Geophys. J. Int. (2006) **165,** 32–46 doi: 10.1111/j.1365-246X.2006.02903.x

Azimuthal flows in the Earth's core and changes in length of day at millennial timescales

Mathieu Dumberry[1,2] and Jeremy Bloxham[1]

[1]*Department of Earth and Planetary Sciences, Harvard University, Cambridge, MA,* 02138, *USA. E-mail: dumberry@earth.leeds.ac.uk*
[2]*School of Earth and Environment, University of Leeds, Leeds,* LS2 9JT, *UK*

Accepted 2005 December 21. Received 2005 December 14; in original form 2005 March 14

SUMMARY

Variations in the length of day (LOD) reconstructed from ancient records of eclipses contain an oscillating component with a periodicity of roughly 1500 yr. A part of the time-dependent variations observed in archaeomagnetic field models consist of episodic eastward and westward motions with a similar periodicity. Using standard inversion techniques, we obtain time-dependent azimuthal flows at the surface of the core that can explain parts of the archaeomagnetic secular variation, and with amplitude and characteristic timescale of variation consistent to explain the LOD signal. On this basis, we argue that exchanges of angular momentum between core flows and the mantle are responsible for the observed millennial oscillations in the Earth's rotation rate. However, we do not obtain a detailed match between the observed changes in LOD and the prediction based on simple flow profiles inside the core. We argue that this is due to the presence of time-dependent azimuthal flows that have an important shear in the direction of the rotation axis. Such flows are interpreted as oscillations in thermal and magnetic winds.

Key words: core dynamics, core–mantle coupling, Earth's magnetic field, Earth's rotation, geomagnetic variation, length of day.

1 INTRODUCTION

The Earth's magnetic field is believed to be generated by convective motion in the fluid core, a process known as the geodynamo. The details involved in this process are not fully understood, partly because we have few observations that can be used to constrain the dynamics. The magnetic field itself, the very product of the geodynamo, has been measured directly at fixed observatories distributed around the globe only since the 1840's (e.g. Stern 2002). The addition of magnetic field data recorded in ship logs and other sources has pushed back the historical magnetic field observations to 1590 (Bloxham *et al.* 1989; Jackson *et al.* 2000). These observations have provided many advances in our current understanding of the geodynamo. However, they cover a time span shorter than the typical 500–1000 yr timescale on which the dynamics responsible for generating and maintaining the magnetic field are believed to operate (e.g. Hollerbach 2003).

Archaeomagnetic field models, on the other hand, offer a much more promising perspective for studying the dynamics involved at the magnetic field generation timescales. These are global time-dependent magnetic field models covering the last few millennia, built from a compilation of palaeomagnetic field data obtained from archaeomagnetic artefacts, lake sediments and lava flows (Daly & Le Goff 1996; Hongre *et al.* 1998; Constable *et al.* 2000; Korte & Constable 2003; Korte *et al.* 2005; Korte & Constable 2005). Although these models only capture some of the crudest features of the field, they extend our continuous record back to 7000 yr before present. Since the early pioneering models (Marton 1970; Kolomiytseva & Pushkov 1975; Braginsky & Burlatskaya 1979; Sakai 1980), there has been a considerable improvement and extension of the database from which they are constructed (Korte *et al.* 2005). There is hope that the models are now sufficiently accurate to provide observational constraints that can be used to extract some of the physical processes involved in the geodynamo. For example, the models suggest the presence of quasi-stationary high-latitude magnetic flux patches in the Northern hemisphere (Constable *et al.* 2000; Korte & Constable 2003, 2005) that are correlated with those seen in the historical models (Bloxham & Gubbins 1985). A similar time-dependent and time-averaged magnetic field morphology can be reproduced by numerical models of the geodynamo when the imposed heat flow at the core–mantle boundary (CMB) is derived from lower-mantle seismic tomography (Bloxham 2002; Olson & Christensen 2002). If these numerical results are correct, they suggest that the presence and behaviour of these patches may be related to thermal core–mantle interactions.

In this study, we further attempt to relate a part of the archaeomagnetic field to dynamic processes in the core. We focus on one particular aspect of the time-dependent field: the longitudinal motion of some of the magnetic field structures—including the high-latitude flux patches—which alternate between westward and eastward motion with a periodicity on the order of a thousand years. These oscillating drifts have been previously interpreted as a superposition of propagating MAC waves at the surface of the core, the latter involving a balance between Lorentz, Coriolis and buoyancy forces

(Braginsky 1972, 1974; Burlastkaya & Braginsky 1978; Braginsky & Burlatskaya 1979). Here, we investigate the possibility that they are instead due to oscillating mean azimuthal flows in the core.

Additional information on core flows at millennial timescales and, therefore, additional constraints for our hypothesis, may be contained in a different geophysical data set: the observed millennial variations in the Earth's rotation rate, recorded as changes in length of day (LOD) (Stephenson & Morrison 1995; Morrison & Stephenson 2001). Direct measurements of the variations in the amplitude of the Earth's rotation has been made possible by the advent of atomic clock in 1955 (e.g. Munk & MacDonald 1960; Lambeck 1980). Telescopic observations of the occultation of stars by the Moon and of the transits of Mercury across the Sun's face can be used to extend the continuous record back to 1620, albeit only for the largest fluctuations of decade to hundred-year timescales (e.g. Stephenson & Morrison 1984). Observations of solar and lunar eclipses documented by the ancient and medieval civilizations of Babylon, China, Europe and Arabia provide a means to reconstruct the variations in LOD further in the past. By integrating the celestial motions of the Earth–Moon–Sun system backward in time and requiring that the eclipses occur at the location and time they were recorded, it is possible to retrace the last 2700 yr of the Earth's rotation rate (Stephenson & Morrison 1984, 1995; Morrison & Stephenson 2001). The eclipse record successfully captures the lengthening of the day at the rate predicted by the measured tidal dissipation between the Earth and the Moon (and a smaller contribution from the Sun) (Christodoloudis *et al.* 1988), minus the effects from the decrease of the Earth's oblateness (Yoder *et al.* 1983; Rubincam 1984; Cheng *et al.* 1989) resulting from post-glacial rebound (O'Connell 1971; Wu & Peltier 1982; Mitrovica & Peltier 1993). More importantly for our present purpose, the eclipse record also suggests the presence of an oscillation in the LOD about this gradual increase, with a semi-amplitude of about 4 ms and a periodicity of roughly 1500 yr.

The similarity between this periodicity and that of the oscillating longitudinal drifts of the archaeomagnetic secular variation (ASV) is tantalising in that it may indicate that both observations result from a common mechanism originating in the fluid core. Indeed, if the millennial LOD variations are due to angular momentum exchange between the mantle and core, the changes in core angular momentum are carried by time-dependent mean azimuthal flows, the very type of flow consistent with the observed periodic drifts of magnetic field structures. The goal of this study is to investigate this hypothesis. If time-dependent mean azimuthal flows in the core can consistently explain both the LOD changes and a part of the ASV, these flows would then reveal important information about dynamical processes involved in the geodynamo at a timescale of a thousand years.

Considerable success has been achieved by applying similar ideas at decade timescales. Torsional oscillations, a component of the flow predicted by theory which consists of azimuthal oscillations of rigid coaxial cylindrical surfaces (Taylor 1963; Braginsky 1970), can be inferred from historical geomagnetic secular variations (Zatman & Bloxham 1997, 1998; Pais & Hulot 2000; Hide *et al.* 2000). The predicted changes in LOD, calculated on the basis of angular momentum exchange between these torsional oscillations and the mantle, are in good agreement with the observed LOD variations, at least from about 1900 onward (Jault *et al.* 1988; Jackson *et al.* 1993). The conclusions that can be drawn from this result are manyfold. First, it confirmed that the decade variations in the LOD are due to exchanges of angular momentum between the core and the mantle. Secondly, it confirmed the theoretical prediction of torsional oscillations and showed that they are well recovered in core flows. This is important, as it indicates that the geodynamo is in a quasi-Taylor state (Taylor 1963) and, therefore, provides constraints for theoretical and numerical models. And third, it adds credence to the quality of the LOD data, the historical magnetic field models and the core flows derived from these models.

Here, we want to verify whether similar ideas can be extended to millennial timescales. One immediate difficulty for doing so is that, in contrast to decade timescales, a well defined framework for interpreting the ASV does not currently exist. In the case of the historical geomagnetic field models, their ongoing refinement has sparked the development of tools for the interpretation of the models in terms of dynamics in the core. This is the case for example of the reconstruction of the time-dependent core surface flow by inversion from the observed secular variation (e.g. Bloxham & Jackson 1991). The archaeomagnetic field models being relatively recent, these tools have not yet been extended for the analysis of the longer timescale secular variation. Similarly, the calculation of the decade changes of angular momentum in the core based on the historical flow models rests on the well defined theory of torsional oscillations. At millennial timescale, we do not know at present of an analogous simple core flow on which we can base the angular momentum calculation.

The goal of this study is not to present a thorough extension of the core surface flow inversions to millennial timescales, nor is it to build a complete theory for zonal flows inside the core at these timescales. Rather, this work is a first step toward extending to millennial timescales the existing framework appropriate for decade to hundred year timescales. For this reason, we found convenient to separate our study into two parts. In the first, using simple theoretical arguments, we investigate the expected millennial timescale behaviour of the part of the flow that carries angular momentum in the core. In the second part, we attempt to find evidence for this dynamical prediction by seeking to explain a part of the ASV and changes in LOD with simple core flows obtained by a regularized inversion method.

2 RIGID AND NON-RIGID AZIMUTHAL FLOWS IN THE CORE

In this study, we make the assumption that a part of the ASV is due to azimuthal flows near the surface of the core. Based on these flows, we wish to calculate the changes in core angular momentum and compare this prediction with the changes required to explain the observed millennial changes in LOD. However, the angular momentum calculation involves flows throughout the core, not only flows near its surface. Therefore, we need to find a way to relate the surface flows to the flows that carry angular momentum in the whole of the core.

