



# Influence of lock aspect ratio upon the evolution of an axisymmetric intrusion

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Through theory and numerical simulations in an axisymmetric geometry, we examine evolution of a symmetric intrusion released from a cylindrical lock in stratified fluid as it depends upon the ambient interface thickness,  $h$ , and the lock aspect ratio  $R_c/H$ , in which  $R_c$  is the lock radius and  $H$  is the ambient depth. Whereas self-similarity and shallow-water theory predicts that intrusions, once established, should decelerate shortly after release from the lock, we find that the intrusions rapidly accelerate and then enter a constant-speed regime that extend between  $2R_c$  and  $5R_c$  from the gate, depending upon the relative interface thickness  $\delta_h \equiv h/H$ . This result is consistent with previously performed laboratory experiments. Scaling arguments predict that the distance,  $R_a$ , over which the lock fluid first accelerates increases linearly with  $R_c$  if  $R_c/H \ll 1$  and  $R_a/H$  approaches a constant for high aspect ratios. Likewise in the constant-speed regime, the speed relative to the rectilinear speed,  $U/U_\infty$ , increases linearly with  $R_c/H$  if the aspect ratio is small and is of order unity if  $R_c/H \gg 1$ . Beyond the constant-speed regime, the intrusion front decelerates rapidly, with power-law exponent as large as 0.7 if the relative ambient interface thickness,  $\delta_h \lesssim 0.2$ . For intrusions in uniformly stratified fluid ( $\delta_h = 1$ ), the power-law exponent is close to 0.2. Except in special cases, the exponents differ significantly from the 1/2 power predicted from self-similarity and the 1/3 power predicted for intrusions from partial-depth lock releases.

**Key words:** geophysical and geological flows, gravity currents

## 1. Introduction

Driven by horizontal differences in density, a gravity current forms when fluid of uniform density flows horizontally into a uniform-density or density-stratified fluid. In the latter case the current can propagate within the interior of the fluid along its level of neutral buoyancy and is referred to as an intrusive gravity current or, simply, an intrusion. For two-layer fluids, if the upper and lower depths are equal and the

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density of the intrusion is the average ambient density, then the intrusion is said to be symmetric.

Most theoretical and experimental studies of gravity currents have examined a constant volume from a lock in a rectilinear geometry (Huppert & Simpson 1980; Maxworthy *et al.* 2002). After a short acceleration time gravity currents are known to propagate at near-constant speed (Benjamin 1968; Shin, Dalziel & Linden 2004) for 6–10 lock lengths. Thereafter the current head height decreases and the speed is predicted to decrease with time as  $t^{-1/3}$ , in what is called the self-similar regime (Rottman & Simpson 1983; Ungarish 2006).

Theoretical predictions of the initial speed of a rectilinear intrusion have been established for both two-layer (Holyer & Huppert 1980; Flynn & Sutherland 2004; Flynn & Linden 2006) and uniformly stratified (Bolster, Hang & Linden 2008) ambients. Experiments have shown that the long-time evolution of rectilinear intrusions is significantly altered by the generation of and interaction with internal waves (Wu 1969; Sutherland, Chow & Pittman 2007; Munroe *et al.* 2009). In particular, rather than entering the self-similar regime, a symmetric intrusion propagates well beyond 10 lock lengths at constant speed, being carried by mode-2 leaky closed-core internal solitary waves at the interface, and then halts abruptly as the lock fluid extrudes completely from the wave (Sutherland & Nault 2007; Munroe *et al.* 2009).

