) Implicit Backward-Difference Method (BTCS). The results are shown graphically as well as tabulated form in different methods. Finally, Von-Neumann stability analysis has considered whether the methods are conditionally or unconditionally stable.
On Construction of Symmetric MRA-based Frame-like Wavelet Systems

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Let $M$ be a matrix dilation, $m = |\det M|$. Mixed Extension Principle is a well known general scheme for the construction of dual wavelet frames. However, generally speaking, this scheme leads to MRA-based dual wavelet systems $\{\psi^{(\nu)}_{ik}\}, \{\tilde{\psi}^{(\nu)}_{ik}\}$ which are not necessary frames in $L_2(R^d)$ and may even consist of tempered distributions. Some properties of these systems such as frame-type expansion (with convergence in different senses) and their approximation order were investigated in [1]. The wavelet systems satisfying these properties were called frame-like.

For an arbitrary matrix dilation, any integer $n$ and any integer/semi-integer $s$, we give an explicit method for the construction of frame-like wavelet systems providing approximation order $n$ such that all wavelet functions $\{\psi^{(\nu)}\}, \{\tilde{\psi}^{(\nu)}\}$ are symmetric/antisymmetric with respect to the point $S = (M - I)^{-1}s$. For some class of $2 \times 2$ dilation matrices, given integer $n$, we construct frame-like wavelet systems with approximation order $n$ and axis symmetry/antisymmetry of all wavelet functions.

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Image Inpainting and Sparse Approximation

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One main problem in data processing is the reconstruction of missing data. In the situation of image data, this task is typically termed image inpainting. Recently, inspiring algorithms using sparse approximations and $\ell_1$ minimization have been developed and have, for instance, been applied to seismic images. The main idea is to carefully select a representation system which sparsely approximates the governing features of the original image – curvilinear structures in case of seismic data. The algorithm then computes an image, which coincides with the known part of the corrupted image, by minimizing the $\ell_1$ norm of the representation coefficients.

In this talk, we will develop a mathematical framework to analyze why these algorithms succeed and how accurate inpainting can be achieved. We will first present a general theoretical approach. Then we will focus on the situation of images governed by curvilinear structures, in which case shearlets will serve as the chosen representation system. Using the previously developed general theory and methods from microlocal analysis, under certain conditions on the size of the missing parts we will prove that such images can be arbitrarily well reconstructed.
Appell Sequences from Scaling Functions

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We consider Appell sequences of polynomials defined by generating functions
\[ \frac{e^{xz}}{\phi(iz)} = \sum_{m=0}^{\infty} \frac{P_m(x)}{m!} z^m, \]
where \( \phi \) is a scaling function. The Appell sequences exhibit some properties reminiscent of Hermite polynomials, which are generated by \( \frac{e^{xz}}{G(iz)} \), where \( G \) is the Gaussian function. For a class of compactly supported sequences of scaling functions that approximate the Gaussian the corresponding Appell sequences of polynomials converge to the Hermite polynomials. The Appell sequence corresponding to a scaling function provides a representation for the recovery of functions from their wavelet transforms with derivatives of the scaling function as the mother wavelets.

Some Results on Low-Rank Matrix Recovery

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In this talk, I shall investigate low-rank matrix recovery problem. Some characterization conditions are obtained to guarantee that a matrix with rank at most \( r \) can be recovered exactly by nuclear norm minimization.

Weighted \( \ell_1 \) Minimization: Support Recovery Guarantees and an Iterative Algorithm

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We study the support recovery conditions of weighted \( \ell_1 \) minimization for signal reconstruction from compressed sensing measurements given multiple support estimate sets with different accuracy. We also propose a sparse recovery algorithm based on iterative weighted \( \ell_1 \) minimization that utilizes multiple weighting sets. We demonstrate experimentally that the proposed algorithm succeeds in recovering sparse signals when standard \( \ell_1 \) minimization fails. Moreover, we demonstrate through extensive experiments that the performance of the proposed algorithm is similar to that of the iterative re-weighted \( \ell_1 \) algorithm proposed by Candés, Wakin, and Boyd. Finally, we identify signal classes for which, given a partial support estimate, weighted \( \ell_1 \) minimization is guaranteed to result in a new support estimate with improved accuracy.

