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ORIGINAL ARTICLE

Generation of internal waves by sheared turbulence: experiments

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Abstract A series of experiments is presented that model the generation of internal gravity waves in the ocean by the forcing of turbulent eddies in the surface mixed layer.The experimental setup consists of a shallow mixed upper layer and a deep continuously stratified lower layer. A source of turbulence is dragged through the upper layer. Internal waves can freely propagate in the lower layer. The internal waves are measured using synthetic schlieren to determine the frequencies of the generated waves. Consistent with other studies, it is found that the characteristic frequencies of internal waves generated by turbulence are an approximate constant fraction of the buoyancy frequency.

Keywords Turbulence · Stratification · Internal waves

1 Introduction

Breaking internal waves in the ocean can generate turbulence which results in localized mixing. In particular this acts to transport heat into the deep ocean and serves in part to close the overturning circulation [6]. In this paper, we investigate the inverse process: how can turbulence generate internal waves? This question focuses on how energy containing turbulent eddies, due to, for example, wind-driven forcing in the oceanic mixed layer, can be a source for internal waves. These internal waves can then propagate downwards and thus transport energy from the surface into the deep ocean. These small-scale processes

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are not resolvable by large-scale general circulation models and must be parametrized. The experiments presented in this paper are part of an effort to develop such a parametrization.

Previous papers have investigated internal waves generated by turbulent wakes from towed spheres [2,3], from a bottom Ekman layer [10], turbulent shear flow [8] and stationary turbulence [4]. Interestingly, although the forcing spans a broad spectrum of time and length scales, it has been found that the frequency, ω, of the resultant internal waves lie in a fairly narrow band in proportion to the buoyancy frequency, N, namely $\omega/N \approx 0.7$. Here, we define the buoyancy frequency as

$$N^2 = -\frac{g}{\rho_0}\frac{d\bar{\rho}}{dz} \tag{1}$$

where g is the acceleration due to gravity, ρ_0 is the density of fresh water, and the background density, $\bar{\rho}$, is a function of depth, z. Recent work [1] looked at the flow of stratified fluid over 'rough topography.' The dominant forcing or excitation frequency, $\omega_{\text{exc}} = Uk$ was related to the speed of the flow, U, and the wavenumber of the topography, k. Topographically generated waves were observed if the forcing frequency was less than the buoyancy frequency. This is a consequence of the dispersion relation,

$$\omega = N\cos\theta \tag{2}$$

where ω is the frequency and θ is the angle made by lines of constant phase to the vertical. Importantly, this relation says that freely propagating internal waves can only exist if $|\omega| < N$. Waves forced at any higher frequency than N will be evanescent. However, those experiments also observed that for supercritical flow, where $\omega_{\text{exc}} > N$, a turbulent wake in the lee of the topography could launch internal waves. As seems to be typical of these types of waves, the frequency is independent of the forcing frequency but given rather by a fixed fraction of the buoyancy frequency.

In this paper, we examine the generation of internal gravity waves by a moving source of turbulence. We have reused the apparatus of the towed topography of [1] but changed the stratification. In our experiments, internal waves in a linearly stratified layer are forced from above by sheared turbulence in a uniform density layer. Thus, we can compare the effect of a moving source of turbulence in a fully stratified fluid, the case presented by [1], to a stratified fluid with a well-mixed upper layer, the case investigated here. Since the physical application of this work is to the turbulence from wind driven eddies in the ocean mixed layer it is more reasonable to use this shear turbulence apparatus as opposed to a grid generated stationary turbulence setup [4]. Note that the focus of the present study is not on topographically generated waves, although the apparatus has been used previously for that purpose. Rather, we are looking at waves launched by turbulent eddies. We view the 'rough topography' acting as an eddy generator due to boundary layer separation and its turbulent wake. We show that for our surface mixed case the generated internal waves have frequencies, as expected, in a narrow range of a fixed factor of the buoyancy frequency.

