

# Interfacial Mixing in Viscous Pipe Flows

## *Interim report to Imperial Oil*

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### 1. Introduction

The cost of energy to pump oil through a pipe line is greatly reduced if the flow is not turbulent but laminar. This is because turbulence results in greater energy losses and drag. Although new pipe lines are being constructed to take advantage of the savings that result from driving laminar flows, preliminary *in situ* measurements have shown that the length of the mixing interface between the two fluids of different viscosities is larger if the flow is laminar rather than turbulent.

This might be expected since in a laminar flow the fluid effectively “sticks” to the pipe walls. As an intruding fluid displaces another it may flow far along the centre of the pipe while the flow near the walls is almost stationary. The batch would be contaminated over long distances in the process. In contrast, a turbulent flow acts to “scrub” the side walls and thus push the whole fluid uniformly through the pipe. The behaviour of a fluid at the turbulent-laminar transition is not well understood at present.

Although the flow of a single fluid in a pipe has been well studied, very little is known about mixing at the interface between two fluids of different viscosities. Research lacks in this area, in part, because oil companies have only recently considered pumping oil through pipes as a laminar flow.

One question of particular interest is the following: if one fluid is of such low viscosity that it is turbulent at a given flow rate while another fluid is of such high viscosity that it is laminar at the same flow rate, will the interface between the two fluids be sharp or diffuse?

The economic implications are obvious. If a pipe line is operated with one fluid turbulent while the other is laminar, then energy costs could be reduced. If in addition this could be done so that the interface is diffuse due to turbulence, then the fluids do not “stick” to the walls and mixing is reduced. This would reduce contamination and increase the net value of the shipment.

We have performed a series of experiments to determine the flow speeds at which the interface between two viscous fluids is laminar, turbulent or transitional. The experiments have been performed in smooth as well as roughened pipes to examine how the roughness affects the turbulence transition and the interface structure.

The experiments have been performed in the Environmental and Industrial Fluid Dynamics Laboratory (EIFL) at the University of Alberta. The laboratory is equipped with cameras and computers running enhanced flow visualisation software with which to measure the flow speeds and interface structure.

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This report discusses the first stage of analysis of the experiments. The laminar to turbulent transition in a smooth and roughened pipe is determined for a variety of flow speeds and viscosity differences.

## 2. Experimental Set-up

A schematic showing the experimental set-up is shown in Figure 1. Two reservoirs are filled with solutions of different viscosities. Typically one reservoir is filled with a glycerol solution, the concentration of which determines its kinematic viscosity,  $\nu$ . Glycerol solutions can have viscosities ranging from  $0.01 \text{ cm}^2/\text{s}$  (1cS) for fresh water to over  $1.0 \text{ cm}^2/\text{s}$  (100cS) for an 86% glycerol solution. The second reservoir is filled with a salt (NaCl) water solution, the concentration of salt being set so that the densities of the two solutions are approximately equal. The kinematic viscosity of salt water varies negligibly for the small salt concentrations used in these experiments. For visualisation purposes, a small amount of dark blue dye is added to the glycerol solution.

The two reservoirs are connected to each other through 3.8 m of clear vinyl tubing and a 2 m long acrylic pipe. Both the tubing and pipe have 1.27 cm (1/2 inch) inner diameter. A valve at one end of the pipe is closed while the reservoirs, tubing and pipe are filled with the glycerol and salt solution. One solution is filled in the reservoir to a greater height than the other so that when the valve is opened a constant pressure gradient is immediately established along the length of the pipe. The pressure gradient acts to accelerate the fluid through the pipe until viscous and turbulent stresses bring the flow to a mean steady state. The horizontal cross-sectional area of the two reservoirs is set to be so large that the height difference between the two fluids does not change by more than 0.2 cm over the course of an experiment. Thus, for typical initial height differences greater than 2 cm, the pressure gradient along the tube is approximately constant during an experiment.

The fluid in the reservoir with the greater height is referred to hereafter as the “intruding fluid” whereas the fluid in the reservoir with the lower height, which also fills the pipe initially, is referred to hereafter as the “ambient fluid”.