Joint work with Ozgur Yilmaz and Michael Friedlander.
Some New Bounds of the R.I.C. in Compressed Sensing

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The restricted isometry constant (R.I.C.) is important in compressed sensing. In this talk, I shall report our recent results on some new bounds of the R.I.C. First we shall give a new bound $\delta_{2s} < 0.4931$ as a sufficient condition for the $\ell_1$-minimization problem to have an $s$-sparse solution. Next, we shall give a new bound $\delta_{s+1} < 1/(\sqrt{s} + 1)$ as a sufficient condition for the orthogonal matching pursuit algorithm to recover each $s$-sparse signal in $s$ iterations. Also, we shall use an example to show that bound is very tight. This example positively verifies the conjecture given by Dai and Milenkovic in 2009. These results are joint works with Song Li and Yi Shen.

Bivariate Empirical Mode Decomposition (BEMD) Based Data Adaptive Approach to EOG Suppression from EEG Signals

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A problem of eye-movement muscular interference removal from electroencephalogram (EEG) recordings is described. In many experiments in neuroscience it is crucial to separate different sources of electrical activity within human body in a situation when a very limited knowledge about nonlinear and nonstationary nature of the mixing process is available. A new two step extension to bivariate empirical mode decomposition (BEMD) is proposed to remove ocular artifacts from EEG. The fractional Gaussian noise (fGn) is used as a reference first to preprocess electrooculogram (EOG) signal, which is next used in the second step as a reference to clean EEG signals. Results with EEG experimental data validate the proposed approach.

System description: This paper focuses on removal of eye movement related artifacts, which carry significant power in from of EOG contaminating much lower in power EEG. To tackle these problems, the current study proposes to use BEMD, a new technique to decompose pairs of signals for which one is introduced as a reference. The aim of this paper is to introduce a novel computational framework based on BEMD, convenient for simultaneous data conditioning and information separation for neurophysiological signals with known interference sources, in particular, to separate eye movements from EEG signals.

BEMD: The Empirical Mode Decomposition (EMD) is a signal processing decomposition technique that decomposes the signal into waveforms modulated in both amplitude and frequency by extracting all of the oscillatory modes embedded in the signal. The waveforms extracted by EMD are named Intrinsic Mode Functions (IMF). Each IMF is symmetric and it is assumed to yield a meaningful local frequency traces. Different IMFs do not exhibit the same frequency at the same time. The traditional EMD is only suitable for univariate signals. The Complex Empirical Mode Decomposition (Complex-EMD) is an extension of the basic EMD suitable for dealing with complex signal representations. The motivation to extend EMD was that a large number of signal processing applications have complex signal representation. In addition, this extension could be applied on both the real and imaginary parts simultaneously because complex signals have a mutual dependence between the real and imaginary parts. With separate decomposition, the mutual dependency is lost. The BEMD
is a more generalized extension of the EMD to complex signals. It decomposes two variables (EEG and fGn) simultaneously based on their rotating properties.

Conclusion: The data-driven adaptive method proposed in this paper allowed us to separate successfully EOG interference from multiple channel EEG recordings where a mixing model was not trivial. The two steps procedure allowed us first to filter out EOG from interfering electrical and in general environmental noise which was modeled with fractional Gaussian noise in form of a reference for first stage BEMD. Such a ”purified” EOG was later used as a reference in a second stage BEMD pairwise with all EEG channels separately preventing any data leakage or crosstalk among them, which is of high importance in biomedical applications.

Multiple Kernel Learning for Scale-Space based Feature Selection
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Scale-space representation of an image is a significant way to generate features for classification. To select only the useful scales in the image scale-space, a framework of Multiple Kernel Learning (MKL) is also proposed in the problem of large lump detection from oil sand. Presence of an undetected large lump can jam crushers and cause undesired production downtime in oil sands mining. Towards detecting large lumps by an automated image analysis method, a novel data-dependent kernel function based on image scale-space features is designed. We utilize a 1-norm support vector machine (SVM) in the MKL optimization problem for sparse selection of scales from the image scale-space. The optimized data-dependent kernel accommodates only a few scales that are most discriminatory according to the large margin principle. With a 2-norm SVM this learned kernel is applied to detect large lumps in oil sand videos. We tested our method on three challenging oil sand data sets. Our method yields encouraging results on these difficult-to-process images.