2 Setup and methods

We performed our experiments in a 197.1 cm long glass tank that is 48.5 cm deep and 17.4 cm wide. A linearly stratified fluid was created in the tank using the standard 'double bucket' technique [7] to a total depth $H = 27$ cm. The typical buoyancy frequency for all experiments in the stratified lower region was $N \approx 1.1\ \text{s}^{-1}$. The density was measured using a Standard Precision Conductivity Probe set up to traverse the entire depth of the fluid. This was the

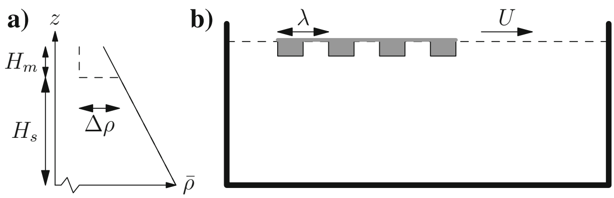

Fig. 1 (**a**) Typical stratifications. The solid line shows the density profile of the uniform stratification experiments. The dashed line shows the density profile of the surface mixed layer experiments. (**b**) Setup of apparatus

stratification used for uniformly stratified experiments and is sketched by the solid line in Fig. 1a. For the surface mixed layer experiments, approximately 5 cm of the fluid was siphoned off and was replaced with an equal volume of fresh water, $\rho_m = 1.0$ g cm^{-3}. This created a density profile as sketched in Fig. 1a using a dashed line. After repeated experiments this mixed region may deepen to approximately 7 cm. A wooden square waveform of the same width as the tank, with peak-to-peak height 2.7 cm and wavelength $\lambda = 13.7$ cm is towed at a constant speed using a pulley system along the top of the tank as shown in Fig. 1b. The topographic wavenumber is $k = 2\pi/\lambda = 0.46$ cm^{-1}.

The goal of this apparatus is to produce a moving patch of turbulent eddies in a mixed layer and then observe internal waves generated in the linearly stratified ambient beneath. The rectangular waveform acts as an eddy generator as fluid separates off the sharp corners and produces vortices. The waveform,which is pulled from left to right, is connected to a motor such that the towing speed is in the range $U = 0.8 - 5.0$ cm s^{-1} to within ± 0.05 cm s^{-1}. Using a characteristic horizontal length scale of the towed object, $L = 14$ cm, and the viscosity of water, $\nu = 0.01\,\text{cm}^2\,\text{s}^{-1}$ we calculate a Reynolds number of $Re = UL/\nu$ as ranging from $Re = 1,100 - 7,000$. With this range of Reynolds numbers we expect to see flow separation and eddies form in the lee of the topography. The waveform, which floats on the surface, is weighted such that the troughs are at the waterline. Different towing speeds excite eddies of different dominant frequencies, namely $\omega_{\text{exc}} = Uk$. It is natural to form a Froude number, $\text{Fr} = Uk/N = \omega_{\text{exc}}/N$ to characterize the towing speed and hence the excitation frequency.

The turbulence in the lee of the towed object can be characterized using particle image velocimetry [5]. Figure 2 shows some details of the structure of the forcing velocity field for the case with a mixed layer. The upper frame shows a typical vector plot of the velocity field obtained in the lee of the towed object as a time series. From this time series, it can be seen that the integral length scale of the dominant eddies is about 6 cm which is the same as the thickness of the mixed layer. The integral time scale is approximately 5 s. The lower frame shows the energy density in the mixed layer as a function of time for three independent experiments.

The turbulent eddies impinge upon the stratified region to force internal waves. The internal waves are measured using ‘synthetic schlieren’ [9]. This optical technique takes advantage of the variation of index of refraction of light as a function of density and allows us to measure non-intrusively the frequency, wavenumbers, and amplitudes of the internal waves.

Figure 3 shows images produced using synthetic schlieren taken from typical experiments. The field shown is N_t^2 which is proportional to the vertical displacement of the fluid. Although the entire length of the tank is almost 2 m, the camera’s field of view is only 27.5 cm wide by 20 cm high. We define a world coordinate system where $x = 0$ cm corresponds to the left edge of the field of view and $z = 0$ cm is the surface. In this region of interest, located at the middle of the tank, we assume the end walls of the tank are far enough away that reflection of