To examine the flow structures, it is necessary to reduce the effect of parallax. When looking through the side of the pipe, the clear pipe wall acts as a lens that differentially magnifies and distorts images flow in the pipe. The effect of parallax is reduced by constructing a 20 cm long rectangular trough around the pipe in a test section near its middle (see Figure 1). The section is filled with a clear fluid whose index of refraction is close to that of the acrylic pipe ( $n \simeq 1.5$ ). For this purpose we use a saturated salt solution. Thus light follows an approximately straight path from the fluid in the pipe through the fluid in the trough to an observer. The test section is illuminated from behind by a 250 Watt halogen light source placed approximately 0.5 m away. Mylar (semi-translucent) paper is attached to the rear of the test section to provide an approximately uniform background illumination.

A series of experiments have been performed in pipes with smooth inner walls and with uniformly rough inner walls. In the latter case, a 10 cm region in the middle of the pipe is left unroughened so that the flow patterns may clearly be observed in the test section.

To record the flow structures, a digital camera is positioned approximately 1.5–2.0 m from the pipe and is focussed on the 20 cm wide test section. The images are later digitised, manipulated and analysed on a computer equipped with frame grabber card and flow visualisation software.

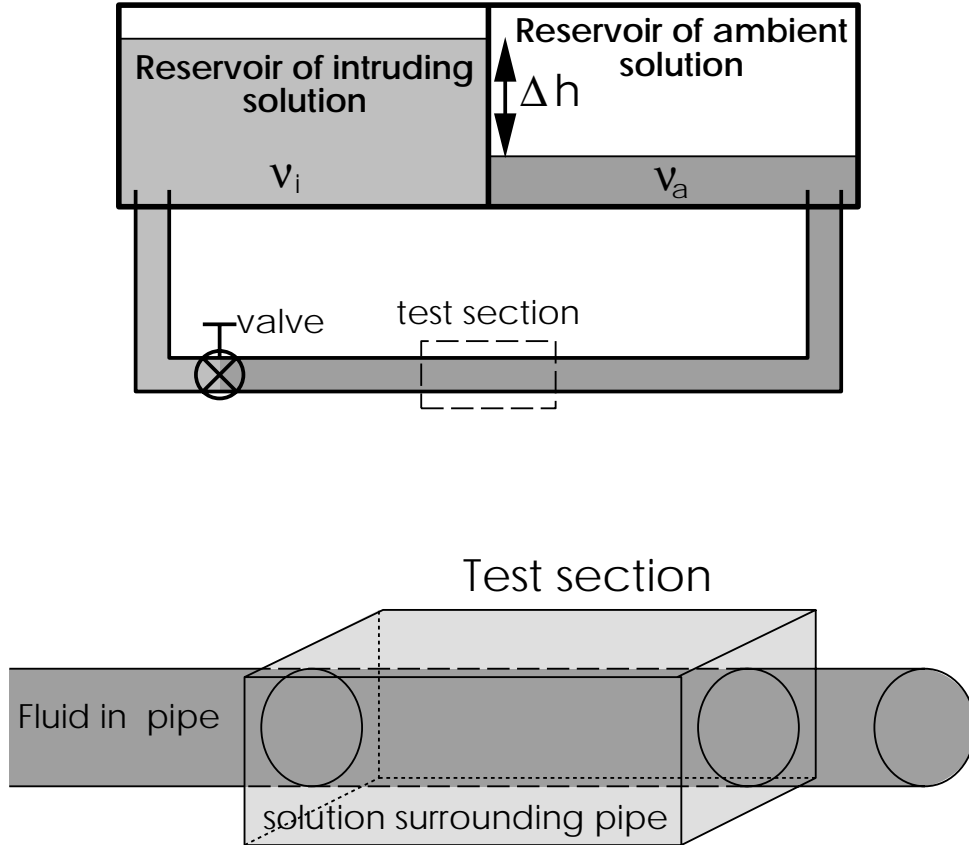


FIG. 1. Set-up of experiment: once the valve is opened, a fluid of kinematic viscosity  $\nu_i$  intrudes along an acrylic pipe filled initially with fluid of kinematic viscosity  $\nu_a$ . In the test section, the pipe is immersed in sugar solution of such large concentration that its index of refraction matches that of the acrylic pipe. The effects of parallax are thus reduced.

In some of the analyses reported here, vertical time series are extracted from the digitised movies of the experiments. In this procedure, the intensity of light from a vertical cross section of the pipe is recorded over time. The time difference between successive vertical “slices” may be as small as 0.017 seconds.

