Application of Hidden Markov Models in Ecology

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Outline

Research question

Stationary Markov models

Hidden Markov models

Implementation of hidden Markov models

Examples of species dynamics

Summary

How can hidden Markov models (HMMs) be useful in studying vegetation dynamics?

Objectives

1. Examine the feasibility of applying HMMs in ecology
   - Possible ecological analogues to model components
2. Compare the success of stationary Markov models (SMMs) vs. HMMs in describing and predicting responses of vegetation along environmental gradients
   - Build on experience with stationary Markov models (SMMs) in Sudbury, ON, Canada

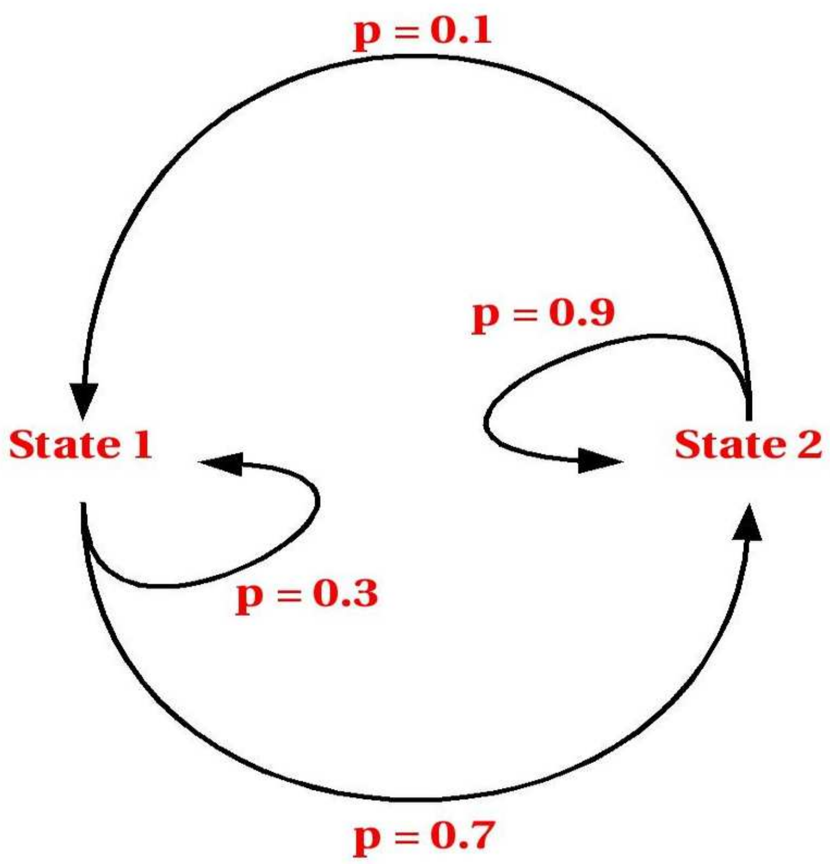
Stationary Markov Model

Vector (X_t) of n states is determined by previous state vector (X_{t-1}) and transition matrix (A)