There have been relatively few studies examining the speed of radially spreading gravity currents released from a cylinder (Huppert 1982; Hallworth, Huppert & Ungarish 2001). Because geometry and mass conservation dictate that the head height must decrease as the radius increases, the front is expected to decelerate shortly after release, the speed changing with time as  $t^{-1/2}$ . If the current is released from a partial-depth lock, shallow-water theory predicts that its speed decreases as  $t^{-2/3}$  after propagating sufficiently far from the lock (Zemach & Ungarish 2007). However, recent experimental and numerical studies in approximately two-layer fluids have shown that symmetric intrusions in an axisymmetric geometry propagate at near-constant speed long distances from the lock even though their head heights decrease (Sutherland & Nault 2007; Munroe *et al.* 2009; McMillan & Sutherland 2010). Like symmetric rectilinear intrusions, the collapsing lock fluid excites a mode-2 cylindrical solitary wave that then transports the lock-fluid radially outward at constant speed until the fluid in the leaky core is depleted and the head abruptly halts. This result was extended to examine full- and partial-depth lock-release experiments in stratified ambients (Holdsworth, Barrett & Sutherland 2012). Because mode-2 internal waves in uniform stratification have faster radial phase speeds than the speed of full-depth intrusions the waves were able to carry energy away from the intrusion more efficiently and, as a consequence, the intrusion stopped a shorter distance from the lock.

In all these experimental and numerical studies of axisymmetric intrusions, the lock aspect ratio (radius,  $R_c$ , to height,  $H$ ) was unity or greater. In the case of high-aspect-ratio locks, the intrusion speed was found to be close to the rectilinear speed, as expected because the speed is established before the intrusion is influenced by the curvature of the lock. One expects that the speed should be slower in small-aspect-ratio locks in which case the curvature of the lock influences the speed even as the intrusion accelerates to the constant-speed regime.

The theoretical understanding of axisymmetric gravity currents in uniform-density and stratified ambients is based primarily upon box models and shallow-water theory, which does not model well the initial non-hydrostatic collapse of a low-aspect-ratio lock, nor does it account for the, albeit weak, non-hydrostatic processes associated

with internal solitary wave generation (Huppert 1982; Ungarish & Zemach 2007). It is for these reasons that shallow-water theory does not capture the long constant-speed regime observed in experiments of symmetric intrusions. Because the structure of fluid released from a cylinder continuously deforms in time, it remains a theoretical challenge to predict the speed and consequent evolution.

In § 2 we review established theories for the speed and evolution of symmetric intrusions in stratified fluid and we present simple scaling theories to predict their speed after release from a low-aspect-ratio cylindrical lock in a two-layer ambient. The numerical model, described in § 3, is set up to examine ambient fluid with a piecewise-linear density profile such that the interface depth in different runs ranges between small (approximately two-layer) and large (uniform stratification). The theoretical and numerical results are compared in § 4. The simulations go on to explore the evolution of the intrusion beyond the steady-speed regime. In this way we are able to assess the success and limitations of shallow-water theory as discussed in § 5.

## 2. Theory

In the limit of high-aspect-ratio locks, the steady-state intrusion speed is expected to be equivalent to that for a rectilinear intrusion. Assuming a full-depth lock release in a Boussinesq stratified ambient of total depth  $H$ , the speed of an energy-conserving symmetric intrusion in a two-layer fluid is (Benjamin 1968; Cheong, Kuenen & Linden 2006)

$$U_0 = \frac{1}{4} \sqrt{g'H}, \quad (2.1)$$

in which  $g' \equiv g(\rho_L - \rho_U)/\rho_0$  is the reduced gravity based upon the difference of the lower and upper layer ambient densities. For a symmetric intrusion each layer has depth  $H/2$  and the intrusion density is  $\rho_{avg} \equiv (\rho_L + \rho_U)/2$ . In a uniformly stratified fluid with buoyancy frequency  $N = \sqrt{-(g/\rho_0) d\bar{\rho}/dz}$ , in which  $\bar{\rho}(z)$  is the background density, the intrusion speed is predicted to be (Bolster *et al.* 2008; Munroe *et al.* 2009)

$$U_N = \frac{1}{8} NH. \quad (2.2)$$

More generally, we consider the speed of an symmetric intrusion along a thick interface of depth  $h$  prescribed by the piecewise-linear background density profile

$$\bar{\rho}(z) = \begin{cases} \rho_U, & \frac{H+h}{2} < z \leq H \\ \rho_{avg} + \frac{(z-H/2)}{h}(\rho_U - \rho_L), & \frac{H-h}{2} \leq z \leq \frac{H+h}{2} \\ \rho_L, & 0 \leq z < \frac{H-h}{2}. \end{cases} \quad (2.3)$$

Defining  $\delta_h = h/H$ , we see that (2.3) corresponds to a two-layer density profile if  $\delta_h = 0$  and it corresponds to a uniformly stratified fluid if  $\delta_h = 1$ .