As the ambient fluid initially flows through the test section, there is little change in the light intensity. However, as the dyed, intruding fluid enters the test section, the intensity of light reaching the camera decreases. The vertical time series thus records the arrival time of intruding fluid reaching the test section. In a turbulent flow the intruding fluid is expected to reach the test section at the same time over the whole depth of the fluid, whereas in a laminar flow the fluid is expected to arrive first near the centre of the pipe and later near the pipe walls.

The varying intensity over time at a fixed vertical position also gives a measure of the

degree of mixing between the two fluids at the interface. A gradual change in intensity would occur if the interface between the ambient and intruding fluid is diffuse. A rapid change would occur for a sharp interface.

### 3. Flow Structures

The across-pipe structure of the flows is visualised by taking vertical time series at a position approximately 1 m along the pipe from the valve. The images are filtered and enhanced to illustrate the flow structures more clearly. At low flow rates the flow is laminar, as shown for example in Figure 2a. The horizontal axis represents the arrival time of intruding fluid released after a valve is opened 1 m upstream. The intruding fluid arrives first at the centre of the pipe and arrives at successively later times with distance from the centre. The approximately parabolic profile of arrival times is consistent with the expected parabolic velocity profile of laminar pipe flow.

In contrast, if the pressure gradient, and hence the flow speed, in the pipe is very large, then the flow is turbulent. A time series of fully turbulent flow is shown in Figure 2c. In this case the upstream fluid intrudes with approximately constant velocity over the cross-section of the pipe. Of course the flow would decrease rapidly in the boundary layers near the pipe walls, but these layers are so small as to be negligible. The horizontal bands are an artifact of the image filtering.

Figure 2b shows a transitional case of moderately fast flow in which the laminar flow is intermittently disturbed by fast time-scale bursts. The bursts occur due to shear instabilities resulting from the large shear stresses within the fluid.

### 4. Flow Regimes

A series of experiments have been performed in which a fluid of one viscosity  $\nu_i$  intrudes into a fluid of another viscosity  $\nu_a$ . The density difference between the two fluids is negligible (less than 0.1 percent). The pressure gradient along the tube, and hence the velocity of fluid in the tube is increased by increasing the initial difference of depths,  $\Delta h$ , between the two fluids in the reservoirs.

In laminar pipe flow, the mean velocity of uniform fluid over a cross section of the pipe is predicted to be

$$\bar{U} = -\frac{R^2}{8\mu} \frac{\partial p}{\partial x}$$

in which  $\mu$  is the molecular viscosity of the fluid,  $R$  is the radius of the pipe and  $\frac{\partial p}{\partial x}$  is the constant pressure gradient along the pipe. The pressure gradient is negative, consistent with flow moving from high to low pressure. If the distance along the pipe between the two reservoirs is  $L$ , then the mean velocity is

$$\bar{U} = \frac{R^2}{8\nu} \frac{g\Delta h}{L},$$

in which  $\nu$  is the kinematic viscosity. The maximum flow speed in the pipe is twice this value.

We define a Reynolds number for the ambient fluid in the pipe to be

$$\text{Re}_a = \frac{\bar{U}R}{\nu_a} = \frac{R^3}{8\nu_a^2} \frac{g\Delta h}{L}. \quad (1)$$