On Stationary and Nonstationary Biorthogonal Wavelets
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Basis and approximation properties of stationary and nonstationary biorthogonal compactly supported wavelets in different function spaces are considered. In particular stationary biorthogonal compactly supported wavelets preserving localization with the growth of smoothness is examined. Two examples of infinitely differentiable nonstationary biorthogonal compactly supported wavelets are investigated also.

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Constructing Parametric Surfaces

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Choice of atlas, in the form of patch layout, and change of variables, in the form of reparameterization across patches, make the construction of parametric free-form surfaces for design an interesting and challenging problem even before determining the shape. This talk will focus on recent insights into preferred patch layout, least reparameterizations, least degree of spline representations and progress in design with rational splines.

Sparse Data Representation by the Easy Path Wavelet Transform

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In the talk, recent methods for adaptive data representation will be summarized with a focus on the Easy Path Wavelet Transform (EPWT) and its generalizations. The EPWT has recently been proposed as a tool for sparse representations of bivariate functions from discrete data, in particular from image data. The EPWT is a locally adaptive wavelet transform. It works along pathways through the array of function values, and it exploits the local correlations of the given data in a simple appropriate manner. In particular, we show that the EPWT leads, for a suitable choice of the pathways, to optimal $N$-term approximations for piecewise Hölder smooth functions with singularities along curves.

The results have been obtained jointly with Armin Iske (Hamburg), Daniela Roșca (Cluj-Napoca), and Stefanie Tenorth (Göttingen).

Root-Exponential Accuracy for Coarse Quantization of Finite Frame Expansions

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In this talk, we show that by quantizing the $N$-dimensional frame coefficients of signals in $\mathbb{R}^d$ using higher order Sigma-Delta quantization schemes, it is possible to achieve root-exponential accuracy in the oversampling rate $\lambda := N/d$. In particular, we construct a family of finite frames tailored specifically for Sigma-Delta quantization that admit themselves as both canonical duals and Sobolev duals. Our construction allows for error guarantees that behave as $e^{-c\sqrt{\lambda}}$, where under a mild restriction on the oversampling rate, the constants are absolute. Moreover, we show that harmonic frames can be used to achieve the same guarantees, but with the constants now depending on $d$ and that random frames achieve similar, albeit slightly weaker, guarantees (with high probability). Time permitting, we will discuss stability to non-quantization noise.

This is joint work with, in part with F. Krahmer and R. Ward, and in part with S. Güntürk and Ö. Yılmaz.
Automated feature discovery is a fundamental problem in data analysis. Although classical feature discovery methods do not guarantee optimal solutions in general, it has been recently noted that certain subspace learning and sparse coding problems can be solved efficiently, provided the number of features is not restricted beforehand. I will discuss an extended characterization of this optimality result and describe the nature of the solutions under an expanded set of practical conditions. In particular, I will demonstrate how the framework can be applied to "semi-supervised" prediction, and demonstrate that feature discovery can co-occur with input reconstruction and supervised training while admitting globally optimal solutions. A comparison to existing semi-supervised feature discovery methods shows improved generalization and efficiency.

Joint work with Xinhua Zhang, Yaoliang Yu, Martha White and Ruitong Huang.

Sparse Signal Representation and the Tunable Q-factor Wavelet Transform

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For the sparse wavelet representation of a signal, the Q-factor of the wavelet transform should be chosen so as to match the signal's oscillatory behavior. This talk describes a new wavelet transform, the 'tunable Q-factor wavelet transform' (TQWT), for which the Q-factor is continuously tunable. Therefore, the wavelet can be chosen according to the oscillatory behavior of the signal, so as to enhance the sparsity of a sparse representation. The TQWT is well suited for iterative algorithms for sparse representation as it is a fully-discrete tight frame which can be efficiently implemented using radix-2 FFTs. Sparse TQWT representations obtained by $\ell_1$-norm minimization will be shown. [The Q-factor of a waveform is defined as the ratio of its center frequency to its bandwidth.]