To do so, it is convenient to conceptually divide the core into cylindrical surfaces and consider the flow on such surfaces. Let us define the axisymmetric azimuthal velocity $\overline{v}_\phi(s, z, t)$ at any point on a given cylinder as

$$\overline{v}_\phi(s, z, t) = \{\overline{v}_\phi(s, t)\} + \overline{v}_\phi^{NR}(s, z, t), \tag{1}$$

where $\{\overline{v}_\phi(s, t)\}$ is the spatially averaged azimuthal velocity of the cylinder (i.e. a rigid flow), and $\overline{v}_\phi^{NR}(s, z, t)$ is the departure from that mean and includes axial gradients (i.e. a non-rigid flow). By separating $\overline{v}_\phi(s, z, t)$ in this fashion, the angular momentum carried by each cylinder is taken entirely by the rigid flow. If the non-rigid part of the flow is zero, then the flow at the end of the cylinder (i.e. the flow at the surface of the core) is equal to the rigid flow and the calculation of the core angular momentum from the surface flow is straightforward. On the other hand, if the non-rigid flows are important, then a knowledge of the axial gradients everywhere along the height of the cylinder is necessary. Unfortunately, the theory of the geodynamo is not sufficiently advanced to predict how the flow in the interior behaves in relation to surface flows, and the calculation of angular momentum is not possible.

In what follows, we concentrate not on attempting to establish the exact form of the flow gradients inside the core, but solely on trying to determine the relative importance between rigid and non-rigid flows for changes that have a typical timescale of 1500 yr. This will provide guidelines for interpreting our flow inversion results and the connection with the changes in LOD.

2.1 Magnetostrophic balance

Fluid parcels in the core must obey the momentum equation. When using the radius of the core r_c as a typical length scale, the decay time of magnetic field $\tau_\eta = r_c^2/\eta$ as a typical timescale, and $\mathcal{B} = \sqrt{2\Omega\rho\mu\eta}$ as a magnetic field scale (where ρ is density, Ω is the Earth's rotation frequency, μ is the permeability of free space, $\eta = 1/\mu\,\sigma$ is the magnetic diffusivity and σ is electrical conductivity), the non-dimensional momentum equation, in the Boussinesq-anelastic approximation, is (e.g. Gubbins & Roberts 1987)

$$R_o\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) + \mathbf{e_z}\times\mathbf{v} = -\nabla P + (\nabla\times\mathbf{B})\times\mathbf{B} + R_a\Theta\mathbf{r} + E\nabla^2\mathbf{v}. \tag{2}$$

In the above equation, the vectors **v**, **B**, **r** and $\mathbf{e_z}$ are, respectively, the velocity field, the magnetic field, the radius vector and the unit vector in the direction of the Earth's rotation, t is time, P is pressure and Θ is the deviation in temperature from a background adiabatic state. The non-dimensional numbers R_o, E and R_a are, respectively, the Rossby, Ekman and Rayleigh numbers and are defined as

$$R_o = \frac{\eta}{2\Omega r_c^2}, \quad E = \frac{\nu}{2\Omega r_c^2}, \quad R_a = \frac{g\alpha\beta r_c^2}{2\Omega\eta}, \tag{3}$$

where ν is the kinematic viscosity, g is the gravitational acceleration at the core–mantle boundary (CMB) and α is the coefficient of volume expansion. β is a typical value of the temperature gradient and consequently, the typical scale of Θ is βr_c. Using typically adopted values of $\eta \approx 1$ m^2 s^{-1} and $\nu \approx 10^{-6}$ m^2 s^{-1} for Earth's core (e.g. Gubbins & Roberts 1987), we get

$$R_o \approx 10^{-9}, \quad E \approx 10^{-15}. \tag{4}$$

This suggests that, to a first order, one may neglect the influence of inertia and viscous forces in the force balance, which leads to the so-called magnetostrophic balance,

$$\mathbf{e_z}\times\mathbf{v} = -\nabla P + (\nabla\times\mathbf{B})\times\mathbf{B} + R_a\Theta\mathbf{r}. \tag{5}$$

2.2 Non-rigid azimuthal flows

The dynamics responsible for generating flow gradients in z, and hence non-rigid flows, can be addressed through the above magnetostrophic balance. Taking the axisymmetric part of $\mathbf{e}_\phi\cdot\nabla\times(5)$, we get

$$-\frac{\partial \overline{v}_\phi}{\partial z} = [\overline{\nabla\times\mathbf{F}}]_\phi, \tag{6}$$

where the over-bar denotes an axisymmetric (ϕ-averaged) quantity and where **F** is the sum of the Lorentz and buoyancy forces,

$$\overline{\mathbf{F}} = \overline{(\nabla\times\mathbf{B})\times\mathbf{B}} + R_a\overline{\Theta}\mathbf{r}. \tag{7}$$

When the right-hand side of eq. (6) vanishes, $\overline{v}_\phi$ cannot have gradients in z and must be constant on cylindrical surfaces aligned with the rotation axis. This result is the well-known Proudman–Taylor constraint (Proudman 1916; Taylor 1917), which restricts flows to be 2-D in rapidly rotating fluids. In the Earth's core, the added constraint of the spherical boundary geometry limits the 2-D flows to rigid rotations of cylindrical surfaces (Bullard & Gellman 1954).

Using eq. (1), only $\overline{v}_\phi^{NR}$ enters the left-hand side of eq. (6): it represents a prescription for non-rigid flows. The non-rigid flow associated with the buoyancy force in eq. (7) is referred to as the thermal wind and is related to the temperature by

$$\frac{\partial}{\partial z}\overline{v}_\phi^{NR} = R_a[\mathbf{r}\times\nabla\overline{\Theta}]_\phi, \tag{8}$$

$$= R_a\frac{\partial\overline{\Theta}}{\partial\theta}, \tag{9}$$

where θ is colatitude. Hence, provided $\overline{\Theta}$ varies with colatitude, azimuthal flows with gradients in z are expected. We are interested here in the time-dependency of this thermal wind flow. Taking the time derivative of both sides of the thermal wind balance (eq. 9),

$$\frac{\partial}{\partial t}\frac{\partial}{\partial z}\overline{v}_\phi^{NR} = R_a\frac{\partial}{\partial t}\frac{\partial\overline{\Theta}}{\partial\theta}, \tag{10}$$

the variations in time in the axial gradients of the azimuthal flows are related to time-dependent variations of latitudinal gradients in $\overline{\Theta}$. (The inertial and viscous terms neglected earlier remain negligible compared to the terms in (eq. 10) when we take $\partial/\partial t$.) The latitudinal gradients in $\overline{\Theta}$ are governed by the evolution of the axisymmetric temperature which, when neglecting thermal diffusion and internal heating, is given in non-dimensional form by

$$\frac{\partial}{\partial t}\overline{\Theta} = -\overline{\mathbf{v}\cdot\nabla\Theta}, \tag{11}$$

$$= -\overline{\mathbf{v}}_p\cdot\nabla_p\overline{\Theta} - \overline{\mathbf{v}'\cdot\nabla\Theta'}, \tag{12}$$

where the subscript p refers to the poloidal part and where the primed quantities are non-axisymmetric components. For convecting flows in a rapidly rotating system, we expect the axisymmetric part of the poloidal flow to be small compared with other components. Hence, we expect $\mathbf{v}' \gg \overline{\mathbf{v}}_p$, a view supported by core flow inversions and numerical models of the geodynamo. These models also tend to suggest that $\overline{\Theta} \sim \Theta'$. Therefore, the temporal changes in $\overline{\Theta}$ (and hence, temporal changes in thermal wind profile) are mostly governed by the second term of eq. (12) and thus reflect globally integrated changes in temperature that result from advection by non-axisymmetric convective motions.

The timescale over which the changes in thermal wind become significant can be determined as follows. Taking typical velocity of fluid parcels in convective cells to correspond to core surface velocities, about 10 km yr^{-1}, and assuming the typical size of these cells to be the thickness of the fluid core, gives a turnover timescale of roughly 500 yr. We then expect that, at timescales of 500 yr and longer, changes in the temperature perturbation due to convective motions are significant enough to alter $\overline{\Theta}$. The typical timescale of changes in thermal wind, therefore, should be 500 yr and longer. If convective rolls inside the core are smaller, or alternatively, if the typical temperature perturbation length scale is smaller, then we expect this timescale to be correspondingly smaller.

A similar argument can be constructed around the Lorentz force and its associated magnetic wind. Using the solenoidal condition on the magnetic field, $\nabla\cdot\mathbf{B} = 0$, we can write the Lorentz force as

$$\overline{(\nabla\times\mathbf{B})\times\mathbf{B}} = \overline{\mathbf{B}\cdot\nabla\mathbf{B}} - \frac{1}{2}\nabla(\overline{\mathbf{B}\cdot\mathbf{B}}). \tag{13}$$

Taking $\mathbf{e}_\phi \cdot \nabla\times$ (13) eliminates the $\nabla(\overline{\mathbf{B}\cdot\mathbf{B}})$ term, and the Lorentz part of eq. (6) is

$$\frac{\partial}{\partial z}\overline{v}_\phi^{NR} = -[\nabla\times\,(\overline{\mathbf{B}\cdot\nabla\mathbf{B}})\,]_\phi, \tag{14}$$

$$= -[\nabla\times\,(\overline{\mathbf{B}}\cdot\nabla\overline{\mathbf{B}}+\overline{\mathbf{B}'\cdot\nabla\mathbf{B}'})\,]_\phi. \tag{15}$$

We are concerned with temporal changes of the magnetic wind,

$$\frac{\partial}{\partial t}\frac{\partial}{\partial z}\overline{v}_\phi^{NR} = -\left[\nabla\times\left(\frac{\partial\overline{\mathbf{B}}}{\partial t}\cdot\nabla\overline{\mathbf{B}}+\overline{\mathbf{B}}\cdot\nabla\frac{\partial\overline{\mathbf{B}}}{\partial t}+\frac{\partial}{\partial t}\overline{\mathbf{B}'\cdot\nabla\mathbf{B}'}\right)\right]_\phi, \tag{16}$$

where the evolution of the magnetic field is governed by the (non-dimensional) induction equation,

$$\frac{\partial\mathbf{B}}{\partial t} = -\mathbf{v}\cdot\nabla\mathbf{B}\,+\,\mathbf{B}\cdot\nabla\mathbf{v}\,+\,\nabla^2\mathbf{B}. \tag{17}$$

Using a similar argument as for the the thermal wind part, we expect that in an overturn timescale of 500 yr, the temporal changes in $\mathbf{B}'$ and $\overline{\mathbf{B}}$ resulting from advection and shear by the flow produce significant changes in non-rigid flows in the form of magnetic wind. Again, for smaller convective eddies or for a smaller typical magnetic field length scale, this timescale becomes smaller.

