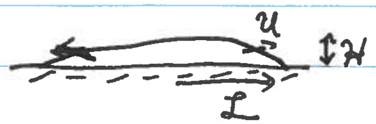
Assignment 2: Solutions

1a)



$\mathcal{U} \sim 1\,\text{mm/s} = 10^{-3}\,\text{m/s}$

$\mathcal{L} \sim 1\,\text{cm} = 10^{-2}\,\text{m}$

$\mathcal{H} \sim 1\,\text{mm} = 10^{-3}\,\text{m}$

$\Rightarrow Re \equiv \frac{\mathcal{U}\mathcal{L}}{\nu} \sim (10^{-3}\,\text{m/s})(10^{-2}\,\text{m})/(10^{-3}\,\text{m}^2/\text{s}) = 10^{-2} \Rightarrow$ VISCOSITY IMPORTANT

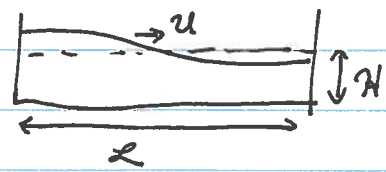
$Fr \equiv \mathcal{U}/\sqrt{g\mathcal{H}} \sim 10^{-3}/\sqrt{10 \cdot 10^{-3}} = 10^{-2}$

ALSO $\alpha \equiv \mathcal{H}/\mathcal{L} = 0.1$

$\Big\} \Rightarrow \alpha^2 Fr^2 = 10^{-6} \Rightarrow$ HYDROSTATIC

$Ro \equiv \mathcal{U}/f_0\mathcal{L} \sim 10^{-3}/(10^{-4} \cdot 10^{-2}) = 10^{3} \Rightarrow$ ROTATION UNIMPORTANT

b)



$\mathcal{U} \sim 1\,\text{m/s}$

$\mathcal{L} \sim 1\,\text{m}$

$\mathcal{H} \sim 0.1\,\text{m}$

$\Rightarrow Re \equiv \mathcal{U}\mathcal{L}/\nu \sim (1)(1)/10^{-6} = 10^{6} \Rightarrow$ VISCOSITY UNIMPORTANT

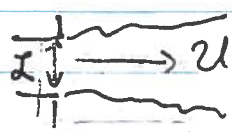
$Fr \equiv \mathcal{U}/\sqrt{g\mathcal{H}} \sim 1/\sqrt{10 \cdot 0.1} = 1$

AND $\alpha \equiv \mathcal{H}/\mathcal{L} = 0.1$

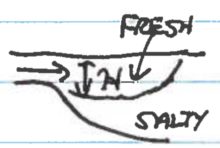
$\Big\} \Rightarrow \alpha^2 Fr^2 = 10^{-2} \Rightarrow$ HYDROSTATIC

$Ro \equiv \mathcal{U}/f_0\mathcal{L} \sim 1/(10^{-4} \cdot 1) = 10^{4} \Rightarrow$ ROTATION UNIMPORTANT

c)



TOP VIEW



SIDE VIEW

$\mathcal{U} \sim 0.1\,\text{m/s}$

$\mathcal{L} \sim 10\,\text{km} = 10^{4}\,\text{m}$

$\mathcal{H} \sim 100\,\text{m}$

$\Rightarrow Re \equiv \mathcal{U}\mathcal{L}/\nu \sim (0.1)(10^{4})/10^{-6} = 10^{10} \Rightarrow$ VISCOSITY UNIMPORTANT

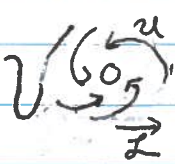
$Fr \equiv \mathcal{U}/\sqrt{g'\mathcal{H}} \sim 0.1/\sqrt{\left(10 \cdot \frac{1.03-1.0}{1.0}\right)100} = \frac{0.1}{\sqrt{30}} \sim 0.02$

AND $\alpha \equiv \mathcal{H}/\mathcal{L} = 10^{-2}$

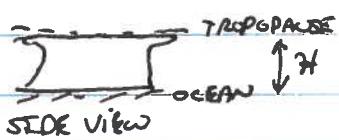
$\Big\} \Rightarrow \alpha^2 Fr^2 = 4 \times 10^{-8} \Rightarrow$ HYDROSTATIC

$Ro \equiv \mathcal{U}/f_0\mathcal{L} \sim 0.1/(10^{-4} \cdot 10^{4}) = 0.1 \Rightarrow$ ROTATION IMPORTANT

d)



TOP VIEW



SIDE VIEW

$\mathcal{U} \sim 30\,\text{m/s},\ \mathcal{L} \sim 100\,\text{km} \sim 10^{5}\,\text{m}$