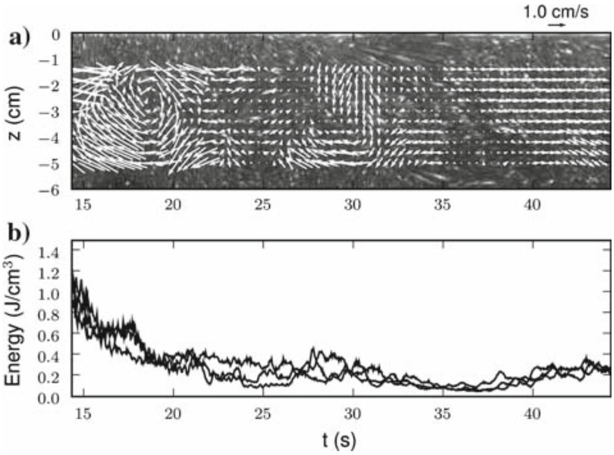

Fig. 2 The lee of the towed object for Fr = 1.46 showing (**a**) velocity field as a vertical time series and (**b**) the energy density of the mixed layer as a function of time for three separate runs of the experiment

waves is unimportant. Reflection of waves off the bottom of the tank still occurs. Figure 3a, c, d shows an experiment with $\mathrm{Fr} = 0.53 < 1$. This is a subcritical experiment since the forcing frequency is less than the buoyancy frequency. Figure 3b, d, e shows an experiment with $\mathrm{Fr} = 1.73 > 1$. This is a supercritical experiment since the forcing frequency is greater than the buoyancy frequency. Vertical (Fig. 3a, b) and horizontal (Fig. 3c, d) timeseries show how the waves in these two cases are generated and propagate over time. In the vertical timeseries taken at $x = 13.7$ cm, which is halfway through the field of view, we can see that there is not much signal directly over the topography. In the wake of the topography, where there is more turbulence visible, strong vertically propagating waves can be seen. On the upper axis is the equivalent horizontal spatial scale defined as taking time and multiplying by the towing speed. It is difficult to isolate individual waves in a vertical timeseries since waves generated at different times are visible in the same image. However, using a horizontal timeseries, taken at $z = -15$ cm, it is much clearer to see waves produced at the same time.

The window selected in time for the timeseries ranges from $t = 0$ s to $t = 60$ s. Here, $t = 0$ s is defined as the time at which the waveform started moving. The frequency and the wavenumber of the internal waves were calculated using a discrete Fourier transform. Examples of power spectra measured are given in Fig. 3e, f. Note that the resolution is proportional to the inverse of the domain of our field of view and can not easily be improved without using a larger tank and apparatus. We estimated the location of the peak and the error using a Gaussian best fit. The largest peak is indicated by the intersection of dashed lines. The width of Gaussian fit gives the standard deviation in the error which is typically 10% of the frequency measurement. Comparing the different experiments, the spectra are noticeably broader for supercritical forcings than for subcritical forcings. As can been seen in Fig. 3e, there may be more than one peak in the power spectrum. Being more restrictive in choosing the window in time on a per-experiment basis could be done to isolate waves generated directly beneath the topography from those generated by the lee of the topography. Reducing the length of the window in time, however, reduces the resolution in determining the frequency. We define the dominant wave frequency as the frequency associated with the largest peak in the power spectrum and hence the waves with the largest energy. The consistent approach of using a window in time with a fixed length was applied to all experiments.