Automatic Removal of Ocular Artifacts from EEG Signals Using S-transform

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Artifacts in electroencephalograph (EEG) signals are unwanted but unavoidable disturbances in the recording process due to several reasons such as eye movements, body movements, power line interference etc. Among them the ocular artifacts are most significant. These are characterized by high amplitude but have overlapping frequency band with the useful signal. Hence, it is difficult to remove the ocular artifacts by traditional filtering methods.
This paper proposes a new approach of artifact removal using $S$-transform (ST). It provides an instantaneous time-frequency representation of a time-varying signal and generates high magnitude $S$-coefficients at the instances of abrupt changes in the signal. A threshold function has been defined in $S$-domain to detect the artifact zone in the signal. The artifact has been attenuated by a suitable multiplying factor. The major advantage of ST-filtering is that the artifacts may be removed within a narrow time-window, while preserving the frequency information at all other time points. It also preserves the absolutely referenced phase information of the signal after the removal of artifacts. Finally, a comparative study with wavelet transform (WT) demonstrates the effectiveness of the proposed approach.

MRA Based Wavelet Frame and Applications
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One of the major driving forces in the area of applied and computational harmonic analysis during the last two decades is the development and the analysis of redundant systems that produce sparse approximations for classes of functions of interest. Such redundant systems include wavelet frames, ridgelets, curvelets and shearlets, to name a few. This talk focuses on tight wavelet frames that are derived from multiresolution analysis and their applications in imaging.

The pillar of this theory is the unitary extension principle and its various generalizations, hence we will first give a brief survey on the development of extension principles.

The extension principles allow for systematic constructions of wavelet frames that can be tailored to, and effectively used in, various problems in imaging science. We will discuss some of these applications of wavelet frames. The discussion will include frame-based image analysis and restorations, image inpainting, image denosing, image deblurring and blind deblurring, image decomposition, segmentation and CT image reconstruction.

Concentration Estimates for Learning with $\ell^1$-Regularizer and Data Dependent Hypothesis Spaces
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We consider the regression problem by learning with a regularization scheme in a data dependent hypothesis space and $\ell^1$-regularizer. The data dependence nature of the kernel-based hypothesis space provides flexibility for the learning algorithm. The regularization scheme is essentially different from the standard one in a reproducing kernel Hilbert space: the kernel is not necessarily symmetric or positive semi-definite and the regularizer is the $\ell^1$-norm of a function expansion involving samples. The differences lead to additional difficulty in the error analysis. We apply concentration techniques with $\ell^2$-empirical covering numbers to improve the learning rates for the algorithm. Sparsity of the algorithm is studied based on our error analysis. We also show that a function space involved in the error analysis induced by the $\ell^1$-regularizer and non-symmetric kernel has nice behaviors in terms of the $\ell^2$-empirical covering numbers of its unit ball.
Resolution of the Gibbs Phenomenon

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The expansion of an analytic nonperiodic function on a finite interval in a Fourier series leads to spurious oscillations at the interval boundaries referred to as the Gibbs phenomenon. The present paper summarizes a new method for the resolution of the Gibbs phenomenon [1] which follows on the reconstruction method of Gottlieb and coworkers [2,3] based on Gegenbauer polynomials orthogonal with respect to weight function $$(1-x^2)^{\lambda-1/2}$$. We refer to their approach as the direct method and to the new methodology as the inverse method. Both methods use the finite set of Fourier coefficients of some given function as input data in the re-expansion of the function in Gegenbauer polynomials or in other orthogonal basis sets. The finite partial sum of the new expansion provides a spectrally accurate approximation to the function [4]. In the direct method, this requires that certain conditions are met concerning the parameter $\lambda$ in the weight function, the number of Fourier coefficients, $N$ and the number of Gegenbauer polynomials, $m$. We show that the new inverse method can give exact results for polynomials independent of $\lambda$ and with $m = N$. The paper presents several numerical examples applied to a single domain or to subdomains of the main domain so as to illustrate the inverse method in comparison with the direct method.


A Note about Non Stationary Multiresolution Analysis

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An orthonormal basis of wavelets of $L^2(\mathbb{R})$ is an orthonormal basis of $L^2(\mathbb{R})$ of type

$$\psi_{j,k} = 2^{j/2} \psi(2^j \cdot -k), \quad j, k \in \mathbb{Z}.$$  

A classical method to obtain such bases consists in constructing a multiresolution analysis. When the mother wavelet $\psi$ depends on the scale (i.e. the index $j$), a non stationary version of multiresolution analysis is then used. We generalize different characterizations of orthonormal bases of wavelets to the non stationary case (as main reference for the stationary case, we used results presented in “A First Course of Wavelets” of E. Hernández and G. Weiss).
Approximation by Wavelet and Scaling Expansions

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We study MRA-based compactly supported wavelets which are redundant representation systems but not frames. Frame-type expansions with respect to such wavelet systems (with convergence in different senses) and their approximation order are investigated. The advantage of these systems is in the simplicity of their construction. The wavelet functions should not have vanishing moments, which is necessary for frames. Starting with any appropriate scaling function or scaling mask one can find a dual scaling mask and all wavelet masks by explicit formulas for matrix extension. Also we investigate scaling expansions for band-limited functions, in particular, the classical sampling theorem is extended to a wide class of functions.