$\mathcal{H} \sim 10\,\text{km} \sim 10^{4}\,\text{m}$

$\Rightarrow Re \equiv \mathcal{U}\mathcal{L}/\nu \sim 30 \cdot 10^{5}/10^{-5} = 3 \times 10^{11} \Rightarrow$ VISCOSITY UNIMPORTANT

$Fr \equiv \mathcal{U}/N\mathcal{H} \sim 30/(10^{-2} \cdot 10^{4}) = 0.3$

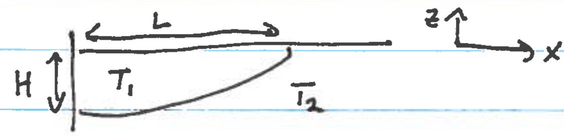
AND $\alpha \equiv \mathcal{H}/\mathcal{L} = 0.1$

$\Big\} \Rightarrow \alpha^2 Fr^2 \sim 10^{-3} \Rightarrow$ HYDROSTATIC

$Ro \equiv \mathcal{U}/f_0\mathcal{L} \sim 30/(10^{-4} \cdot 10^{5}) \sim 3 \Rightarrow$ ROTATION SOMEWHAT IMPORTANT.

2) THERMAL WIND BALANCE FOR NORTHWARD FLOW IS

$$\frac{\partial v}{\partial z} = -\frac{1}{f}\frac{g}{\rho_0}\frac{\partial \rho}{\partial x}$$



FROM LINEAR APPROXIMATION TO DENSITY OF SEA WATER ESTIMATE

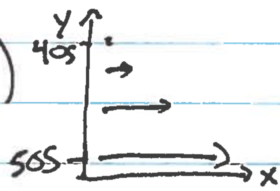
$$\Delta\rho \approx \rho_0[-\beta_T \Delta T] \approx \rho_0[-2.1\times10^{-4}\,K^{-1}\,(10\,K)] \sim -2\times10^{-3}\rho_0$$

IS DENSITY DIFFERENCE BETWEEN GULF STREAM AND ATLANTIC OCEAN

$$\text{So} \quad \left\|\frac{\partial v}{\partial z}\right\| \sim \frac{-1}{10^{-4}\,s^{-1}}\;\frac{10\,m/s^2}{\rho_0}\;\underbrace{\left(\frac{-2\times10^{-3}\rho_0}{200\times10^3\,m}\right)}_{\Delta\rho/L} = 10^{-3}\,s^{-1}$$

$$\text{So} \quad \Delta v \sim \left\|\frac{\partial v}{\partial z}\right\| H = (10^{-3}\,s^{-1})(10^3\,m) = 1\,m/s$$

3)



a) $$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \approx 0 - \frac{u|_{40S} - u|_{50S}}{10^\circ \times (111\,km/^\circ)}$$

$$= (0.20\,m/s)/(1110\times10^3\,m)$$

$$\approx 1.8\times10^{-7}\,s^{-1}$$

b) $$H_E = \sqrt{2\nu/|f_0|} \simeq [2(10^{-2}\,m^2/s)/(10^{-4}\,s^{-1})]^{1/2}$$

$$\simeq 14\,m$$

c) $$W_E = \frac{1}{2}H_E\,\zeta$$

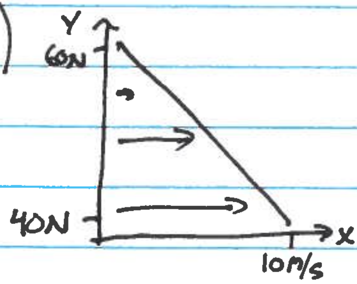
$$\simeq \frac{1}{2}(14\,m)(1.8\times10^{-7}\,s^{-1})$$

$$\simeq 1.3\times10^{-6}\,m/s$$

So SPIN-DOWN TIME IS $\frac{H}{W_E} \simeq \frac{4000\,m}{1.3\times10^{-6}\,m/s} \simeq 3.1\times10^9\,s$

$\simeq 3.6\times10^4$ DAYS

$\simeq 100$ YEARS

4) 

a) $W_E = \frac{1}{\rho_0 f_0}(\nabla \times \underline{\tau})\cdot\hat{z} = \frac{1}{\rho_0 f_0}\left[\frac{\partial}{\partial x}\cancelto{0}{\tau_{yz}} - \frac{\partial}{\partial y}\tau_{xz}\right] = \frac{1}{\rho_0 f_0}\left[-\frac{\tau|_{60N} - \tau|_{40N}}{L_y}\right]$

IN WHICH $\tau|_{60N} = 0$

$\tau|_{40N} = C_D \rho_{AIR} (u|_{40N})^2 \approx 10^{-3}(1.29\ kg/m^3)(10\ m/s)^2$

