

# Even and Odd functions

## Definitions:

1. A function  $f(x)$  is even if  $f(-x) = f(x)$ .
2. A function  $f(x)$  is odd if  $f(-x) = -f(x)$ .

## Properties of even functions:

1. Some even functions are:  $1, x^2, x^4, \cos(x), \sin(x^2), x \sin(x), \sin(ax) \sin(bx), \cosh(x)$
2. The graphs of even functions are symmetric about the y-axis. That is, the graph is unchanged by reflection about the y-axis (see left diagram below).

3. If  $f$  is even,

$$\int_{-L}^L f(x) dx = 2 \int_0^L f(x) dx.$$

## Properties of odd functions:

1. Some odd functions are:  $x, x^3, x^5, \sin(x), \sin(x^3), x \cos(x), \sin(ax) \cos(bx), \sinh(x)$
2. The graphs of odd functions are symmetric about the origin. That is, the graph is unchanged by reflections about both the x-axis and the y-axis (see right diagram below).

3. If  $f$  is odd,

$$\int_{-L}^L f(x) dx = 0.$$