There is no rigorous theory to predict the symmetric intrusion speed in non-uniformly stratified fluid (Ungarish 2005). However, by assuming that the speed is set by the density difference between the mean density of the ambient over the head height and the density of the intrusion head, McMillan & Sutherland (2010) proposed an extension of (2.1) and (2.2) to obtain a prediction for the rectilinear intrusion speed

in an ambient with density profile given by (2.3):

$$U_\infty = U_0 \begin{cases} (1 - \delta_h)^{1/2}, & 0 \leq \delta_h \leq 0.5 \\ \frac{1}{2}\delta_h^{-1/2}, & 0.5 < \delta_h \leq 1. \end{cases} \quad (2.4)$$

This reproduces the speeds in two-layer and uniformly stratified fluid in the limits  $\delta_h \rightarrow 0$  and  $\delta_h \rightarrow 1$ , respectively. The infinity subscript on  $U$  anticipates the limit of speeds observed for high-aspect-ratio ( $R_c/H \rightarrow \infty$ ) cylindrical lock-release experiments.

In agreement with theory (Huppert & Simpson 1980), bottom-propagating axisymmetric gravity currents decelerate beyond 1–3 lock radii such that their position changes in time as  $r_N(t) \propto t^{1/2}$ . However, experiments (Sutherland & Nault 2007; Holdsworth *et al.* 2012) and simulations (McMillan & Sutherland 2010) have shown that doubly symmetric intrusions at an interface propagate at constant speeds for longer distances then stop abruptly. However, these were restricted to the case of lock aspect ratios ( $R_c > H$ ) of order unity or greater. At sufficiently high aspect ratios, the intrusion speed is expected to be predicted approximately by the rectilinear speed (2.4).

For low-aspect-ratio locks, the speed is expected to be relatively smaller. An estimate of the radius,  $R_a$ , at which the intrusion has accelerated to near-constant speed,  $U$ , and speed  $U$  itself can be estimated by a scaling analysis that balances the conversion of available potential energy to kinetic energy taking into account mass conservation.

The time scale for acceleration of the current to speed  $U$  over a distance  $R_a$  is  $R_a/U$ . This is proportional to the time scale for collapse due to gravity over the lock depth:  $R_a/U \propto \sqrt{H/g'}$ . After propagating a small distance  $R_a$  compared to  $R_c$ , the loss of available potential energy per unit mass is of order  $E_p \sim g'H^2 R_c R_a$ . For low-aspect-ratio locks, the kinetic energy,  $E_k$ , is dominated by the vertical speed which, though the incompressibility condition, is proportional to  $U^2 H^3$ . Equating this with the available potential energy gives another relationship between  $U$  and  $R_a$ . Thus we find that  $R_a \propto R_c \propto U\sqrt{H/g'}$ . The same relationship is deduced if the vertical velocity is confined to a cylindrical shell of radius  $R_c$  and thickness  $R_a$ , in which case the kinetic energy scales as  $E_k \sim U^2 H^3 R_a/R_c$ .

For high lock aspect ratios, the acceleration time is longer and the horizontal velocity dominates the kinetic energy. In this case the kinetic energy scales as  $E_k \sim U^2 H R_c R_a$ . Balancing this with the available potential energy predicts that  $R_a/H \equiv R_{a\infty}/H$  is a constant, independent of  $R_c$ . Also in the  $R_c/H \gg 1$  limit we expect the speed of the current to approach the  $R_c$ -independent speed predicted by rectilinear theory (2.4).