2.3 Rigid azimuthal flows

The dynamics responsible for rigid azimuthal flows can also be deduced with the magnetostrophic balance as a starting point. Upon integration of the azimuthal component of eq. (5) on cylindrical surfaces aligned with the rotation axis, the Coriolis, pressure and buoyancy terms all vanish identically. This then leads to a morphological constraint on the magnetic field,

$$\int_\Sigma((\nabla\times\mathbf{B})\times\mathbf{B})\,d\Sigma\;=\;0, \tag{18}$$

where $d\Sigma = s\,d\phi\,dz$ and $(s,\,\phi,\,z)$ are cylindrical coordinates. In other words, the axial Lorentz torque on cylinder surfaces must cancel. This result is known as Taylor's constraint (Taylor 1963).

One expects that the complex dynamics involved in the geodynamo lead to a time-dependent magnetic field, which cannot satisfy this constraint at all times and produces non-vanishing Lorentz torques. These torques must then be balanced by one of the terms that were left out of the magnetostrophic balance. Reinstating the acceleration term and the viscous friction (through Ekman pumping in boundary layers at the CMB), a modified Taylor's constraint can be written as (e.g. Jault 1995; Fearn 1997; Bloxham 1998)

$$R_o\frac{\partial}{\partial t}\{\overline{v}_\phi\}\,+\,\frac{E^{1/2}}{z^{3/2}}\{\overline{v}_\phi\} = \frac{1}{4\pi sz}\int_\Sigma((\nabla\times\mathbf{B})\times\mathbf{B})\,d\Sigma. \tag{19}$$

The relative importance of inertial acceleration versus viscous forces to balance the Lorentz torque depends on the timescale of the magnetic field variations considered. For changes that occur on a timescale shorter than the spin-up time $\tau_v = R_oE^{-1/2}$, inertia is dominant. Using the values in eq. (4), we get $\tau_v \sim 10^4$ yr. At shorter timescales, non-vanishing Lorentz torques are balanced by inertial accelerations and the integral constraint becomes

$$R_o\frac{\partial}{\partial t}\{\overline{v}_\phi\} = \frac{1}{4\pi sz}\int_\Sigma((\nabla\times\mathbf{B})\times\mathbf{B})\,d\Sigma. \tag{20}$$

The inertial acceleration thus induced has a feedback on the Lorentz torque: it produces changes in the azimuthal component of the magnetic field and in so doing produces a secondary Lorentz torque that opposes the original torque. The above system allows oscillatory solutions of $\{\overline{v}_\phi\}$, that is, azimuthal motions of rigid cylinder surfaces, about a position where Taylor's constraint is satisfied. These are known as torsional oscillations and their typical periodicity is $\tau_{to} = R_o^{1/2}/B_s$ (Braginsky 1970). For a typical magnetic field strength of 0.5 mT (or 5 Gauss), this leads to periods with a characteristic timescale of decades.

Mathematically, eq. (20) does not require the azimuthal velocity to be rigid, but only that its integrated average over the cylinder balances the Lorentz torque. In other words, it does not preclude the presence of a non-rigid component of the flow participating in torsional oscillations. However, the amplitude of the non-rigid flows involved in torsional oscillations is much smaller than the rigid flows. This can be verified by estimating the amplitude of the magnetic wind produced by the shearing action of a rigid azimuthal flow. The latter is given by a linearized form of eq. (15),

$$\frac{\partial}{\partial z}\overline{v}_\phi^{NR} = -2\frac{\partial}{\partial z}\left(\frac{\overline{B_\phi^o}\,\overline{B_\phi^\omega}}{s}+\frac{\overline{B_\phi^{o\prime}\,B_\phi^{\omega\prime}}}{s}\right), \tag{21}$$

where $\overline{B_\phi^o}$ and $B_\phi^{o\prime}$ are, respectively, the axisymmetric and non-axisymmetric azimuthal part of a steady background magnetic field, and $\overline{B_\phi^\omega}$ and $B_\phi^{\omega\prime}$ are the time-dependent parts induced by a periodic rigid flow. The change in the azimuthal component of the magnetic field resulting from torsional oscillations is (Braginsky 1970)

$$\overline{B_\phi^\omega} = \frac{s\overline{B_s^o}}{i\omega}\frac{\partial}{\partial s}\frac{\{\overline{v}_\phi\}}{s}, \tag{22}$$

and similarly for $B_\phi^{\omega\prime}$ except replacing $\overline{B_s^o}$ by $B_s^{o\prime}$. Using $2\pi\,\|B_s^o\|/\sqrt{R_o}$ as an estimate of the torsional oscillations frequency ω, where $\|B_s^o\|$ is the largest of $\overline{B_s^o}$ and $B_s^{o\prime}$, an order of magnitude of $\|B_\phi^\omega\|$ (the largest of $\overline{B_\phi^\omega}$ and $B_\phi^{\omega\prime}$) is given by

$$\|B_\phi^\omega\| \sim \sqrt{R_o}\{\overline{v}_\phi\}. \tag{23}$$

Using this order of magnitude in eq. (21) gives an estimate of the amplitude of the non-rigid versus the rigid flow,

$$\frac{\overline{v}_\phi^{NR}}{\{\overline{v}_\phi\}} \sim 2\sqrt{R_o}\frac{\partial}{\partial z}\|B_\phi^o\|. \tag{24}$$

For this ratio to be $\mathcal{O}(1)$, the azimuthal component of the field must be $\mathcal{O}(R_o^{-1/2})$, which in dimensional units correspond to a field of $\sim 10^4$ mT. This is a value larger by a few orders of magnitude than the most generous estimate of B_ϕ in the core. Alternatively, eq. (24) is $\mathcal{O}(1)$ if the characteristic length scale of B_ϕ along the rotation axis is of the order of 100 m. However, if this was the case, the associated ohmic dissipation would exceed the most generous estimate of heat flux coming out of the core (e.g. Buffett 2002). Therefore, $\overline{v}_\phi^{NR} \ll \{\overline{v}_\phi\}$, which reflects that the Lorentz force resulting from the action of a rigid azimuthal flow is almost entirely in the azimuthal direction, which implies that the right-hand side of eq. (6) is very small. Torsional oscillations are indeed oscillations of rigid cylindrical surfaces.

Rigid flow motions are not restricted to torsional oscillations as Taylor's constraint remains valid at timescales outside the periodicity spectrum of torsional oscillations. For instance, the changes in thermal and magnetic wind with a typical timescale of 500 yr described in the previous section are most probably accompanied by a change in the rigid flow component. This is because local changes in $\overline{v}_\phi^{NR}(s,z,t)$ lead to changes in $\overline{B}_\phi(s,z,t)$ through the induction equation and, therefore, a likely change in the Lorentz torque. If this is the case, then according to eq. (20) a rigid acceleration over a timescale of 500 yr is required in order to balance this torque. Alternatively, if the kinematic viscosity in the fluid core is higher than the estimate given in Section 2.1, or if turbulent mixing is important, the balance is provided by viscous friction through Ekman pumping at the solid boundaries. In any case, these slower changes in rigid flow do not correspond to torsional oscillations. Rather, they represent a readjustment of the mean differential rigid rotation between cylinder surfaces in order to maintain Taylor's constraint. In the language of geodynamo theory, this is a change in the geostrophic flow. We expect these longer timescale rigid flows to be strongly attenuated, even if the acceleration provides the balance, because magnetic field diffusion is important at these timescales.

2.4 Expected dynamics of azimuthal flow in the core at millennial timescales

At decade timescales, observations suggest that rigid flows dominate the time-dependent dynamics of axisymmetric azimuthal flows in the core. This is consistent with the presence of torsional oscillations as described in the previous section. The dominance of rigid over non-rigid flows at decade timescales may simply be because the changes in magnetic and thermal winds are not sufficiently important on such short timescale. An alternative view is that even if non-rigid flows were important, they would be accompanied by changes in rigid acceleration and, therefore, would excite large torsional oscillations very efficiently.

At a timescale of 1500 yr, we do not expect rigid flows to dominate. Clearly, if the changes in millennial LOD result from an exchange of angular momentum with the core, then rigid flows must be non-zero for at least some of the cylinders. Rigid flows at these timescales represent a diffusive readjustment of the background differential rigid rotation between the cylinders. We expect such readjustments to be a consequence of the complex changes in the 3-D structures of the field as a result of convective motions. Hence these changes should also be accompanied by consistent changes in thermal and magnetic winds. In other words, we expect the diffusive readjustment velocity to include gradients in z with amplitudes comparable to the rigid flow.

Although none of the arguments above are definitely conclusive, they tend to support the view that time-dependent changes in azimuthal flows at millennial timescales are comprised of both rigid and non-rigid flows. A more rigorous approach to determine the relative contribution of each would be to investigate the natural modes of oscillation and instability of MAC waves (e.g. Braginsky 1967) that involve the axisymmetric azimuthal part of the flow for simple temperature and magnetic field background profiles. This is beyond the scope of the present paper.