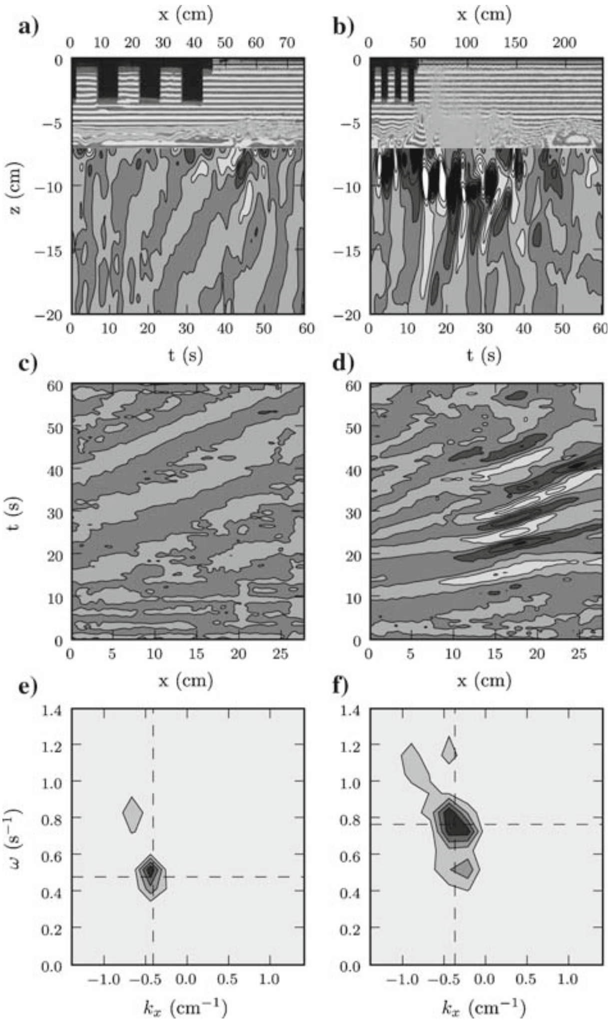

Fig. 3 Turbulently generated internal waves shown using (**a**) vertical timeseries at $x = 13.7$ cm with the equivalent horizontal scale ($x = |U|t$) shown in the top axis, (**c**) horizontal timeseries at $z = -15$ cm, and (**e**) power spectrum of the horizontal time series for an experiment with Fr $= 0.53$. (**b**), (**d**), (**f**) are respectively the same but for Fr $= 1.73$. Contours in **(a–d)** are plotted at values between -0.15 and 0.15 s^{-3} spaced by 0.10 s^{-3}. The spectra are normalized to have maximum value of unity

3 Analysis and results

Figure 4 shows the relative wave frequency, ω/N, as a function of the forcing frequency expressed as a Froude number, Fr $= Uk/N$. The plotted values are the means of dominant frequencies determined according to the method in the previous section for each of three horizontal timeseries at $z = -13$, -15, and -17 cm. The error bars are the standard error in the mean for each experiment. The dotted line shows $\omega = N\text{Fr} = \omega_{\text{exc}}$, which is the wave frequency predicted by linear theory for flow over sinusoidal hills. The vertical dashed line indicates the boundary between subcritical, $\omega_{\text{exc}} < N$ and supercritical, $\omega_{\text{exc}} > N$.

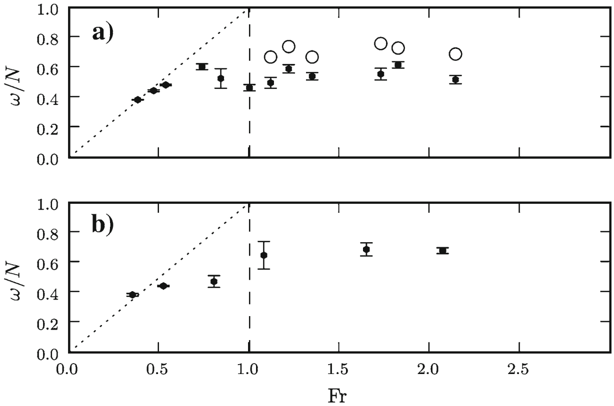

Fig. 4 Plot of measured dominant relative internal wave frequency, ω/N, as a function of $\mathrm{Fr} = \omega_{\mathrm{exc}}/N$ for (**a**) uniformly stratified experiments and (**b**) surface mixed layer experiments. The open circles in (**a**) are waves generated in the far lee by turbulence only

A series of 12 experiments using a uniform stratification over the entire depth [1] is given in Fig. 4a. When $\mathrm{Fr} < 0.7$, topographically generated waves with the same frequency as the forcing frequency are observed as the dominant signal. In the supercritical region, the dominant wave frequency is independent of Fr and has a mean value of 0.55 ± 0.04. It is known from previous work [1] that this frequency corresponds to internal waves generated in the near lee of the towed topography. The near lee is characterized by an undulating "boundary-trapped lee wave" which acts as a source of internal waves. Waves generated by turbulence in the far lee are distinguished from orographic- and lee-generated waves by separation in time. In the far lee, defined as beyond two buoyancy periods after the trailing edge of the topography has passed, the flow is much less coherent and more turbulent. The open circles in Fig. 4a correspond to horizontal timeseries being windowed in time such that only information from the far lee is included. The waves here have a mean frequency of 0.71 ± 0.09.