As corroborated by our experimental data, we suppose that we can merge these asymptotic limits by formulating exponential fits through the values of  $R_a$  and  $U$  as a function of  $R_c/H$ :

$$\frac{R_a}{H} = \frac{R_{a\infty}}{H} \left[ 1 - \exp\left(-\frac{R_c/H}{\sigma_a}\right) \right] \quad (2.5a)$$

and

$$\frac{U}{H} = \frac{U_\infty}{H} \left[ 1 - \exp\left(-\frac{R_c/H}{\sigma_u}\right) \right]. \quad (2.5b)$$

### 3. Numerical model and analysis

Our numerical simulations solve the axisymmetric, Boussinesq, cylindrical Navier–Stokes equations by approximating spatial derivatives using a second-order finite-difference scheme on a staggered grid and leapfrog time stepping. At the free-slip boundaries the no-normal-flow condition was imposed. The numerical model was described in greater detail and tested against experimental data by McMillan & Sutherland (2010). The code has subsequently been validated through comparison with the experiments of Holdsworth *et al.* (2012).

The model captures trends in the data, but overpredicts the speeds. Although the discrepancy is small compared to experimental errors, this difference indicates a bias in the model attributed to the fact that the two-dimensional model cannot fully capture three-dimensional dynamics. In particular, the intrusions may be losing some energy to the generation of turbulence which is not captured by our two-dimensional model.

Physical parameters were chosen by analogy with laboratory experiments of intrusions in salt-stratified fluid. The ambient density field was prescribed by (2.3) with  $\rho_L = 1.0662 \text{ g cm}^{-3}$ ,  $\rho_U = 0.9982 \text{ g cm}^{-3}$  and with a range of relative interface depths  $\delta_h = 0.01, 0.05, 0.1, 0.2, 0.5, 1$ . The fluid depth and lock height was fixed at  $H = 10 \text{ cm}$  and the lock radius was varied over the range  $R_c = 0.5, 1, 2, 3, 6, 9 \text{ cm}$  between different simulations. The radius of the domain was fixed at  $R = 80 \text{ cm}$ , sufficiently large that the boundary did not influence the flow evolution over the simulation time. The values of gravity, kinematic viscosity and diffusivity were taken respectively to be  $g = 981 \text{ cm s}^{-2}$ ,  $\nu = 0.01 \text{ cm}^2 \text{ s}^{-1}$  and  $\kappa = 0.001 \text{ cm}^2 \text{ s}^{-1}$ . The last of these is two orders of magnitude larger than the actual diffusivity of salt water but is set to maintain numerical stability while still being small enough to have negligible influence on the flow dynamics.

The resolution for all of the numerical simulations was  $\Delta t = 0.00125 \text{ s}$ ,  $\Delta x = 0.044 \text{ cm}$ ,  $\Delta z = 0.044 \text{ cm}$ .

The model outputs a passive tracer field that was used to track the front position at the level of neutral buoyancy,  $z = H/2$ . For example, figure 1 shows snapshots of the passive tracer field at three times and the evolution of the front position,  $r_N(t)$ , in two different simulations, one with  $R_c/H = 0.1$  and the other with  $R_c/H = 0.9$ . In both cases  $\delta_h = 0.2$ .

In a short time the front rapidly accelerates to a constant speed. The distance,  $R_d$ , from the lock at which this constant-speed regime is reached is difficult to discern in figure 1(d), but is clear from plots that zoom-in on early times, as shown in figure 2.

The near-constant speed,  $U$ , was calculated as the slope obtained using least-squares linear regression on  $r_N(t)$  over the interval  $[R_d, R_d + 0.5R_c]$ . The interval was chosen for consistency across all of the simulations.

To determine the radial extent of the constant-speed regime, we measured the radius  $R_d$ , at which the best-fit line used to determine  $U$  deviated from the front position of the current by  $0.25R_c$ . Given  $R_d$  and the corresponding time  $T_d$  at which the current began to slow down, we formed log–log plots of the front position, relative to  $R_d$ , versus time, relative to  $T_d$ . From best-fit lines through these plots taken over 1 s after the deceleration begins, we obtain a power law of the form  $r_N(t) - R_d \propto (t - T_d)^p$ . The exponent can then be compared to the predictions assuming self-similarity.