Alternatively, one can use the results of numerical models of the geodynamo as a guide to the theoretical assertions above. Fluctuations about a time-averaged axial zonal wind profile have indeed been observed in some simulations (Aubert 2005). Unfortunately, it is difficult to relate the amplitude, timescale and form of the fluctuations observed in the numerical models directly to those that may occur in the fluid core. This is because the parameter regime that one can attain numerically remains far from that of the Earth. More specific to the issues involved in the current study, the difference in parameter regime affects the force balance on cylindrical surfaces (e.g. Dumberry & Bloxham 2003), and consequently it affects the dynamics controlling rigid and non-rigid flows. Therefore, the current generation of numerical models cannot prove or disprove unambiguously the views we have presented in this section. Nevertheless, numerical models may be the best tool for future studies on the dynamics of rigid and non-rigid flows in the core.

3 INVERSION FOR FLUID FLOWS AT THE SURFACE OF THE CORE

We now proceed to build time-dependent flows at the surface of the core that are consistent with the observed ASV. Flow maps inverted from the historical secular variation of the magnetic field are based on the frozen-flux hypothesis (Roberts & Scott 1965; Backus 1968), which assumes that the advection of the magnetic field by the flow dominates diffusion. The radial component of the induction equation in this diffusionless limit is

$$\frac{\partial B_r}{\partial t} = -\nabla_h \cdot (B_r \mathbf{u}_h). \tag{25}$$

The knowledge of B_r and $\partial B_r/\partial t$ is then used to invert for the horizontal component of the flow $\mathbf{u}_h$. The flow obtained with the above equation is non-unique and additional dynamical assumptions must be specified in order to reduce the ambiguity. Assumptions that are frequently made are that the flow is steady (Gubbins 1982; Voorhies 1986), tangentially geostrophic (Hills 1979; Le Mouël 1984), or purely toroidal (Whaler 1980). For a review of the subject, see Bloxham & Jackson (1991).

In the present study, we are only interested in flows that carry changes in angular momentum: the time-dependent part of the azimuthal component that is both axisymmetric and symmetric about the equator. In the context of historical geomagnetic secular variation, this component of the flow can be obtained for example by extracting it from a time-dependent geostrophic model from which the steady part has been removed (Zatman & Bloxham 1997, 1998; Pais & Hulot 2000; Hide *et al.* 2000). Alternatively, one can invert for a flow for which the time-dependent part is specified entirely in terms of axisymmetric azimuthal flows symmetric about the equator (Bloxham *et al.* 2002). Even though this last method involves a very parsimonious representation of the time varying flow, it successfully explains the part of the secular variation due to torsional oscillations, which is shown to include geomagnetic jerks.

Here, we proceed along the lines of the latter method. We seek flows which can best explain the ASV, but for which the time-dependent part is restricted to contain only the axisymmetric, equatorially symmetric azimuthal component. Using vector spherical harmonics, the time-dependent part of the flow $\mathbf{u}_T(\theta, t)$, where θ is colatitude, can be represented entirely in terms of a toroidal flow decomposition as

$$\mathbf{u}_T(\theta, t) = \nabla \times (\mathcal{T}(\theta, t) r \mathbf{e}_r), \tag{26}$$

with

$$\mathcal{T}(\theta, t) = \sum_{n\,odd} t_n^0(t) P_n^0(\cos\theta), \tag{27}$$

where $\mathbf{e}_r$ is a unit vector in the radial direction, $\mathcal{T}$ is a toroidal scalar and $P_n^0(\cos\theta)$ are Legendre polynomials. The time-dependent flow is then entirely specified in terms of the coefficients $t_n^0(t)$. We restrict n to *odd* values, which forces flow symmetry across the equator. (We note that since we expect flows to have an important shear in z, there is no reason to assume this equatorial symmetry. However, anti-symmetric flows do not carry angular momentum and cannot be constrained with the LOD data. For this reason, we decided not to include *even* time-dependent flow coefficients in our inversion.

We have verified that including these coefficients does not alter the results presented in this study significantly.)

Our flow model is obtained by solving the linear inverse problem, minimizing the misfit to the ASV. All of the flow coefficients are retained in the inversion, although a heavy damping is imposed on the time derivative of coefficients other than $t^0_{n=odd}$ to ensure that they remain steady. Our inversion is regularized temporally by minimizing a norm of the second time derivative, and spatially by minimizing a norm of the second spatial derivatives of the horizontal flow (as in Bloxham 1988, although we define two separate norms: one for the flow coefficients $t^0_{n=odd}$ and one for all others).

Several concerns can be raised about the application of the above method for extracting core flow variations at millennial timescales. First and foremost, for timescales longer than a few centuries, diffusion of the magnetic field is expected to play a role in the dynamics. This compromises seriously the assumption of frozen flux. Indeed, part of the ASV consists of growth or decay in intensity of specific geomagnetic features (Constable *et al.* 2000; Korte & Constable 2003). The dynamics governing these changes certainly involve diffusion, and ideally our inversion procedure should include these effects (e.g. Gubbins 1996). If we were seeking poloidal flows, the breakdown of frozen flux would indeed be a serious concern. However, the restricted time-dependent flow that we seek to recover should not be sensitive to local changes in intensity and should depend only on the part of the secular variation which consists of longitudinal displacements. It is highly unlikely that diffusion can give rise to a form of secular variation which can be aliased into a purely axisymmetric azimuthal flow. At least for the case where a time-dependent axisymmetric zonal flow advects both the magnetic field structure and the underlying dynamics that sustains it (and for which diffusion is important), using the frozen-flux assumption to recover this flow likely remains a valid approximation. This is similar to the argument used in the drifting reference frame flow models (Voorhies & Backus 1985; Davis & Whaler 1996; Holme & Whaler 2001).

Furthermore, neglecting diffusion in the context of our specific scenario can be justified on quantitative grounds. For the axisymmetric azimuthal motions that we seek, the right-hand side of eq. (25) contains only one term: the advection of azimuthal field gradients by the flow,

$$\frac{\partial B_r}{\partial t} = -\frac{\overline{v}_\phi}{r\sin\theta}\frac{\partial B_r}{\partial \phi}. \tag{28}$$

For a characteristic velocity scale $\mathcal{U}$ and a magnetic field scale $\mathcal{B}$, an order of magnitude of the secular variation from eq. (28) is $\mathcal{UB}/\mathcal{L}$, where $\mathcal{L}$ is the length scale over which B_r varies. If the azimuthal displacements of B_r were instead the result of diffusion, an order of magnitude of the secular variation would be $\eta\mathcal{B}/\mathcal{L}^2$. From the ratio of these two estimates, we form a magnetic Reynolds number

$$R_m = \frac{\mathcal{UL}}{\eta}. \tag{29}$$

For a typical value of the flow of 10^{-4} m s^{-1} ($\sim$3 km yr^{-1}, see Fig. 1), a typical length scale of B_r between spherical harmonic degree 2 and 5 or, respectively, $\sim$5000 km and $\sim$2000 km (the scale we can recover in the archaeomagnetic field model, see next section), and $\eta \sim 1$ m^2 s^{-1} (see Section 2.1), we obtain values of R_m between 200 and 500. Hence, for this specific flow geometry, diffusion should not be important. The frozen-flux approximation remains valid, although not in the sense that the flux integrals over null-flux curves remain invariant in time (Backus 1968), but only in the sense that azimuthal displacements of magnetic field can be used as a tracer for axisymmetric azimuthal fluid motion.

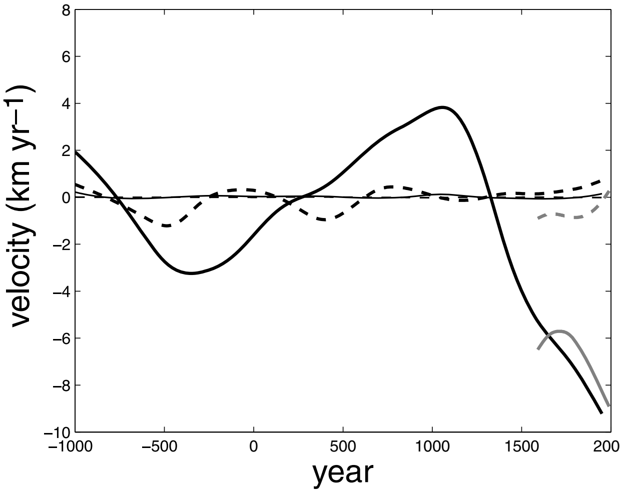

Figure 1. Coefficients t^0_1 (thick solid line), t^0_3 (thick dashed line), t^0_5 (thin solid line) and t^0_7 (thin dashed line) of the inverted millennial flow model. The coefficients t^0_5 and t^0_7 have very small amplitudes and are barely distinguishable from zero. The solid and dashed grey lines between 1590 and 1990 are, respectively, the coefficients t^0_1 and t^0_3 of the flow inversion from GUFM.

A second worry, also related to diffusion, concerns the steady part of the flow. As it has been demonstrated for the case of kinematic dynamos, if the geodynamo is in a nearly steady state, the secular variation at the surface results from a near balance between the right-hand side of eq. (25) and diffusion (Gubbins & Kelly 1996; Love 1999). In this case, the steady part of the flow inverted from eq. (25) is erroneous. One may argue that, for the slowly varying flows we are seeking, diffusion may have time to respond and lead to a similar problem. However, the value of R_m derived above also represents a ratio of diffusion to advective timescales for the azimuthal motion of magnetic field features. Hence, it indicates that, for the part of the dynamics we are trying to recover, the timescale over which diffusion may respond is much greater than that due to advective effects.