$X_t = X_{t-1} \times A$, gives state distribution in time

Transition matrix remains constant

State Transitions



$$\mathbf{A} = \begin{pmatrix} 0.3 & 0.7 \\ 0.1 & 0.9 \end{pmatrix}$$

Limits of Stationary Markov Models

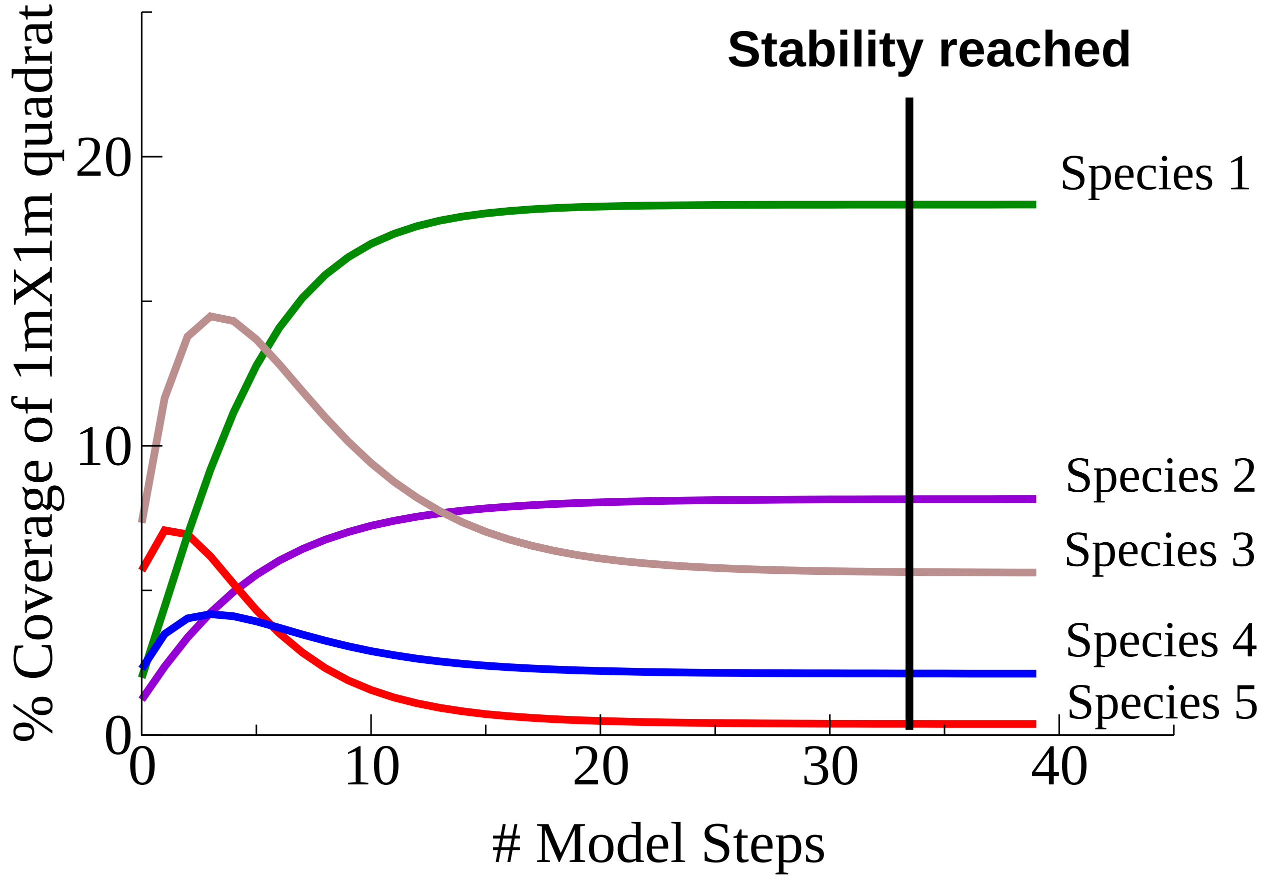
State probability distribution must attain stability

- Useful for looking at Clementsian succession or strong environmental gradients

Assume that the modelled process consists of observable events

- It is likely that observations are the result of hidden states

SMM State Stabilization



Tucker and Anand, 2003

SMM State Stabilization

Discrete Hidden Markov Models

Although underlying Markov state distribution will stabilize, the model observations need not stabilize

Hidden states may allow for the detection and/or description of underlying ecological processes