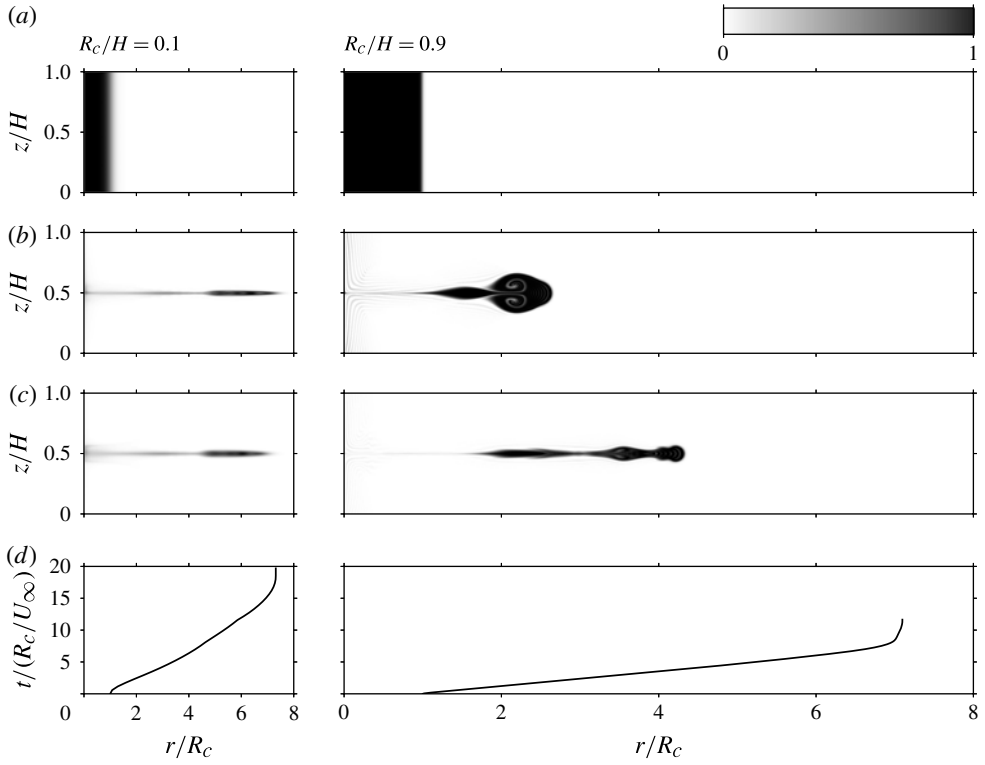


FIGURE 1. Results from two different simulations with  $\delta_h = 0.2$  and  $R_c/H = 0.1$  (left) and  $0.9$  (right). The passive tracer field (equal to 1 for fluid originating in the lock and zero otherwise) is shown at times (a)  $t = 0$  s, (b)  $t = 3$  s, (c)  $t = 6$  s, corresponding to non-dimensional times  $t/(R_c/U_\infty)$  of (a) 0, (b) 17 (left) and 2 (right), and (c) 34 (left) and 4 (right). The front position as a function of time is plotted in (d).

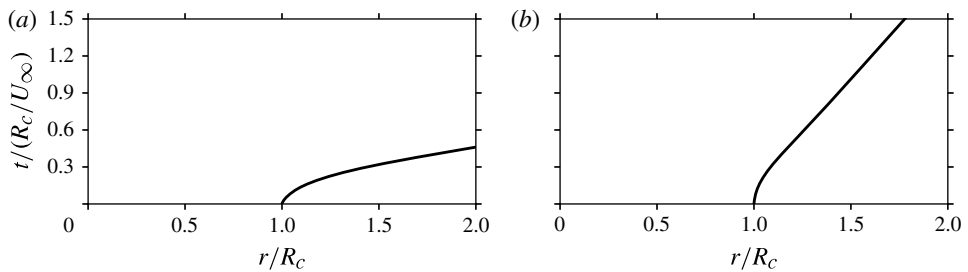


FIGURE 2. Front position at early times taken from the simulations shown in figure 1 with (a) and (b) corresponding to left and right plots in that figure.