Scales in Vision and Immunology

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We will suggest an alternate perspective on multiscale computing with the goal of constructing a “good kernel”.

Factoring Banded Matrices and Matrix Polynomials

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Banded matrices with banded inverses can be factored into tridiagonal matrices with tridiagonal inverses. These matrices are rare but useful – wavelet matrices and ”CMV matrices” are leading examples. (The analysis and synthesis filters are all FIR, even for time-varying filters.) The number of tridiagonal factors depends on the bandwidth and not on the matrix size.

When those matrices are block Toeplitz, with submatrices repeating down each (block) diagonal, all the information is in the matrix polynomial with those submatrices as coefficients. Suppose its determinant is 1. Then we look for linear factors with $det = 1$. This is a start on factoring more general doubly infinite matrices. I will look at the ordinary $A = LU$ (or $A = LPU$) factorization for which the usual elimination process has no reasonable place to start.
Wiener’s Lemma and Two Nonlinear Sampling Problems in Signal Processing

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Wiener’s lemma occurs in many fields of mathematics and engineering, such as functional analysis, harmonic analysis, signal processing, and numerical analysis. In this talk, I will discuss Wiener’s lemma for infinite matrices, and consider its nonlinear extension and applications to nonlinear sampling problems in signal processing.

Wavelets with Crystal Symmetry Shifts

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We introduce the basics for a theory of wavelets on $\mathbb{R}^n$ where the usual lattice of shifts is replaced by a more general noncommutative group and the dilation matrix must be compatible with the shift group. Our motivation is to provide a context which includes much of the interesting work done recently on wavelets with composite dilations, but also admits those crystal symmetry groups of shifts which cannot be covered by the composite dilation theory. We include a number of examples on $\mathbb{R}^2$.

On the Different Kinds of Quasi-interpolation

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To analyze the data stemming from the application, one should find the underlying function via mathematical method such as interpolation or approximation. From the theory of the approximation, we know that the choice of the basis should be data dependent. The quasi-interpolation is a typical method to find the simple basis as well as the approximant to approximate the underlying function. The classical quasi-interpolation are almost all based on the data of function values on grid. In this talk we will introduce the quasi-interpolation for scattered date as well as for linear functional data. Then the quasi-interpolation can be used in the numerical solution of PDEs and different kinds of data analysis.
Noise-shaping Quantizers for Compressed Sensing
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Recent advances have established compressed sensing as an effective sampling theory for acquisition of high dimensional sparse vectors from few linear measurements. The vast majority of the results in the literature assume that the compressive samples are real numbers, and a comprehensive quantization theory for compressed sensing has been missing. In this talk, we show that we can successfully employ noise-shaping Sigma-Delta quantizers for compressed sensing. We prove that, by adopting a two-stage approach involving the use of appropriate Sobolev dual frames in the reconstruction, Sigma-Delta quantizers utilize the inherent redundancy of compressed sensing more efficiently than "any" round-off type quantization algorithm, at least in the case of Gaussian measurement matrices. This framework is especially successful if the underlying signals are exactly sparse. We will then consider the cases when there is measurement noise (in addition to the quantization error) and when the signal to be acquired is not strictly sparse, but compressible. We will introduce alternative reconstruction methods that effectively handle this more general setup.

This is joint work with R. Saab, S. Gunturk, and in part with M. Lammers and A. Powell.

A Survey of Subdivision Algorithms of Manifold-Valued Data
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In this talk, I will first survey a number of different ways to adapt a given linear subdivision rule to manifold-valued data. Of particular interest is the so-called single base-point scheme, which shows up in the construction of wavelet-like transform for manifold-valued data. I shall present a number of smoothness and approximation order results of these various subdivision schemes.