Components of a Hidden Markov Model

Model components

n - # of states in the model

A - state transition matrix

$X_{t=0}$ - initial state probability distribution

B - observation probability distribution for each state

Ecological interpretation

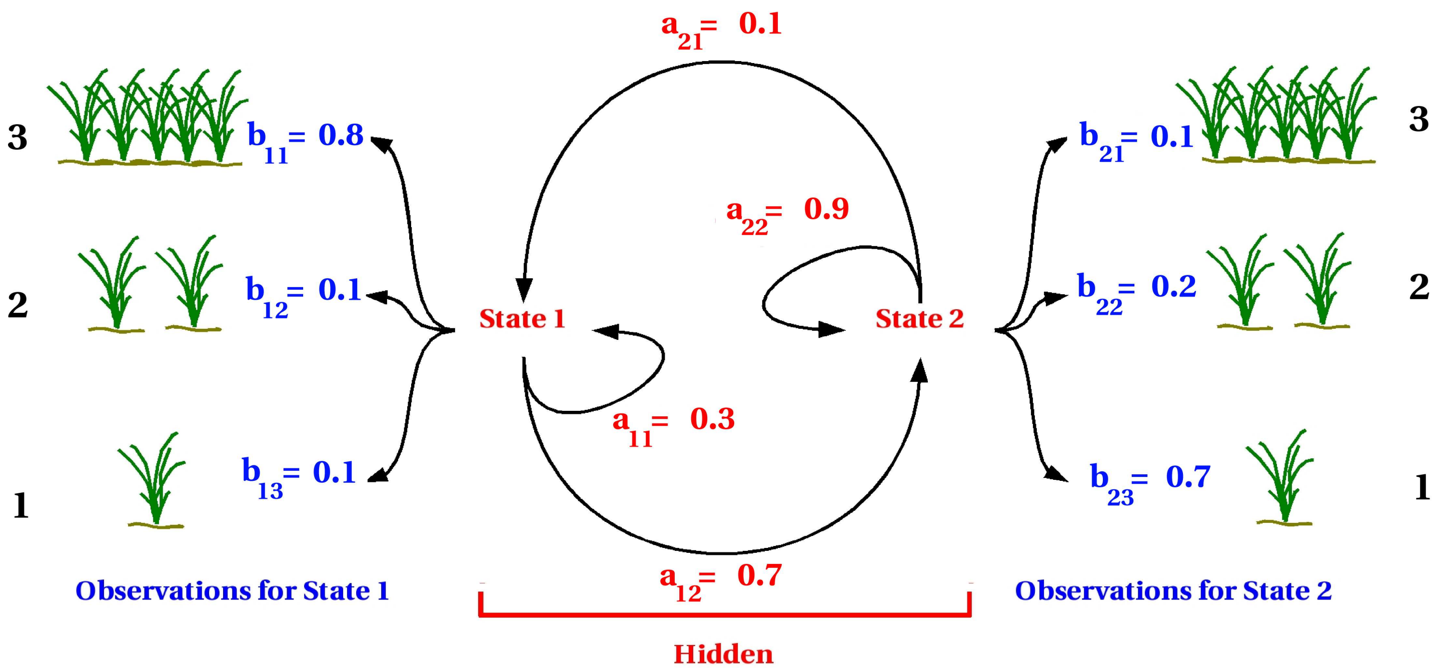
n - # of environmental states

A - environmental state transition matrix

$X_{t=0}$ - initial distribution of states

B - observation probability distribution for each environmental state

2-State HMM with 3 Observations



$$\mathbf{A} = \begin{pmatrix} 0.3 & 0.7 \\ 0.1 & 0.9 \end{pmatrix}$$

State transition matrix

$$\mathbf{B} = \begin{pmatrix} 0.8 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.7 \end{pmatrix}$$

Observation prob. distribution for each state

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Implementation of HMMs

Parameter estimation

- Estimate the model parameters $(A, B, X_{t=0})$

Model size selection

- Decide the size of the model (number of states)

Optimal hidden state sequence

- Find the most likely hidden state sequence that accounts for the data

Parameter Estimation

Estimate the parameters of the model ($\lambda = A, B, X_{t=0}$)

- Find the estimate of $A, B, X_{t=0}$ so that $A, B, X_{t=0}$ result in the highest probability of producing the original data

Baum-Welch iterative algorithm

- Will find the local maximum probability estimate given $a\ priori$ estimates of $A, B, X_{t=0}$
- Different $a\ priori$ estimates may lead to different $a\ posteriori$ estimates
- Train many models with random $a\ priori$ estimates and choose "best" one based on the probability of the model generating the original data ($P(O|\lambda)$)

Model Size Selection

How many states should the model have?