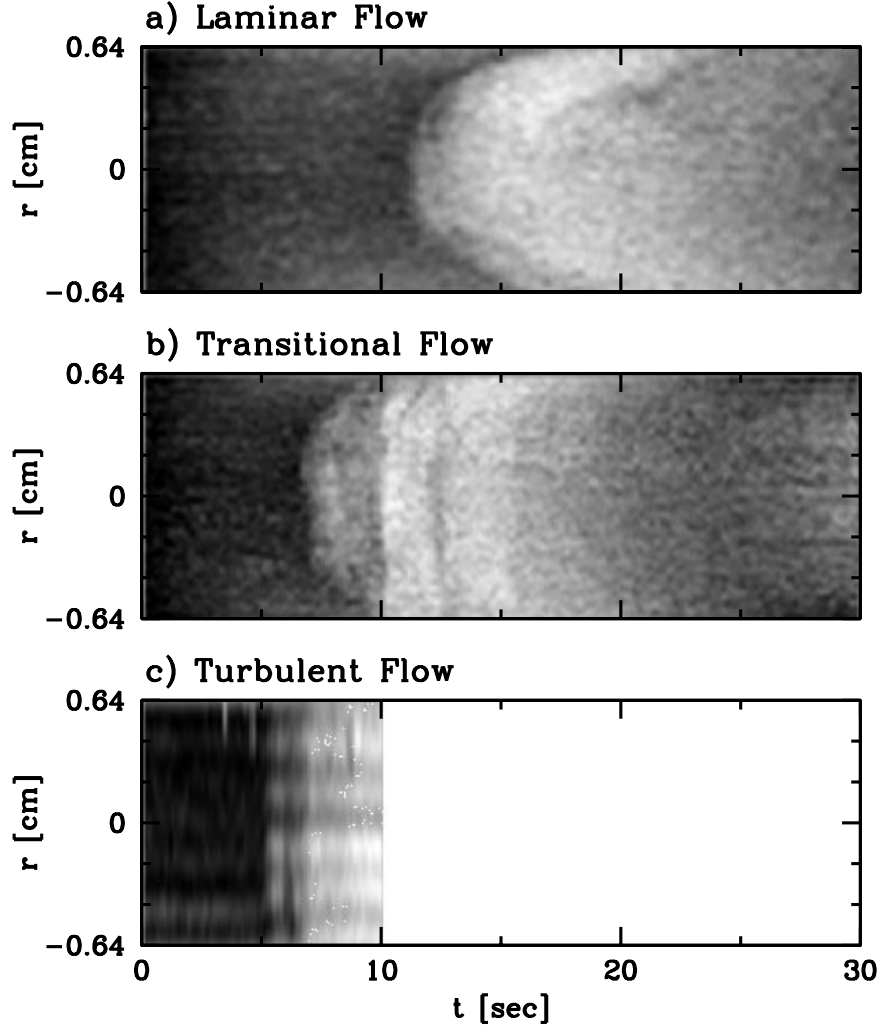


FIG. 2. Vertical time series of a) laminar, b) transitional and c) turbulent flow. The centre of the pipe is at  $r = 0$ . Time  $t = 0$  corresponds to the time at which the valve at one end of the pipe is opened. In c) the time series is recorded only for the first 10 seconds.

Similarly, we define a Reynolds number for the intruding fluid to be

$$\text{Re}_i = \frac{\bar{U}R}{\nu_i} = \frac{R^3}{8\nu_i^2} \frac{g\Delta h}{L}. \quad (2)$$

Whether the pipe flow is laminar or turbulent may thereby be assessed by determining whether both Reynolds numbers are small or large, respectively. A series of experiments have been performed to establish the structure of the interface as a function of the Reynolds numbers of the two fluids.

Figure 3a shows a scatter plot indicating the flow regimes in a smooth pipe. The open circles represent experiments whose flows are observed to be laminar. Crosses are plotted for experiments with fully turbulent flows and solid triangles are plotted for transitional cases. In experiments for which the ambient and intruding fluids have the same viscosity

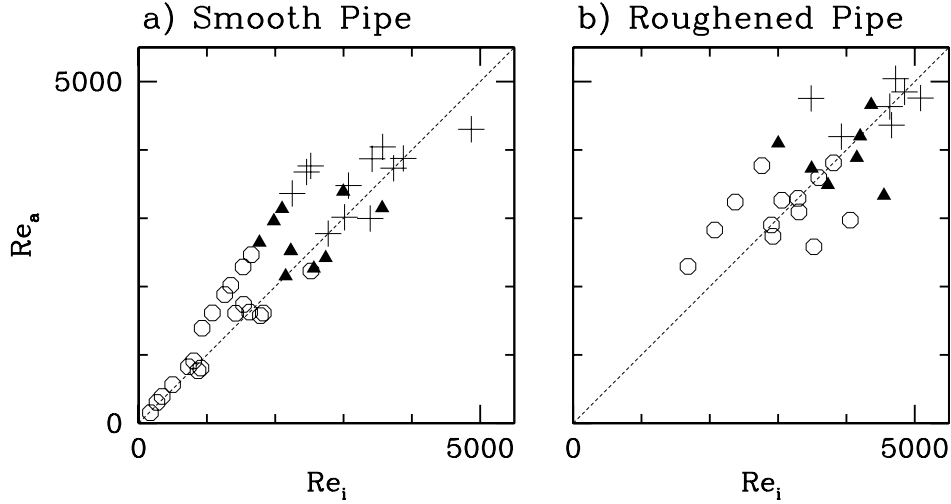


FIG. 3. Stability regimes of viscous flow in a) smooth and b) roughened pipes. Open circles, solid triangles and crosses are plotted for experiments in which the flow is laminar, transitional, and turbulent, respectively. The diagonal line indicates experiments for which the ambient and intruding fluid have the same viscosity.