We note in addition that an inherent assumption in our procedure is that the longitudinal drift of magnetic field features is due to advection by fluid motion. An alternative view is that these may represent the azimuthal propagation of a magnetic wave for which the associated flow is markedly different. Indeed, the historical secular variation probably contains a part which is due to waves rather than fluid motion (Hide 1966; Braginsky 1967; Jackson 2003), and as we mentioned in the introduction, the propagation of longitudinal MAC waves has already been proposed as an explanation for the ASV. If this is the case, provided we have sufficient spatial resolution, an inversion that uses frozen flux and retain all harmonics for the time-dependent flow should in principle be able to recover the flow that is directly associated with the wave. In practise however, it would be difficult to recover this smaller-scale flow because the regularized inversion is biased toward the larger scale, and for this reason it would likely map any non-zero phase propagation into a large-scale flow. In this study, where we retain only the axisymmetric harmonics for the time-dependent flow, we have obviously no chance in recovering the details of the flow associated with longitudinal MAC waves. Thus, if the azimuthal drifts are in reality due to the azimuthal propagation of waves, our inversion mistakenly assigns their phase velocity to a large-scale flow. This is certainly a possibility and the results that we infer in this study are subject to this caveat.

Even if none of the above concerns are a problem, our flow model may still be polluted by the effects of non-uniqueness. Consider for example the case where equal amounts of eastward and westward flow exist at the core surface so that the globally averaged azimuthal flow is zero. If large azimuthal gradients in the radial magnetic field are only located under the regions of westward flow (a real effect, or one due to the limitation of the magnetic field model), the secular variation would consist only of westward motion, which would be aliased into a global axisymmetric westward flow.

Ideally, instead of using an inversion that uses techniques appropriate to study decade to century timescale dynamics, it would be more desirable to develop a new inversion scheme consistent with millennial timescale dynamics. However, here we use this simple method and hope that it will be sufficient for our purpose. All the concerns raised above can be addressed *a posteriori*, once we have verified whether there exists a connection between our inverted flows and the millennial variations in LOD. As a case in point, some of the above concerns also apply to the inversion of zonal flows at decade timescale. Yet, the agreement between the observed variations in the LOD and the prediction based on inverted core flows suggests that the time-dependent part of the zonal flows is well retrieved and indeed represents fluid motion as opposed to waves. The confidence in the time-dependent zonal flows that we invert at millennial timescale is similarly subject to an independent test with the variations in the LOD. Absence of correlation may not unambiguously resolve the issue, as it may indicate that our inverted flows are flawed but could also imply that millennial variations in the LOD are not due to core–mantle angular momentum exchange. A positive correlation on the other hand, despite the chance that it may be fortuitous, will tend to support the validity of our inverted flows.

4 INVERSION RESULTS AND CHANGES IN LOD

4.1 inversion results

We use the archaeomagnetic field model CALS3K.2 (Korte & Constable 2005), which covers the period 1000 BC to 1950 AD. The magnetic field is expanded spatially in real spherical harmonics truncated at degree 10 and is expanded in time on a cubic B-splines basis on a sequence of 53 equally spaced knots. The flow model that we invert is similarly truncated at degree 10 and expanded in cubic B-splines on the same knot sequence. We seek flows that have millennial timescale variations. This is for two reasons. First, to minimize the possibility of modelling parts of the ASV which are due to over-fitting of the data in the archaeomagnetic model itself. Secondly, our aim is to verify a possible correlation between the inverted azimuthal flows and changes in rotation rate of the mantle. The eclipse record, from which the variations in the LOD are reconstructed, is not precise enough to constrain variations that occur on timescales shorter than about a 1000 yr. Therefore, only the part of the azimuthal flow which varies on a millennial timescale can be compared with the changes in rotation rate of the mantle.

The coefficients t_1^0, t_3^0, t_5^0 and t_7^0 of our preferred flow model inversion are shown in Fig. 1. These include both the steady and time-dependent part. The largest part of the time-dependency is carried by t_1^0 alone, a solid body rotation of the whole surface of the core. This is partly a consequence of our choice of damping in the inversion. This may also be partly due to the smoothness in the construction of the archaeomagnetic field model itself, as the lowest degree harmonics that dominate the secular variation require large-scale flows.

On Fig. 2, we show the evolution of a few of the Gauss coefficients of the archaeomagnetic field model (thick solid line) and the fit obtained with our flow model (dashed line). We do not show the zonal Gauss coefficients ($m = 0$) because they are not influenced by purely zonal azimuthal flows and, consequently, our flow model is not expected to reproduce any of the time-dependent features of these coefficients. We note that the magnetic field predicted from the flow model can only be determined up to an arbitrary initial condition value. We chose this value to correspond to the field model at 1000 BC and a different choice can improve the fit slightly.

It is clear that only some features of the long term average trend of the ASV are captured by our parsimonious flow model. Our model also does progressively worse as we consider coefficients with higher harmonic degree. The variance reduction, specified in terms of the time-dependent Gauss coefficients following the definition given by Bloxham (1989), is only 28.6 per cent. However, it is not a surprise that the overall fit is poor: the part of the ASV that our flow can capture—the axisymmetric longitudinal drifts of magnetic features—represents only a small fraction of the total ASV. To put this result in perspective, the secular variation for the 20th century can be explained with a variance reduction above 95 per cent by simple steady flows (Bloxham 1992). A variance reduction of over 98 per cent is achieved when torsional oscillations are added to the steady flows (Zatman & Bloxham 1997). Hence, the time-dependent azimuthal flow motions in historical models are also responsible for only a small part of the secular variation. That the remaining part is explained well with a steady flow is indicative that over that time period the field has not changed in an overly complicated fashion. In contrast, most of the ASV is due to more complicated time-dependent behaviour, including non-axisymmetric and/or meridional drifts, and growth or decay of local flux patches, all of which cannot be explained by simple steady flows. Hence, although our flow model at millennial timescales provides a significantly worse overall fit of the secular variation than a flow model obtained with the same technique over the last hundred years, this is mostly reflecting the inadequacy of the steady part of the flow at explaining the ASV. In fact, the variance reduction for the steady part of our inverted flow is a mere 1.1 per cent. In simple terms, steady flows work well over short time periods and poorly over longer intervals. The time-dependent azimuthal flows, the part of the flow model which we are interested in, may still be well recovered.

To illustrate this point, we produced an inversion for which the constraint on the form of the time-dependent flow is slightly relaxed, allowing components of the flow other than just t_{odd}^0 to vary in time. The fit of the ASV for this flow model is shown in Fig. 2 (thin line). Clearly, a much larger part of the ASV is explained and the variance reduction for this case is 93.8 per cent. (In fact, relaxing the constraint even more, we found solutions that can explain virtually all of the ASV.) However, the general character of the $t_{\mathrm{odd}}^0(t)$ part of the flow remains unaltered. We show in Fig. 3 the first few coefficients $t_{\mathrm{odd}}^0(t)$ for this inversion. The coefficients have different amplitude than in Fig. 1, but their general behaviour remains the same. While this does not constitute an exhaustive proof, it supports our assertions that our parsimonious time-dependent flow model may be valid even though it explains only a small part of the ASV.

(We note that for the inversion with the relaxed constraint on flow geometry, we do not believe that the components of the time-dependent flow model other than the zonal toroidal part have any

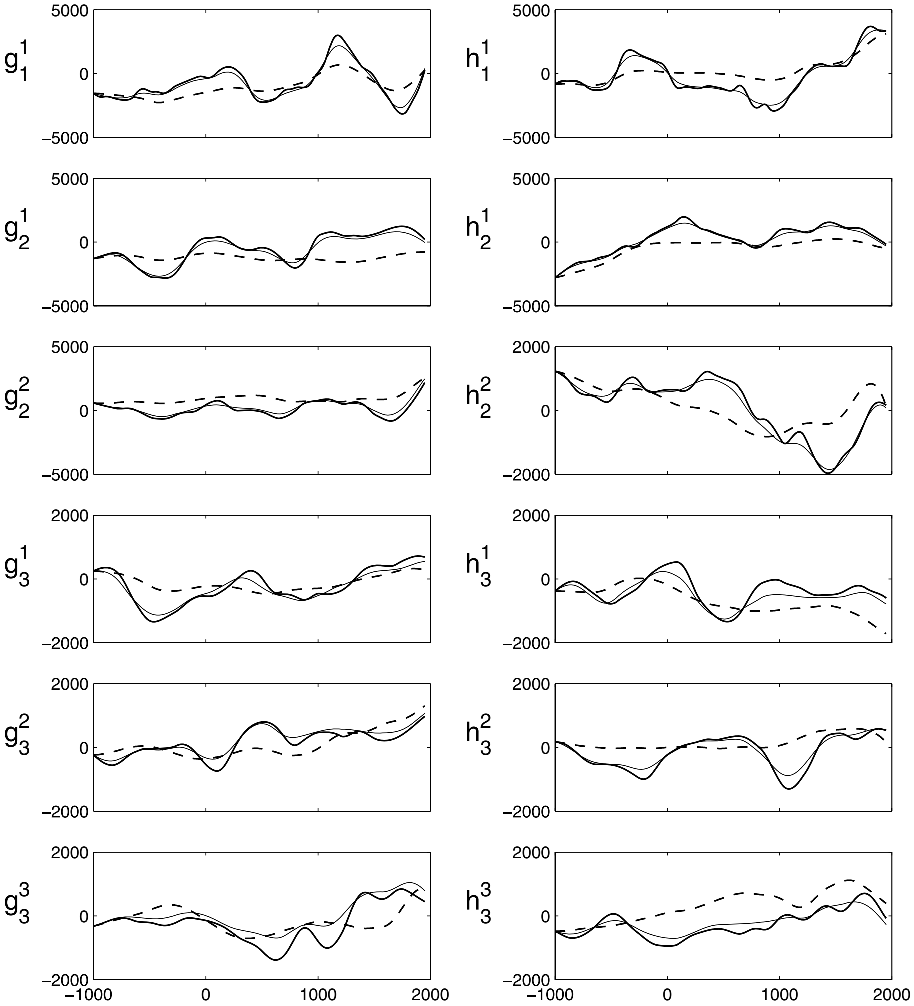

Figure 2. Evolution of some of the Gauss coefficients g_l^m and h_l^m of the archaeomagnetic field model CALS3K.2 (thick solid lines), the fit generated from our inverted flow model when the time-dependent part is restricted to toroidal odd zonal harmonics (dashed lines), and the fit when this restriction is relaxed (thin solid line). Units of the vertical axes are in nT and units of the horizontal axes are in years.

