EAS 475/587: Sample Final Exam

(All questions equal weight, though some questions may be longer than others.)

Give your answers in S.I. units unless otherwise specified.

1. The lower stratosphere has a constant temperature of $-80°$C from $z_1 = 10$ km to $z_2 = 12$ km height. The pressure changes according to $P = P_1 e^{-(z-z_1)/H}$ where $P_1 = 250$ mbar and $H = 5.7$ km.

   (a) What is the potential temperature (in degrees Kelvin) of air at 12 km height?

   (b) If a parcel of this air at 12 km height is brought adiabatically to 10 km, what is its temperature (in ^{o}C)?

   (c) What is the difference in density (in kg/m^3) between that of the air parcel brought adiabatically from 12 km to 10 km and that of the surrounding air at 10 km? Will the displaced parcel feel an upward or downward buoyancy force?

2. (a) A parcel of ocean water at 20°C has 35ppt salinity. Ignoring the influence of pressure, what is its density (in kg/m^3)?

   (b) Where the Columbia River flows into the Pacific ocean, the salinity increases linearly from 0 ppt to 35 ppt from the ocean surface to 50 m depth. Assuming the temperature has a constant value of 20°C over this depth and again ignoring the influence of pressure, find the buoyancy frequency (in radians per second) and the corresponding buoyancy period (in seconds).

   (c) A fluid parcel at 20°C with 35ppt salinity moves adiabatically from the ocean surface to great pressure at 4 km depth. What is its density at that depth (in kg/m^3)?

3. The wind stress acting on the ocean surface is given by $\vec{\tau} = C_D \rho_a |\vec{u}|\vec{u}$ with ρ_a the density of air and $C_D = 10^{-3}$ the drag coefficient. Eastward winds decrease linearly in speed from 10 m/s to 0 m/s going from 40^{o}N to 60^{o}N.

   (a) Estimate the Ekman velocity, w_E (in m/s). Is this upward or downward?

   (b) Estimate the average vertically integrated meridional flow $\mathcal{V}_E$ (in m^2/s) resulting from Sverdrup balance. Is this flow northward or southward?

4. A 200 km wavelength coastal Kelvin wave in a 4 km deep ocean moves northward past San Diego, which is situated at 33°N. Its amplitude (maximum vertical surface displacement) at the coast is 10 cm.

   (a) What are the maximum zonal and meridional velocities (in m/s) at the coast?

   (b) What is the maximum vertical surface displacement (in cm) at a distance 1000 km west of the coast?

5. A storm launches a 60 km wavelength interfacial, northward-propagating Poincaré wave on a 100 m deep thermocline at 45°N in the mid-Atlantic. The density across the thermocline, which can be treated as a thin interface, increases from 1028 kg/m^3 above to 1029 kg/m^3 below.

   (a) What is the (internal) Rossby deformation radius (in km)?

   (b) What is the frequency (in s^{-1}) of the waves?

   (c) What are the phase and group velocities (in m/s) of the wave?

   (d) What is the meridional transport of zonal momentum due to these waves?

Handy Constants

Thermodynamics	
	$0^o\text{C} = 273\text{K}$
for air	$R_a = 287\,\text{J}/(\text{kg K})$, $C_v = 718\,\text{J}/(\text{kg K})$, $C_p = 1005\,\text{J}/(\text{kg K})$
	$H_\rho \simeq R_a T_0/g$ (for isothermal atmosphere)
for water	$\beta_T = 2.1 \times 10^{-4}\,\text{K}^{-1}$, $\beta_S = 7.4 \times 10^{-4}\,\text{ppt}^{-1}$, $\beta_p = 4.1 \times 10^{-10}\,\text{Pa}^{-1}$
	$H_\rho \simeq 200\,\text{km}$, $H_T \simeq 2000\,\text{km}$

Density of dry air at 20^oC and standard pressure

$\rho_a = 1.29\,\text{kg/m}^3$

Density of fresh water at 20^oC and standard pressure

$\rho_w = 998\,\text{kg/m}^3$

Constants for the Earth	
angular frequency	$\Omega_e = 7.3 \times 10^{-5}\,\text{s}^{-1}$
f_0 at midlatitudes	$f_0 = 10^{-4}\,\text{s}^{-1}$
β at equator	$\beta = 2 \times 10^{-11}\,(\text{ms})^{-1}$
standard pressure	$P_0 = 101\,\text{kPa}$
gravity	$g = 9.81\,\text{m/s}^2$
Earth's radius	$R_e = 6.4 \times 10^6\,\text{m}$

1^o latitude is 111 km