$\simeq 0.129\ N/m^2$

$L_y = (60^\circ - 40^\circ) \times 111\ km/^\circ \simeq 2.22 \times 10^6\ m$

$\rho_0 \sim 1030\ kg/m^3$

$f_0 \sim 10^{-4}\ s^{-1}$

$\Rightarrow W_E \simeq \frac{1}{1030\ kg/m^3}\frac{1}{10^{-4}s^{-1}}\left[-(0 - 0.129\ N/m^2)/2.2\times10^6\ m\right]$

$\Rightarrow W_E \simeq \underline{\underline{5.8 \times 10^{-7}\ m/s}}$ ($W_E > 0$, SO SUCTION DRAWS FLUID $\underline{\underline{\text{UPWARDS}}}$)

b) $V_E = \frac{1}{\rho_0 \beta}(\nabla \times \underline{\tau})\cdot\hat{z}$

AT MIDLATITUDES ($\sim 50^\circ N$) $\beta = \frac{1}{R_E}(2\Omega_E \cos 50) \simeq 1.5\times10^{-11}\frac{1}{m\cdot s}$

$\Rightarrow V_E = \frac{1}{1030\ kg/m^3}\frac{1}{1.5\times10^{-11}\frac{1}{m\cdot s}}\left[\frac{0.129\ N/m^2}{2.2\times10^6\ m}\right]$

$\Rightarrow V_E \simeq \underline{\underline{3.9\ m^2/s}}$ ($V_E > 0$, SO FLOW IS NORTHWARD)

5) a) FROM y-MOMENTUM EQUATION HAVE, IN GENERAL,

$$\bar{\rho}\left[\frac{Dv}{Dt} + fu\right] = -\frac{\partial P}{\partial y}$$

ASSUMING GEOSTROPHY ($\left\|\frac{Dv}{Dt}\right\|$ NEGLIGIBLE) AND EQUATORIAL β-PLANE ($f = \beta y$)

$$\Rightarrow \quad \beta y (u_0) = -\frac{1}{\rho_0}\frac{\partial P}{\partial y} \qquad (\text{TREATING } u = u_0,\ \bar{\rho} = \rho_0 \text{ CONSTANT})$$

$$\left(\Rightarrow \quad \beta u_0 = +\frac{1}{\rho_0}\,\partial^2 P/\partial y^2 \right)$$

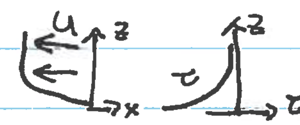
SOLVING $\Rightarrow$ $\underline{\underline{P = +\frac{1}{2}\beta u_0 \rho_0 y^2 + P_0}}$

b) INCLUDING BOTTOM STRESS τ, HAVE

$$\beta y u = -\frac{1}{\rho_0}\frac{\partial P}{\partial y} + \frac{1}{\rho_0}\frac{\partial \tau_y}{\partial z}$$

$$-\beta y v = -\frac{1}{\rho_0}\frac{\partial P}{\partial x} + \frac{1}{\rho_0}\frac{\partial \tau_x}{\partial z}$$

WHERE $\underline{u} = \underbrace{-u_0\hat{x}}_{\text{GEOSTROPHIC}} + u_E$, $\underline{v} = 0 + v_E$



AND τ_x SHOULD INCREASE TO ZERO FROM BOTTOM

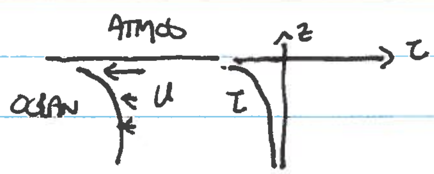
AND $\tau_y = 0$ SINCE NO GEOSTROPHIC FLOW IN Y

$\Rightarrow$ $u_E = 0$ AND $-\beta y v_E = \frac{1}{\rho_0}\frac{\partial \tau_x}{\partial z} > 0$

SO MUST HAVE $v_E < 0$ FOR $y > 0$ AND $v_E > 0$ FOR $y < 0$

I.e. FLOW IS <u>EQUATORWARD</u>

c)



AGAIN HAVE $-\beta y v_E = \frac{1}{\rho_0}\frac{\partial \tau_x}{\partial z}$

BUT NOW $\frac{\partial \tau_x}{\partial z} < 0$

SO $v_E > 0$ FOR $y > 0$ AND $v_E < 0$ FOR $y < 0$

I.e. FLOW IS <u>POLEWARD</u>

(THIS IS ONE REASON WHY OBSERVATIONS OF THE EQUATORIAL OCEAN IS TRICKY ... DRIFTING BUOYS DON'T HANG AROUND)