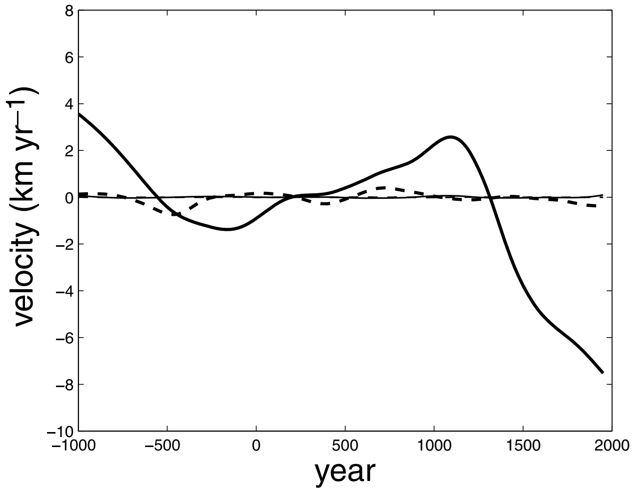

Figure 3. Coefficients t_1^0 (thick solid line), t_3^0 (thick dashed line), t_5^0 (thin solid line) and t_7^0 (thin dashed line) of the flow model for the inversion for which the restriction on the time-dependent flow is relaxed.

real physical significance. These additional components certainly suffer from the caveats of neglecting diffusion as we explained in the previous section.)

A possible way to impose a tighter constraint on the time-dependent azimuthal flows is to isolate from the total ASV the part attributable to azimuthal drifts and constrain the flow only to fit that part of the signal. We have tried this option by using the method devised by James (1970), which separates the secular variation into a solid rotation part, a part due to a stationary growth or decay, and a residue. This method is obviously not ideal as we then lose the part of the drift due to non-solid body rotation of the surface flow, that is, flow components t_3^0, t_5^0, etc. However since t_1^0 is significantly larger than other flow components, one can argue that this may be a good enough approximation. Unfortunately, the results were not improved. This could be because this specific method is not sufficiently refined to allow for a correct separation between the drifting and non-drifting part of the secular variation. Alternatively, it may also reflect the current limitations of the archaeomagnetic field models for extracting core zonal flows.

As a further test to explore the validity of our flow model, we have applied our flow inversion technique to the historical magnetic field model GUFM (Jackson *et al.* 2000). This model covers the time interval between 1590 and 1990 and overlaps with CALS3K.2 for a period of roughly 350 yr. Since the historical and millennial magnetic field models show good agreement during their overlap period (Korte & Constable 2003; Korte *et al.* 2005), the flows inverted from GUFM should in principle be consistent with our inversion of the archaeomagnetic field model. We note that GUFM relies on data that does not include direct magnetic field intensity prior to 1832 and, therefore, flow inversions depend on a scaling factor, which involves assumptions about the axial dipole component of the field (Jackson 2000). However, since the flow coefficients we are interested in do not change the axial dipole and since we impose heavy spatial and temporal damping, this may not be a fatal shortcoming. We have plotted the results of the historical model inversion on Fig. 1. The coefficients t_1^0 and t_3^0 agree well with their counterpart from the flow inversion of CALS3K.2, except that the steady part of t_3^0 is offset by about 1 km yr^{-1}. We note that this offset is of no consequence for our subsequent calculation of the changes in LOD, as the latter only depends on the time variations of the flow coefficients. We note in addition that the dip toward larger negative values seen in t_1^0 of the historical model near 1600 can also be reproduce in the millennial flow model if we reduce the temporal damping.

The correlation with the historical flow models during the last 350 yr obviously does not prove that our millennial flow model is valid for earlier times. Nevertheless, this correlation is encouraging and provides, at the very least, a good test of consistency. Therefore, if the assumptions on which our inversion is based hold, we believe that the flow model in Fig. 1 accurately captures the part of the ASV due to large-scale and slowly varying axially symmetric zonal flows.

4.2 Changes in length of day

We now verify whether our inverted flow model is also consistent with the observed changes in angular momentum of the mantle recorded as variations in LOD. At decade timescales, angular momentum in the core is carried by torsional oscillations and for this specific case, the inverted time-dependent flow at the top of the core allows a straightforward calculation of the angular momentum, which, in a mantle-fixed reference frame, is given by (Jault & Le Mouël 1991; Jackson *et al.* 1993)

$$L_c(t) = \frac{I_c}{r_c}\left(t_1^0(t) + \frac{12}{7}t_3^0(t)\right), \tag{30}$$

where r_c is the radius of the core and I_c its axial moment of inertia. The good agreement between the observed variations in LOD and the prediction determined from eq. (30) (Jault *et al.* 1988; Jackson *et al.* 1993) underpins our understanding of core dynamics at decade timescales. As discussed in Section 2, we do not expect the angular momentum in the core at millennial timescales to be carried by purely rigid flows and hence we do not expect that eq. (30) applies. However, the assumption of rigid flows provides a convenient and instructive first test. On Fig. 4, we compare the observed millennial variations in LOD to the prediction based upon the assumption that the time-dependent flow in Fig. 1 represents rigid cylindrical flows in the core. This prediction is calculated from

$$\Delta LOD(t) = \frac{2\pi}{\Omega^2}\frac{L_c(t)}{(I_m + I_c)}, \tag{31}$$

where Ω is the Earth's frequency of rotation, I_m is the axial moment of inertia of the mantle and $L_c(t)$ is given by eq. (30). The two curves on Fig. 4 have similar amplitudes and periodicities, although there is a clear misalignment between them.

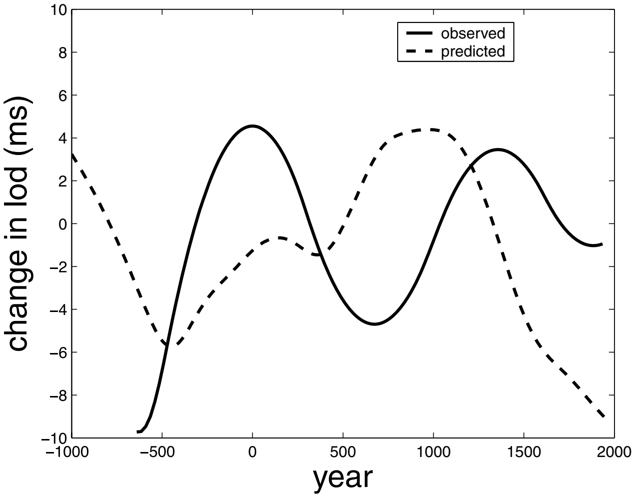

Figure 4. Comparison between the observed variations in the length of day obtained from historical eclipses (solid line) and the prediction from our inverted flow model (dashed line) based upon the assumption of rigid cylindrical flows in the core and conservation of angular momentum with the mantle. The part of the observed LOD curve after 1620 has been obtained by fitting a parabola to the telescopic and modern observations.

The similarity in amplitude and timescale between the observed and predicted variations in LOD is indicative of a connection between azimuthal core flows and the observed changes of angular momentum of the mantle. Hence, it suggests that, as for decade timescales, the variations in rotation rate of the mantle are due to exchanges of angular momentum with the core. However, because the curves are not in phase, it indicates that rigid cylindrical flows do not dominate the dynamics in the core at millennial timescales. Hence, if millennial changes in LOD are indeed due to core angular momentum changes, and if our inverted surface azimuthal flows are correct, non-rigid flows inside the core must be as important as rigid flows. This is consistent with our theoretical discussion in Section 2.

Unfortunately, it is difficult to establish the presence of non-rigid flows on a more robust basis. This is because if non-rigid flows are important, in order to demonstrate the correlation between the LOD and core angular momentum, we need to know the full velocity profile in z at all times on each cylinder. In the absence of a theory for the leading behaviour of core flows at millennial timescales, this is information we do not have.

To illustrate the relative importance between rigid and non-rigid flows, it is instructive to consider the following simple case. Let us assume that the flow profile is such that the angular momentum carried by each cylinder in the core is *equal and opposite* to that calculated based on the inverted azimuthal flow near the CMB. In other words, that the time-dependent flow near the ends of cylinders is equal and opposite to the azimuthal flow averaged over the cylinder (i.e. the rigid flow). For example, this would correspond to a flow profile such as that of Fig. 5. We emphasize that this idealized flow profile is not based on physical arguments but simply chosen because it is a simple case that satisfies the above requirements on the rigid and boundary flows.

The comparison of the observed changes in LOD and the prediction based on such a velocity profile is shown in Fig. 6. Admittedly, the correlation remains far from perfect, especially before 500 BC

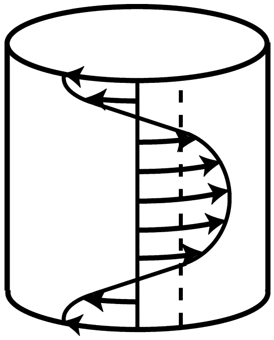

Figure 5. Schematic of the snapshot in time of a hypothetical time-dependent azimuthal velocity profile (black curved line) on a cylinder surface for which the boundary flow is equal and opposite to the rigid part of the flow. The black arrows represent the direction of the flow. Half of a period later, the flow profile and the average azimuthal velocity are reversed.