Mathematical Modeling and Multilevel Computation of Dispersed Drug Release from Swellable and Erodible Polymeric Matrix Systems
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Many drugs possessing low to mediate water solubility would experience gradual dissolution in modified release matrix systems. Thus drug release kinetics may be controlled by both dissolution and diffusion. A mathematical model of drug release from swellable and erodible matrix systems with initial drug loading higher than solubility was developed. The model was verified by the existing exact solution with the assumption of dissolution much faster than diffusion as a special case. Multilevel methods were introduced to solve the governing system of diffusion equations in order to achieve better approximation with lower computational costs.
Interpolation of Missing Values in Times Series Based on its Periodicity

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In this talk, a method to deal with missing data in times series is introduced. The application of information from frequency domain to the estimation of missing values is the guideline of this method. The actual data and simulation data are used to evaluate the efficiency of filling, and to explore the necessary conditions of the process. The research involves time-domain information extraction, frequency-domain information extraction, simulation, periodicity-weighted method, and evaluation to effectiveness of filling. We found that, in randomly missing pattern, periodicity method and spline method are both good, and the values of RMSE and NRMSE are both small; while in continuously missing pattern, periodicity method is more accurate than spline method.

Error Analysis and Sparsity of Some Learning Algorithms

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Learning theory studies learning function relations or data structures from samples. In this talk we shall first introduce mathematical analysis of some learning algorithms generated by regularization schemes in reproducing kernel Hilbert spaces. Then we shall discuss two classes of kernel-based learning algorithms which produce sparse approximations for regression. The first class is of kernel projection machine type and generated by least squares regularization schemes with $\ell^q$-regularizer ($0 < q \leq 1$) in a data dependent hypothesis space based on empirical features (constructed by reproducing kernels and samples). The second class is spectral algorithms associated with high-pass filter functions. Learning rates and sparsity estimations will be provided based on properties of the kernel, the regression function, and the probability measure.
Iterative Filtering, EMD and Instantaneous Frequency Analysis

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The empirical mode decomposition (EMD) was a method pioneered by Huang et al as an alternative technique to the traditional Fourier and wavelet techniques for studying signals. It decomposes a signal into several components called intrinsic mode functions (IMF), which have shown to admit better behaved instantaneous frequencies via Hilbert transforms. In this talk we present our recent progress on an alternative algorithm for EMD based on iterating certain filters.

This approach yields similar results as the more traditional sifting algorithm for EMD. In some cases the convergence can be rigorously proved. The method is highly data dependent. It is designed to complement the classical Fourier transform or wavelets to better treat non-stationary, nonlinear processes commonly used in practice. It returns adaptive sparse representations of original signals. Each component in the representations reflects certain intrinsic properties at any given moment. It has been demonstrated through many applications that such type of algorithms compute effectively structured decompositions that provide outstanding de-mixing ability to separate tangled signal components such as noise and background information. (This presentation is based on collaborative work with Luan Lin (Brown), Jingfang Liu (Georgia Tech) and Yang Wang (Michigan State)).

The Fast Digital Shearlet Transform and Applications to Denoising and Coarse Quantization

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In signal processing, one of the primary goals is to obtain a [representation of the digital] signal of interest that is suitable for processing, storage, transmission, and recovery. One of the key features of a suitable representation system is redundancy, providing a representation which is robust under noise, quantization, and data loss. Due to the fact that most high-dimensional signals exhibit anisotropic features, directional representation systems like shearlets and curvelets play a significant role, since they] have been proved to provide optimally sparse approximations of [such signals in contrast to] wavelet systems.

In this talk, we introduce and discuss a Fast Digital Shearlet Transform (FDST) which is a faithful digitization of the continuum domain shearlet transform. For this, we first introduce pseudo-polar grids with oversampling along with the fast pseudo-polar Fourier transform. We show that there exist weightings of the pseudo-polar grid, which gives rise to an isometric pseudo-polar Fourier transform, allowing inversion by taking its adjoint. Based on this pseudo-polar Fourier transform, we shall provide a rationally designed FDST. We will prove that it is indeed the exact digitization of the shearlet transform in continuum domain, thereby showing that shearlet theory provides a unified treatment of both the continuum and digital realms. In addition, we shall discuss the software package ShearLab that implements FDST alongside with various quantitative measures allowing one to tune parameters and objectively improve the implementation as well as compare with other directional transform implementations. Finally, we will discuss several applications of the FDST, in particular, to denoising and coarse quantization.