#### 4. Results

In all of our simulations, the time for collapse of the lock fluid to form an intrusion propagating at steady speed occurred in less than 1 s. Once the maximum speed was reached the intrusion propagated radially at near-constant speed for between  $0.8R_c$  and  $4.6R_c$ , depending upon the lock aspect ratio and ambient density. Thereafter the

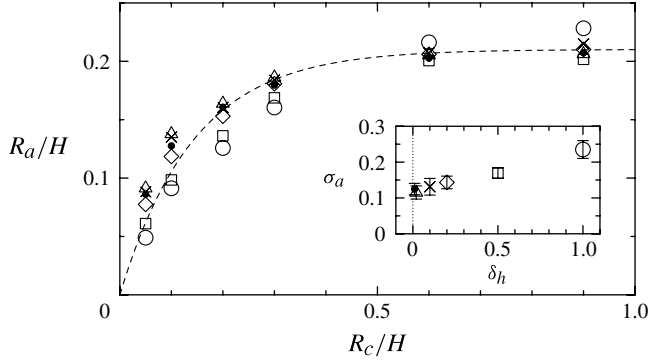


FIGURE 3. The radial distance at which the current accelerates to near-constant speed plotted against the lock radius. Both are normalized by the domain height. Values are shown for a range of relative interface depths  $\delta_h = 0.05, 0.1, 0.2, 0.5$  and  $1.0$ , as indicated by the different symbols – see inset. The dashed line shows the best-fit exponential, in the form of (2.5a), through the points with  $\delta_h = 0.2$ . The values of the e-folding scale,  $\sigma_a(\delta_h)$ , with associated errors are shown in the inset.

intrusion front rapidly decelerated to a halt. This behaviour is illustrated in figure 1 for the case of relative interface depth  $\delta_h = 0.2$ . In the case of a low-aspect-ratio lock  $R_c/H = 0.1$  (left plots), the intrusion advances and halts abruptly after 3 s after propagating a total distance of  $7.5R_c$ . The intrusion from the higher-aspect-ratio lock  $R_c/H = 0.9$  (right plots) shows the front advancing a longer distance at near-constant speed before rapidly decelerating to a halt. In neither case do we observe the self-similar prediction that the front position increases with time as  $t^{1/2}$ .

From analyses of a range of simulations we compute the distance,  $R_a$ , travelled by the intrusion from the lock before reaching the constant-speed regime. The results are plotted in figure 3. Consistent with scaling theory, the results show that, for fixed  $\delta_h$ ,  $R_a$  increases with increasing aspect ratio for small  $R_c/H$  and asymptotes to a constant value for  $R_c/H \gtrsim 0.6$ . For the range of  $\delta_h$  between 0.05 and 1 the asymptotic value is  $R_a/H = 0.21 \pm 0.01$ .

The initial rate of increase of  $R_a/H$  to  $R_c/H$  is larger if  $\delta_h$  is smaller. This rate is found explicitly by determining the best-fit exponential, which gives the e-folding rate,  $\sigma_a(\delta_h)$ . This is found to change with  $\delta_h$  with  $\sigma_a \simeq 0.12 \pm 0.02$  for approximately two-layer fluids and increasing linearly to  $\sigma_a \simeq 0.24 \pm 0.02$  for uniformly stratified fluids. Thus for  $R_c/H \ll 1$ , the distance to attain constant speed is  $R_a \simeq (R_{a\infty}/\sigma_a)R_c$ , which ranges from  $\simeq 2R_c$  to  $R_c$  as  $\delta_h$  increases from 0.05 to 1. Bottom-propagating axisymmetric gravity currents in uniform-density fluid have been observed to enter the self-similar regime after propagating 1–3 lock radii (Huppert 1982). That we find the constant-speed regime being attained for intrusions at thin interfaces after 2 lock radii explains why a constant-speed regime is not easily discerned for gravity currents in uniform-density ambients, for which  $\delta_h = 0$ .