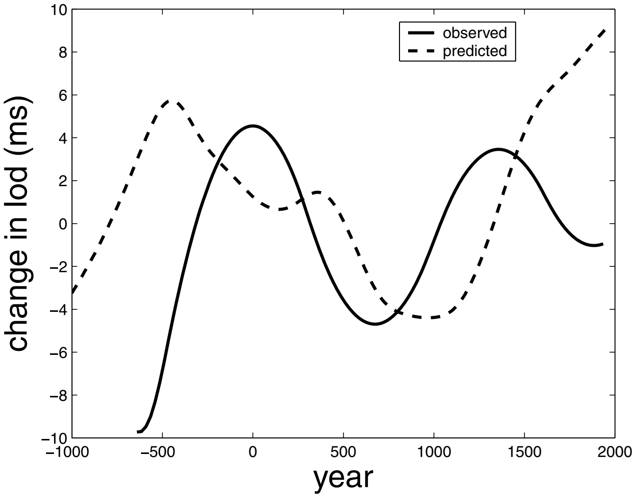

Figure 6. Comparison between the observed variations in the length of day obtained from historical eclipses (solid line) and the prediction from our inverted flow model (dashed line) for the hypothetical case where the azimuthal flow at the ends of cylinders is equal and opposite to the average velocity over the cylinder.

and after 1500 AD. However, between 200 BC and 1500 AD, the predicted and observed LOD variations are in good agreement.

We do not argue here that the thousand year timescale dynamics is carried by the flow pictured in Fig. 5. The exact relationship between the surface and rigid flow likely varies with both cylinder radius and time and an infinite number of flow profiles can be chosen in order to match perfectly the changes in LOD. The important point we want to make is simply that a larger portion of the LOD curve may be readily explained with a simple flow structure on cylinders for which the flow at the ends significantly departs from the rigid component.

4.3 Inversions constrained to satisfy LOD changes

As a further attempt to extract the relative importance of rigid and non-rigid flows and to provide better constraints for these flows, we performed the following experiment. We inverted for flows which are not only constrained to provide the best fit to the ASV, but are also constrained to carry changes in core angular momentum that match exactly the observed changes in LOD. We did this for the two different assumptions of flow profiles used in the previous section: the first is assuming purely rigid flows; the second assumes the flow at the ends of cylinders to be equal and opposite to the rigid part of the flow (such as in the idealized profile of Fig. 5).

In both cases, we were able to find simple flows that matched perfectly the observed LOD variations and yet also explained a part of the ASV. The time-dependent part of the flow coefficients for the rigid and non-rigid case are shown in Fig. 7. Upon a simple inspection of the flows in Fig. 7 versus those in Fig. 1, one cannot determine which model is more realistic from a physical point of view. The variance reduction for the purely rigid and non-rigid cases are, respectively, 24 and 14 per cent.

Although our flow model in Fig. 1 captures a larger part of the ASV, it does not do significantly better than the two models of Fig. 7. In other words, a purely rigid flow regime and one with both rigid and non-rigid flows are equally compatible with the observations. This illustrates the non-uniqueness problem when inverting for core flows (Backus 1968; Holme 1998a). In the present case, it also reflects that our flows are not well constrained by the archaeomagnetic field model.

The above results are a reminder that it may not be possible to unambiguously extract the time-dependent azimuthal flow dynamics from the current archaeomagnetic field models. This is a limitation that we cannot easily cast aside. And although it is not necessarily the case, the flows of Fig. 1 may not be very robust. The encouraging correlation between the predicted core angular momentum and the observed LOD in Fig. 4 may then be fortuitous and our tentative conclusion about the presence of large non-rigid flows at millennial timescales remains speculative.

5 DISCUSSION

5.1 Torque between the core and mantle

Our discussion on core–mantle dynamics at millennial timescales has so far focused on the angular momentum budget. If the LOD changes are due to flows in the core, there has to be a physical mechanism that applies a torque on the mantle. Another way to investigate whether core–mantle angular momentum exchanges are responsible for the millennial LOD changes is then to determine if a torque of the correct amplitude can be readily produced. As an

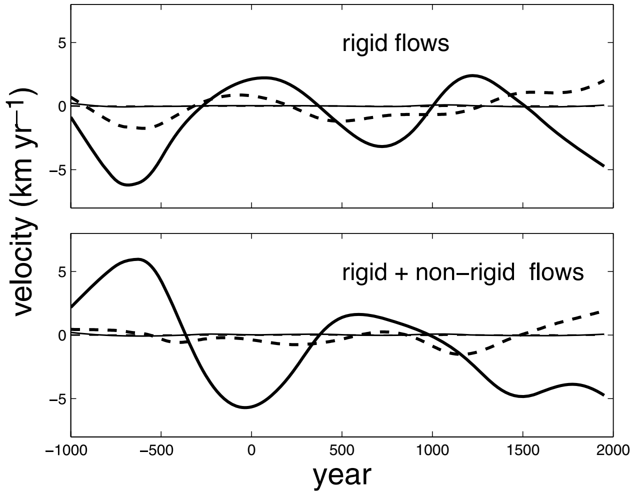

Figure 7. Coefficients t_1^0 (thick solid line), t_3^0 (thick dashed line), t_5^0 (thin solid line) and t_7^0 (thin dashed line) of the flow models that reproduce the exact changes in LOD, when the flow inside the core is rigid on cylinders (top), and when the flow has the profile of Fig. 5 (bottom).

analogy, the decade timescale angular momentum balance between the core and the mantle is well established, but the nature of the torque remains contentious as different mechanisms can reproduce the required torque on the mantle. In a sense this is good news, because if no mechanisms could account for the observed torque, one could feel sceptical about the core–mantle angular momentum budget. Here we wish to determine if we can draw similar conclusions at millennial timescales, and thereby solidify our assertion about the angular momentum budget.

In order to produce the observed millennial changes in LOD, a torque of roughly 10^{16} N m must be applied on the mantle. One possibility is electromagnetic coupling at the CMB (Bullard *et al.* 1950; Rochester 1960), which is efficient if there is a layer of highly conducting material at the base of the mantle (e.g. Buffett 1992). This coupling mechanism may play a role at decade timescales (Holme 1998a), and if it is the case, we expect it to be also important at millennial timescales. The amplitude of the electromagnetic torque is proportional to

$$\Gamma \propto u_\phi \sigma \Delta, \tag{32}$$

where σ and Δ are, respectively, the conductivity and thickness of the conducting layer at the bottom of the mantle. In order to fully account for the torque between the core and the mantle at decade timescales ($\sim 5 \times 10^{17}$ N m), the conductance of the layer ($\sigma\Delta$) must be above 10^8 S (Holme 1998b). Since the amplitude of u_ϕ in our millennial flow is similar to that of torsional oscillations at decade timescales, one may remark that the electromagnetic torque would then be too large at millennial timescales. However, this is not necessarily the case because the leakage part of the torque which we cannot observe may provide a counter effect, and the proper inclusion of diffusion in the dynamics governing this torque would decrease its efficiency.

Another possibility is that the torque is produced by viscous stresses at the CMB (Hide 1977). This torque is important when the viscous spin-up time is similar to the timescale considered. A spin-up time of 1500 yr requires an effective Ekman number of $\sim 10^{-14}$, corresponding to an effective kinematic viscosity of $\sim 2 \times 10^{-5}$ m^2 s^{-1}, a value only one order of magnitude larger than the molecular estimate (Poirier 1988; Alfè *et al.* 2000; Dobson 2002).

Topographic coupling from the perturbation of the mean azimuthal flows by the presence of bumps on the CMB is also a possible mechanism (Anufriyev & Braginsky 1977). The amplitude of this torque is proportional to

$$\Gamma \propto \frac{u_\phi \{\overline{B}_\phi^2\}}{\omega}, \tag{33}$$

where ω is the frequency of oscillation of u_ϕ. This torque is too small to account for decade exchange of angular momentum unless $B_\phi \sim 10$ mT inside the core (Buffett 1998), a value which is large but not unreasonable. However, values of B_ϕ on the order of 1 mT can easily accomplish a torque of 10^{16} N m for the lower frequency motion of the millennial variations. Diffusion of the magnetic field was not taken into account in the above relationship and would need to be included to give a more accurate amplitude of this torque.

A second kind of topographic coupling arises from the interaction of the non-axisymmetric part of the meridional flow with the bumps on the CMB (Hide 1969; Jault & Le Mouël 1989). This torque involves a part of the flow, which is not included in our inversion. For a CMB topography on the order of a kilometre, the amplitude of this torque is actually too large by almost two orders of magnitude than the torque required at decade timescales. Whether this type of topographic torque is adequately modelled remains a controversial issue (Kuang & Bloxham 1997; Kuang & Chao 2001). However, we note that if there are millennial timescales meridional flows with the same amplitude as those inferred from present-day core flow maps, then this torque would be larger by three to four orders of magnitude than the required millennial torque.

A volume torque in the form of gravitational coupling between the density heterogeneities $\Delta\rho_m$ in the mantle and the small density heterogeneities $\Delta\rho_c$ that drive convection in the fluid core (Jault & Le Mouël 1989) is probably too small at decade timescales. The amplitude of this torque is proportional to the angular displacement of $\Delta\rho_c$ as they are carried by azimuthal flows. At decade timescales, a typical torsional oscillation flow velocity of 10^{-5} m s^{-1} advecting $\Delta\rho_c$ for 10 yr produces an angular displacement of 10^{-3} rad, which leads to a torque of the order of 10^{14} N m, too small to explain the decade torque. However, the typical millennial core flows that we have recovered (also 10^{-5} m s^{-1}) produce an angular displacement 100 times larger, which leads to a torque of 10^{16} N m, of the correct magnitude to explain the millennial torque. Indeed, this torque has already been suggested as a candidate for the millennial LOD changes (Rubincam 2003).