(those lying on the diagonal line in the figure) the flow is observed to be transitional for  $Re_c = 2100 \pm 500$ . When the viscosity of the two fluids differs, the flow is found to be laminar if the Reynolds numbers of both fluids are less than this critical value. It is interesting to note that if the intruding fluid, for example, has Reynolds number moderately smaller than  $Re_c$ , then the interface remains laminar even if the ambient fluid is moderately larger than  $Re_c$ . Likewise if the ambient fluid has Reynolds number less than  $Re_c$ , then the interface remains laminar even if the intruding fluid has Reynolds number moderately larger than  $Re_c$ .

In summary, the interface is generally stabilized by the presence of at least one fluid in laminar flow.

Figure 3b shows a scatter plot indicating the flow regimes in a uniformly roughened pipe. The plot shows that the flow remains laminar at much higher Reynolds numbers. When the viscosity of the two fluids is equal, the critical Reynolds number is approximately  $Re_c = 4200 \pm 300$ .

This behaviour is expected since the rough walls on the pipe introduce a boundary layer on the scale of the wall roughness. Because viscous drag is exerted primarily in this small layer the stress within the body of the pipe is reduced. Thus the flow remains laminar at faster flow rates. (One might draw an analogy between this behaviour and a that of putting sawdust on a shuffleboard table. A rock smoothly glides over the sawdust, whereas it might skip at the same velocity if in direct contact with the smooth table surface.)

As in the case of the smooth pipe, when the viscosity of the two fluids differs, it appears that the interface is stabilized by the presence of one of the fluids being in laminar flow.

## 5. Discussion and Conclusions

In the flow of two fluids of different viscosity through a pipe, the mixing length is least if both fluids are turbulent. However, if driven by a sufficiently small pressure gradient that the flow is transitional between laminar and turbulent, the energy costs required to pump the fluid is reduced while the mixing at the interface between the fluid moderately increases. To determine whether such a circumstance is profitable, first it is necessary to determine at what flow rates and viscosities the flow is transitional.

Based on the experimental data collected so far, a crude estimate of the critical flow speeds may be found from

$$Re_a = 2Re_c - Re_i$$

in which  $Re_a$  and  $Re_i$  are the Reynolds numbers of the ambient and intruding fluids respectively and  $Re_c$  is the critical Reynolds number determined experimentally. In terms of the static head  $\Delta h$ ,

$$\Delta h = 16Re_c \frac{L}{R^3 g} \left( \frac{1}{\nu_a^3} - \frac{1}{\nu_i^3} \right)^{-1}$$

in which  $L$  is the length of the pipe,  $R$  its inner radius,  $g$  the acceleration of gravity, and  $\nu_a$  and  $\nu_i$  the kinematic viscosities of the ambient and intruding fluids, respectively. In experiments of flow through a smooth pipe  $Re_c \simeq 2100$ . In a uniformly roughened pipe  $Re_c \simeq 4200$ .

In ongoing research, accurate profiles of the horizontal flow speeds are being determined through analysis of the horizontal time series of the experiments. Together with the vertical time series, these analyses will be used to measure the mass transport rates and to determine the mixing lengths between the ambient and intruding fluids as a function of the pressure gradient.

The experiments and analyses will be repeated for pipes with different types of roughening including along-pipe striations and spiral striations similar to rifling.

In order to extend these results more reliably to the large scale flows in typical operational circumstances, a numerical model of the flow of two fluids in a pipe will be developed. The model will be tested and calibrated by running simulations similar to the laboratory experiments.

In the course of running these experiments, it has been noticed that density differences as small as 0.5% between the ambient and intruding fluids significantly changes the structure of the laminar and transitional flows. Further research is necessary to characterise the turbulent transition in such cases.

## Acknowledgements

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