We can compare these uniform stratified experiments with our surface mixed experiments to determine the importance of the turbulent forcing being located in a mixed layer as opposed to a stratified layer. The relative wave frequency, as shown in Fig. 4b, increases until $\mathrm{Fr} \approx 0.7$. In the supercritical region, the wave frequency is independent of Fr and has a mean value is 0.69 ± 0.03. This is the same as the frequency observed in the far lee of the uniformly stratified experiments. Note that in this case no windowing in time has been used to differentiate the near lee versus far lee. This suggests that the dominant frequency observed is due to waves generated by shear turbulence. Towing rough topography through a mixed layer precludes the development of a boundary-trapped lee wave and leaves only the turbulence in the wake as a source for internal wave generation. The horizontal wavenumber is a constant for all experiments.

This wavenumber, $|k_x| = 0.46\ \mathrm{cm}^{-1}$ is coincidentally the same as the wavenumber given by the periodicity of the rectangular topography. However, length scales of topographic amplitude and wave length and mixed layer depth are all similar for these experiment so it not clear what sets the horizontal length scale of the waves. From other experiments performed (not shown) using a towed plate with an effective wavelength of $\lambda = 0\,\mathrm{cm}$ and the same amplitude as the rectangular topography we have observed generated waves have the same horizontal length scale and frequency but much larger amplitude.

Further experiments to distinguish which length scale sets the length scale of the dominant eddies of turbulence in the lee and thus the length scale of the internal waves are on-going. The results presented here show that the frequency of the waves depends weakly on the thickness of the mixed layer.

The amplitude of the waves, computed as the root mean square of the horizontal timeseries, increases with towing speed. The dependence seems to be a fractional power law and further analysis is currently being performed. The wave amplitude also depends on the topographic amplitude [1] and may depend on the thickness of the mixed layer.

4 Discussion and conclusions

Experiments with a surface mixed layer above uniformly stratified fluid demonstrate that the frequency of turbulently generated internal waves occur in a narrow band around a fixed fraction of the buoyancy frequency.

The observation that the wave frequency matches the forcing frequency up until Fr $\simeq 0.7$ and then remains roughly constant at $\omega/N \simeq 0.7$ is consistent with previous experimental studies of turbulence generated internal waves [1,4]. It has been suggested by Dohan and Sutherland [4] that waves of this frequency are dominant because waves with $\omega/N = 1/\sqrt{2} \simeq 0.71$ are the ones that have the maximum vertical flux of horizontal momentum and thus there is an interaction between the wave field and the turbulent region which excite waves close to this frequency. Another explanation is proposed by Taylor and Sarkar [10] where it is suggested that waves of this frequency endure the least amount of viscous damping and thus are the dominant waves observed.

Internal waves from both sheared turbulence, such as in this experiment, and stationary turbulence, such as the experiments by Dohan and Sutherland [4] are observed to have the same characteristic frequencies. From observations in [4], a broad range of turbulent eddies with different characteristic velocity and length scales generates a relatively narrow band of internal waves in terms of frequency and length scales. This suggests the properties of these turbulence generated waves are universal and independent of the forcing mechanism.

We are progressing in computing the energy and momentum flux of these turbulently generated internal waves. We are currently comparing the amplitude of turbulently generated internal waves between the paper by Aguilar and Sutherland [1] and this work. This will demonstrate the differences in wave generation especially with regard to energy and momentum transport and illustrate any important differences in having the turbulent source in a uniformly mixed layer as opposed to being in a uniformly stratified region. Also, additional experiments will be performed using a continuous stratification to address the effect of having a density discontinuity in the stratification.

Future work on this topic will involve the use of a particle image velocimetry system to characterize the sheared turbulence and the length scales associated with the eddies. The eventual goal is to create a parametrization relating the energies of the internal waves generated by shear turbulence to the characteristics of the turbulent eddies found in the wind driven ocean mixed layer. This parametrization would be useful to understanding ocean mixing in large scale numerical simulations.

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