Problem: Different observation sequences may be the result of different environmental state sequences

- Different species in the community may react to different numbers of environmental states
- Often the goal is to infer the number of hidden states

Solution: Estimate models of many sizes

- Normally, B is set by data collection method, therefore cannot change
- N, the number of states, can be varied
- Choose the best model from group of models with different numbers of states

Information Criteria for Model Size Selection

Bayesian and Akaike's Information Criteria (BIC & AIC)

- Evaluate the model based on the probability of its generating the data but penalize for model complexity
  $BIC = -2\log L + 2n_p$
  $AIC = -2\log L + n_p \log(T)$

For hidden Markov models use Adjusted BIC and AIC

- Do not include parameters where $p = 0$ (provide no information about process)
- Do count parameters where $p = 1$
  $A - BIC = -2\log L + 2(n_p - n_{p=0} + n_{p=1})$
  $A - AIC = -2\log L + (n_p - n_{p=0} + n_{p=1}) \log(T)$

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Optimal Hidden State Sequence

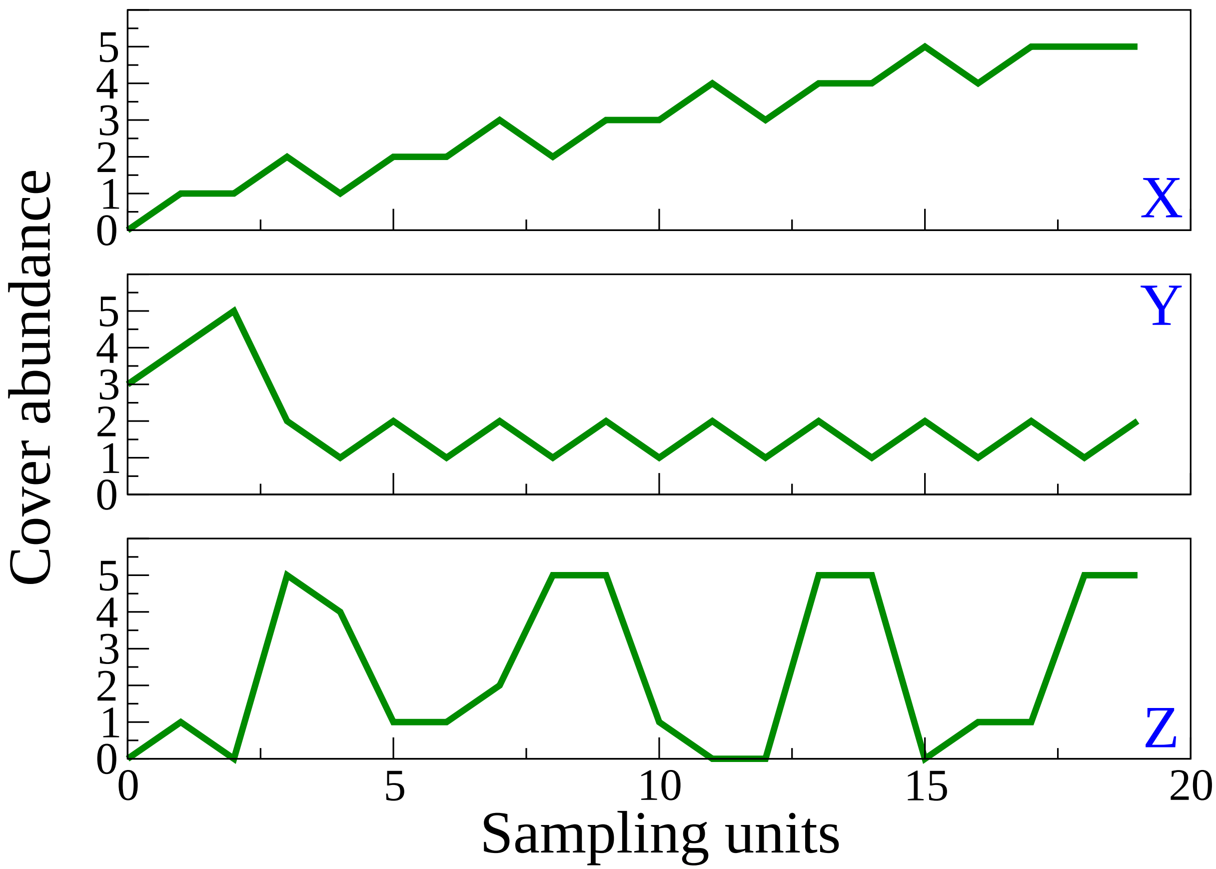
Determine the hidden state sequence that would have the highest probability of generating the observation data