For high-aspect-ratio locks, intrusion speeds decreased monotonically with increasing  $\delta_h$  in accordance with the rectilinear prediction given by (2.4) although the speed of intrusions with  $\delta_h \ll 1$  was 10–15% smaller than  $U_\infty$ . Because intrusions released from low-aspect-ratio locks propagate 2 lock radii before attaining constant speed, the height of the lock fluid decreases substantially during the acceleration phase. Thus the steady speed relative to  $U_\infty$  is slower in these cases. This is shown in



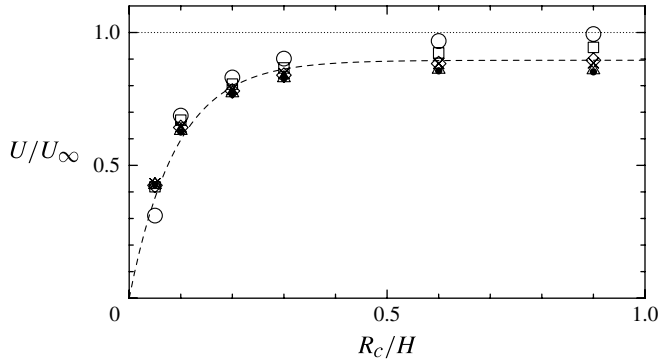


FIGURE 4. The intrusion speed normalized by the theoretical speed of a rectilinear intrusion, given by (2.4), plotted as a function of the lock aspect ratio. The symbols correspond to the value of  $\delta_h$ , as indicated in the inset of figure 3. The dashed line shows the best-fit exponential through data in all simulations with  $\delta_h \leq 0.2$  according to (2.5b), but with  $U_\infty$  taken to be the value of  $U$  for  $R_c/H = 0.9$  for each  $\delta_h$ .

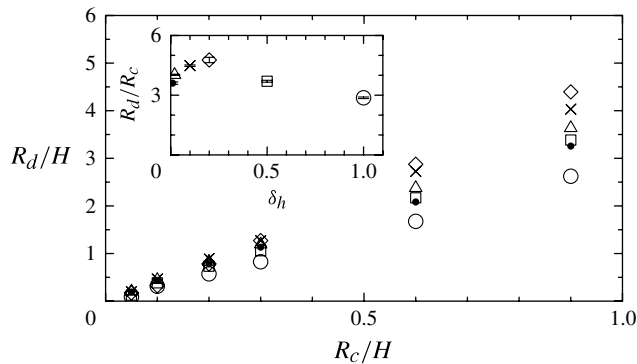


FIGURE 5. The normalized distance at which the intrusion begins to decelerate plotted against the aspect ratio. The symbols are the same as those indicated in the inset of figure 3. The inset plot shows the deceleration radius relative to the lock radius,  $R_d/R_c$ . These are determined from the best-fit lines through  $R_d/H$  versus  $R_c/H$  data at fixed values of  $\delta_h$ .

figure 4, which plots the relative intrusion speed,  $U/U_\infty$ , as a function of  $R_c/H$ . The speed is found to increase exponentially with lock aspect ratio. For  $\delta_h \leq 0.2$  we find the e-folding depth to be  $\sigma_u = 0.091 \pm 0.003$ .

Whereas shallow-water theory predicts that intrusions should begin to decelerate a short distance (less than 3 lock radii) from the gate (Ungarish 2006), consistent with laboratory experiments, we find that the intrusions propagate many lock radii from the gate. This is shown in figure 5 which plots the relative distance from the lock,  $R_d/H$ , at which the intrusion begins to decelerate. For given  $\delta_h$ , we find that  $R_d/H$  increases approximately linearly with  $R_c/H$ . The slope of the best-fit lines passing through the origin for each set of points gives the relative deceleration distance  $R_d/R_c$ . These values, shown in the inset of figure 5, reveal that deceleration begins up to  $4.8 \pm 0.1$  lock radii from the gate (for  $\delta_h = 0.2$ ). In the extremes approaching the circumstance of a two-layer fluid and uniformly stratified fluid, we



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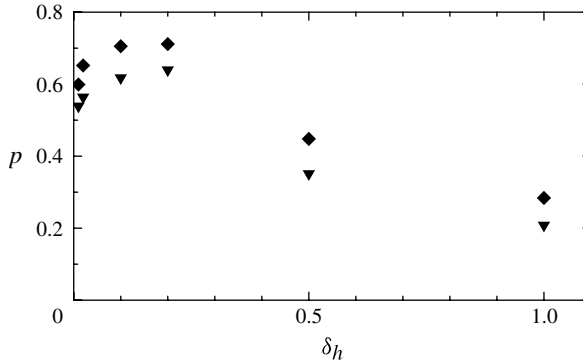