- Viterbi algorithm

Ecological relevance of state sequence

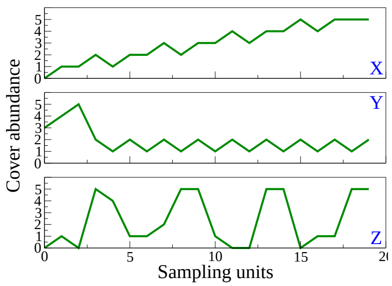
- Changes of environmental state along spatial/temporal gradients
- Oscillating pattern of dominant processes

Example observed species dynamics

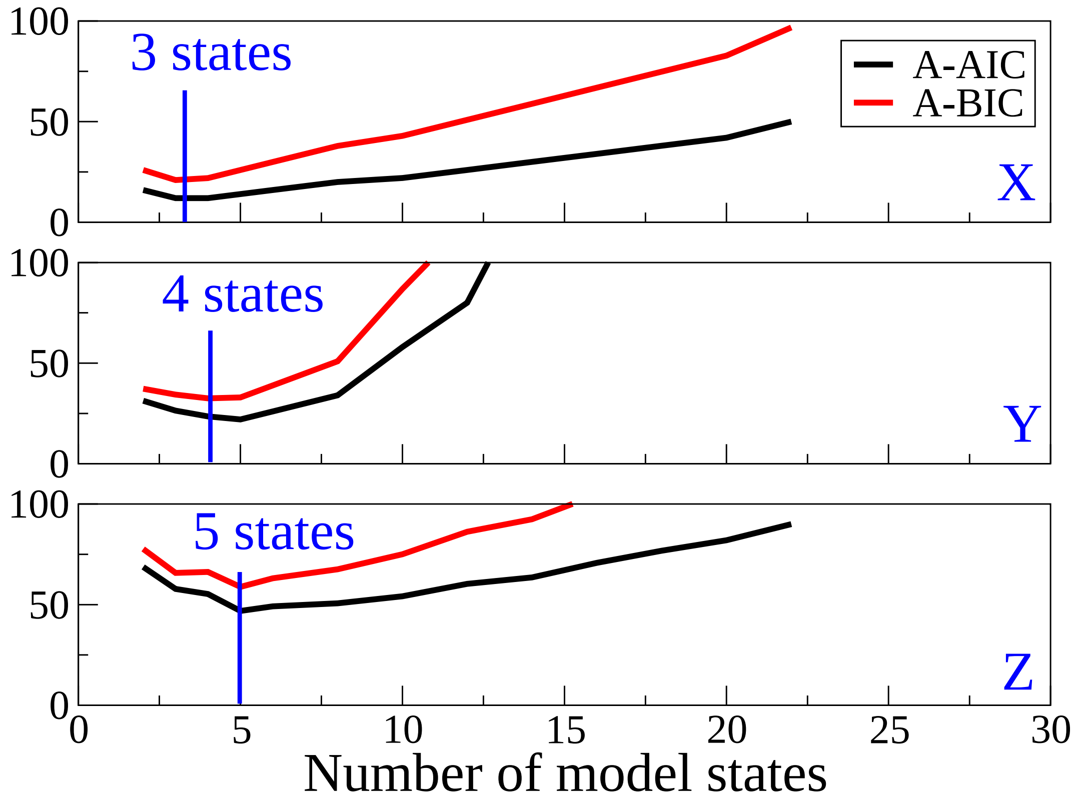


Consider 3 plant species sampled in 5 classes in 20 sampling units

Model size selection

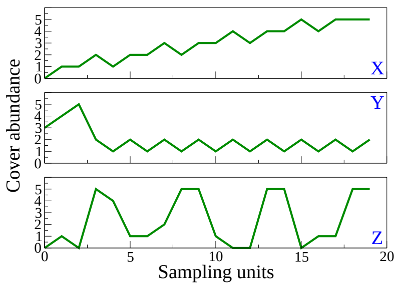


Observed data

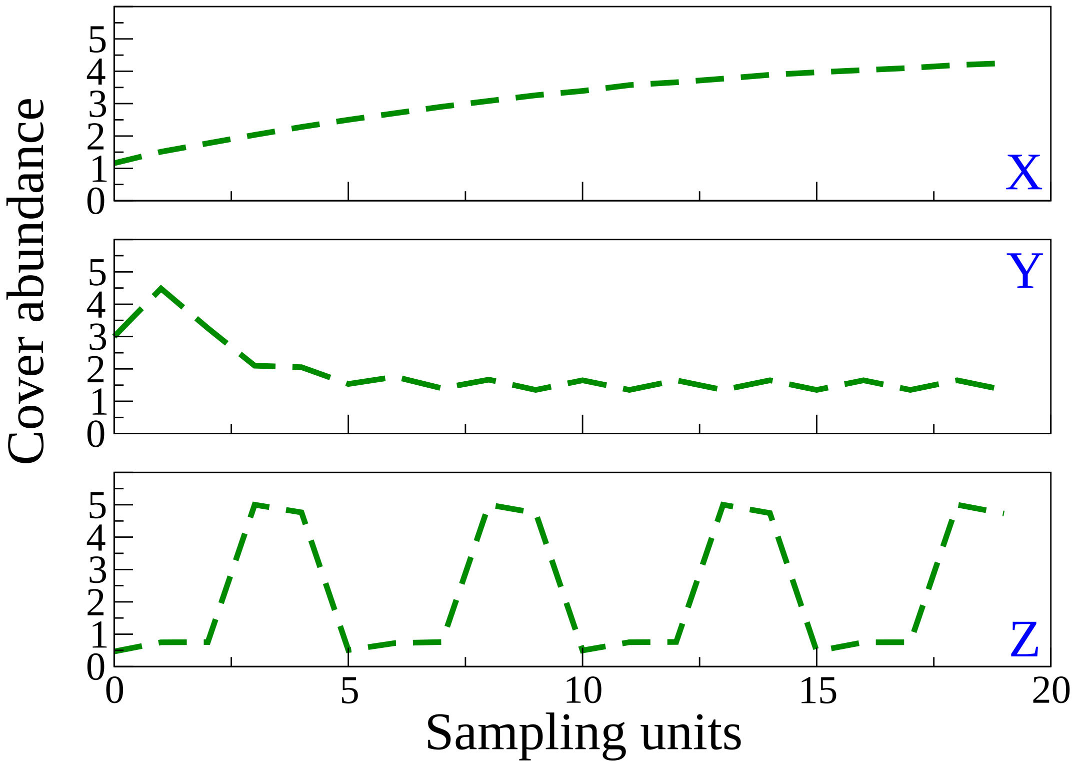


Different species dynamics require models of different size

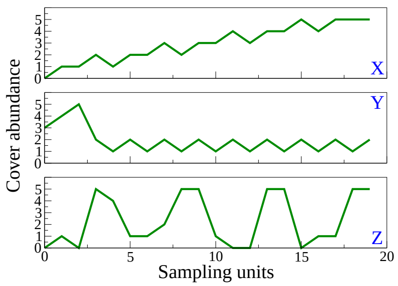
Projected species dynamics



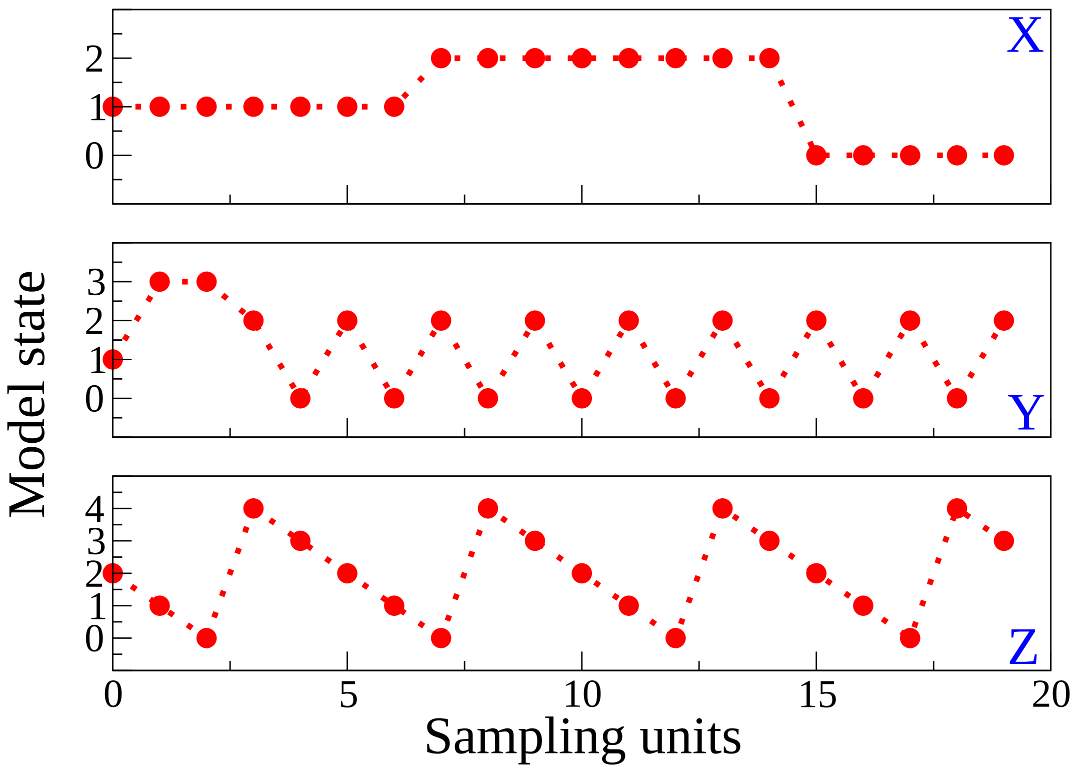
Observed data



Optimal hidden state sequence



Observed data



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EMAI: an extendable DHMM software package

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Terminal
File  Edit  View  Terminal  Go  Help
                                EMAI v.4.1

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       EMAI comes with ABSOLUTELY NO WARRANTY. This is free
       software, and you are welcome to redistribute it under
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Menu

1.      Load file
2.      Transformations
3.      Information
4.      Estimate DHMM with Baum Welch
5.      Sample DHMM
6.      Aproximate randomization to assess sig. of model mean and variance
7.      Bootstrap for model parameter confidence intervals
8.      Approximate randomization to assess sig. of model goodness of fit
9.      Optimal state sequence via the Viterbi algorithm
99.     Quit

Selection:
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Summary and a look ahead

Transition matrix of SMM must reflect the ecological observations

Transition matrix of HMM can reflect abstract or phyical "causes" of the ecological observations

May be a valueable data mining tool for use with old ecological datasets

Application of HMMs to natural and anthropogenic perturbation gradients in Sudbury, ON, Cananda

Completion of DHMM software package

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