FIGURE 6. Power-law exponent,  $p$ , for front position  $r - R_d \propto (t - t_d)^p$ , determined in the deceleration regime computed over 1 s for  $r > R_d$ . Only values computed in simulations with  $R_c/H = 0.1$  (triangles) and  $R_c/H = 0.9$  (diamonds) are shown for a range of  $\delta_h$ . Typical errors in the value of  $p$  are 5 %.

respectively find  $R_d/R_c = 3.6 \pm 0.1$  for  $\delta_h = 0.05$  and  $R_d/R_c = 2.87 \pm 0.04$  for  $\delta_h = 1$ . The peak distance for  $\delta_h = 0.2$  corresponds to the circumstance when the intrusion speed is critical with respect to the speed of shallow mode-2 internal waves at the interface. For smaller  $\delta_h$ , the intrusion excites a solitary wave which carries the lock fluid long distances at constant speed even as the head height of the intrusion decreases (Sutherland & Nault 2007; McMillan & Sutherland 2010). For larger  $\delta_h$ , the intrusion excites internal waves and, more importantly, the return flow into the lock excites internal waves that propagate radially outward eventually catching up with the intrusion head and halting its advance (Holdsworth *et al.* 2012).

For an axisymmetric gravity current that begins to decelerate, self-similarity theory predicts that the intrusion nose should advance with time as  $r \propto t^p$  with  $p = 1/2$  (Huppert 1982). Shallow-water theory for intrusions resulting from partial-depth lock releases in stratified fluid predicts  $p = 1/3$ . As shown in figure 6, we do not observe this power-law behaviour except in special circumstances. In the approximately 2-layer cases ( $\delta_h = 0.05$ ), the power-law exponent is moderately larger than 0.5 if  $R_c/H = 0.1$ , but is significantly larger for the higher-aspect-ratio lock case. The largest exponents are measured if  $\delta_h \simeq 0.2$ . These are the currents that propagate farthest at constant speed and stop because the lock fluid is depleted out of the solitary wave that transports the fluid. For larger  $\delta_h$ , the deceleration exponent is slower because energy is gradually lost to internal waves generated ahead of the intrusion head. In uniformly stratified fluid ( $\delta_h = 1$ ), we observe exponents  $p \simeq 0.21$  for  $R_c/H = 0.1$  and  $p \simeq 0.28$  for  $R_c/H = 0.9$ .

## 5. Conclusions

We have shown that the relative acceleration distance and constant speed of a symmetric intrusion vary as an exponential with lock aspect ratio: the dependence is linear if  $R_c/H \ll 1$  and asymptotes to a constant if  $R_c/H \gtrsim 0.6$ . The constant-speed regime can persist up to 5 lock radii from the gate and thereafter decelerates before halting abruptly.

These dynamics differ qualitatively from the predictions of shallow-water theory. Experimental evidence indicates that the presence of stratification, even at a thin

interface, introduces non-hydrostatic dynamics associated with the generation of mode-2 internal waves (Sutherland & Nault 2007; Munroe *et al.* 2009). At a thin interface the waves are nonlinear forming leaky closed-core varicose solitary waves (McMillan & Sutherland 2010). In nearly uniformly stratified fluid, the intrusion excites an upstream-propagating wave and the return flow excites internal waves in the lee of the intrusion which then catch up with the intrusion, halting its advance (Holdsworth *et al.* 2012).

Although the study of symmetric intrusions originating from a cylindrical lock release is highly idealized, the contrast between these results and shallow-water theory serves as a caution in the interpretation of hydrostatic models in the study of intrusions in stratified fluid. For example, the influence of non-hydrostatic wave generation should be considered when modelling river plumes interacting with a thermocline (e.g. Nash & Moum 2005) and large volcanic eruptions that spread radially in the stratosphere (e.g. Baines & Sparks 2005).

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