A different gravitational torque mechanism involves coupling between density heterogeneities in the mantle and an inner core with longitudinal variations in topography (Buffett 1996). At decade timescales, this mechanism may be the most efficient one (Buffett 1998; Mound & Buffett 2003). However it may be less important at millennial timescales because of viscous relaxation of the inner core topography. Following the mapping of Buffett (1997), a characteristic relaxation time of 1000 yr corresponds to an inner core viscosity of 5×10^{19} Pa s. Estimates for the viscosity of the inner core range anywhere between 10^{11} Pa s (e.g. Van Orman 2004) and 10^{21} Pa s (e.g. Yoshida *et al.* 1996) and remains one of the least known parameters of the Earth's interior. However, if it is smaller than 5×10^{19} Pa s, this coupling mechanism is not effective.

In summary, the candidates for the torque at millennial timescales are many, including electromagnetic, viscous, topographic and gravitational. Since so many possibilities exist, it is difficult to imagine that we have overestimated the amplitude of the torque for all of the above cases. This suggests that a torque on the order of 10^{16} N m between the core and the mantle is readily achieved and provides an additional argument that the millennial changes in LOD are indeed due to core–mantle interactions. Further modelling efforts are required to asses each of the above possibilities in a more rigorous quantitative way. This is also desirable since such an investigation is likely to illuminate the issue of the torque at decade timescales.

As an additional observation, we note that if the torque results from one of the surface processes at the CMB (either from electromagnetic, viscous or topographic coupling of the first kind), and if our inverted flows are correct, then this provides an additional argument in favour of the presence of non-rigid flows in the core. Consider a change in azimuthal flow that results in a net change in core angular momentum in one direction. To conserve angular momentum, an equal change in mantle angular momentum in the opposite direction must take place. This implies that the change in rotation rate of the mantle must be necessarily in the opposite direction to the azimuthal flow that carries the change in core angular momentum. Yet, if the torque is due to a form of traction at the CMB, the azimuthal flow near the CMB must be in the same direction as the change in mantle rotation rate. In other words, the flow that accomplishes the torque must be in the reverse direction to the flow that carries the angular momentum of the core. These two requirements can only be simultaneously satisfied if the flow has a shear in either the s- or z-direction, or both.

At decade timescales, with torsional oscillations dominating the angular momentum transport, the flow has a shear in the s-direction alone. The cylinders that are the most efficient at transmitting the torque (for example those with $s \leq r_{ic}$ if the torque is from gravitational coupling between the inner core and the mantle (Buffett 1996), or those with $s \approx r_c$ if the torque is due to electromagnetic coupling at the CMB (Rochester 1960) and provided the rms B_r is everywhere equal on the CMB) are different from those that carry the most angular momentum (middle of the core). In fact, the requirement of satisfying both the torque and the angular momentum budget determines the resulting normal modes of torsional oscillations (Buffett 1998; Mound & Buffett 2003). The inverted historical time-dependent rigid zonal flow is indeed suggestive that this is the case. The change in core angular momentum as given by eq. (30) is the result of an almost perfect cancellation between the larger individual contribution from $t_1^0(t)$ and $t_3^0(t)$ (Jackson *et al.* 1993; Jault *et al.* 1996). The fact that other flow coefficients ($t_5^0(t)$, $t_7^0(t)$, etc.) are also of similar magnitude than $t_1^0(t)$ and $t_3^0(t)$ is indicative that the cylindrical flow structure is accomplishing the difficult task of simultaneously satisfying the angular momentum balance while providing the torque on the mantle.

At millennial timescales, our flow inversion suggests that the situation may be different. Most of the time-dependent variations in the core surface flows are carried by $t_1^0(t)$ alone, an almost solid body rotation of the whole surface flow. In other words, a surface flow which has little shear in s. In order to apply a surface torque and conserve the angular momentum budget, azimuthal flows must necessarily have a shear in z.

5.2 Exchange of angular momentum between the mantle and the fluid envelope at the Earth's surface

If the changes in LOD are not due to core–mantle interactions, then they must result from exchanges of angular momentum between the mantle and the fluid envelope at the surface of the Earth. Therefore, an additional indirect way of assessing the validity of our results is by investigating whether surface processes participate in the axial angular momentum balance.

Angular momentum changes in the fluid envelope can be produced by two different processes: a change in the mean zonal fluid velocities, or latitudinal mass displacement. An estimate of these effects can be obtained. Consider first the possibility that zonal accelerations are responsible for the millennial variations in LOD. Observations of the changes in zonal wind velocities during the interval 1949–1997 as a result of the global warming trend indicate that the axial angular momentum of the atmosphere has been increasing (Abarca del Rio 1999). The corresponding rate of change in LOD is on the order of 0.56 milliseconds/century (ms/cy). The global rate of increase in temperature during that interval is $\sim$0.79 °C/cy. Adopting for simplicity a linear correspondence between the two implies an increase of 0.7 ms in LOD for each degree C change in temperature. Therefore, in order to explain the observed millennial LOD variations (peak-to-peak $\sim$10 ms), an oscillation in global temperature of $\sim$13 °C is required, an unlikely scenario. An alternative estimate of the change in LOD due to the effect of global warming has been obtained using a suite of coupled ocean-atmosphere general circulation models (de Viron *et al.* 2002). The dominant effect is produced by changes in zonal wind velocities and a mean rate of increase in LOD of 0.11 ms/cy is obtained from a doubling of CO_2 in the atmosphere, although it is as large as 0.44 ms/cy for one of the model. Even for this most extreme case, the concentration of CO_2 would need to change by a factor 5–6 in order to produce the observed millennial variations in the LOD, an equally unlikely scenario. Obviously, such a simple analysis does not completely rule out a possible contribution from variations in wind speed. For instance, the change in the zonal winds during El Niño events can produce a change in the LOD of the order of 1 ms (Rosen 1993). If a millennial periodicity in climate is associated with a change in wind pattern of a similar order as those observed during El Niño, this could explain the observed LOD.

The second possible surface mechanism is through latitudinal mass displacements, which produce changes in the axial moment of inertia of the whole Earth. A corresponding change in the rotation rate must take place to conserve angular momentum. The largest contribution to this effect is most probably from a mass exchange between oceans and high latitude glaciers that result from climate variations. A change in ocean mass lead to a change in global sea level and the effect of the latter on the Earth's rotation is well known: a sea-level change of 1 cm produces a change in LOD of 2.75×10^{-3} ms (Chao & O'Connor 1988). To that we must add the effect of the change in ice mass, which, for simplicity, we can take to be about equivalent. This means that in order to produce the observed peak-to-peak millennial changes in LOD of 10 ms, we need peak-to-peak variations in global sea level of about 20 m. This is roughly three times the sea-level change produced by an entire melting of the Greenland Ice Sheet (e.g. Alley *et al.* 2005), and more than an order of magnitude larger than any record of sea-level changes during the last millennia (Tanner 1992; Kearney 1996; Nunn 1998). However, because we may have overestimated the required sea-level change, let us proceed further with this hypothesis. For the sake of the argument, suppose that a change on the order of a couple of °C is sufficient to produce the required sea-level change and mass redistribution to explain the millennial LOD variations. The observed changes in temperature for the past few millennia (e.g. from Greenland ice sheet boreholes (Dahl-Jensen *et al.* 1998)) may then be compatible with this scenario. However, the warmest period is observed to be between 500 and 1000 AD, which should correspond to an increase in sea level and a maximum in the change in LOD. Yet, the observed variation in LOD is at a minimum during this time interval (see Fig. 4), indicating that this mechanism is probably not the main cause of the observed changes. If this effect participates, it must be compensated by a larger inverse effect.

Although the above estimates remain crude, they indicate that the observed millennial changes in LOD cannot easily be accounted for by surface processes. This is an additional argument in favour of core–mantle angular momentum exchanges. However, if surface processes were to participate in the angular momentum balance, an interesting consequence is that these would then have the ability to influence dynamics in the core, and vice versa.

5.3 Flow, waves and westward drift

We have assumed throughout this study that the large scale oscillating longitudinal drifts of the magnetic field are the result of advection by flow. Of course, it is also possible that these are in fact due to the azimuthal propagation of MAC waves near the surface of the core. If this is the case, because we have limited the time-dependent flow in our inversion to contain only the axisymmetric azimuthal part, the azimuthal phase velocity of these waves would have been aliased into a flow. The results of our study are hampered by the difficulty in resolving these two possible scenarios.

For the variations at millennial timescales described in this paper, we favour an explanation in terms of flow motion. This is mainly because of the match in amplitude and periodicity between the observed changes in LOD and the prediction based on this hypothesis. Moreover, if the ASV are instead due to propagating waves, the oscillation on Fig. 1 must be accounted for by changes in the direction of the phase velocity. While it is possible that these can arise as a result of small changes in the background state or from the specific details of the evolving instability supporting the waves, the explanation in terms of flow remains a simpler hypothesis.

According to the core surface flow model in Fig. 1, during the last few hundred years t_1^0 is negative, implying westward motion, with an amplitude of the same order as the observed westward drift in core flow models inverted from historical secular variations, as illustrated by the good fit with our flow inversion from GUFM. Interestingly, our flow model in Fig. 1 suggests that for about one thousand years between 300 and 1300 AD the drift was eastward, as t_1^0 was then positive. This indicates that the present-day westward drift may be a temporary phenomenon only reflecting the secular variation of the field since 1300 AD.

However, the reality may be more complex. The global eastward and westward motion that we resolve in our inversion is due to the coherent displacement of large-scale features. The present-day westward drift on the other hand is mainly the result of flux bundles of azimuthal wavenumber between 5 and 8 that are confined to low latitudes in the Atlantic hemisphere and moving at roughly 20 km yr^{-1} (Finlay & Jackson 2003; Jackson 2003) (when averaged over the entire surface of the CMB, its rate is then divided by about a factor 2 and corresponds to the velocity on Fig. 1). It is doubtful that such features could be observed in the archaeomagnetic field model, as only the spherical harmonic degrees 5 and lower are resolved reliably (Korte & Constable 2005). Furthermore, features with azimuthal wavenumber 5 and larger, travelling at 20 km yr^{-1}, would be averaged out in roughly 150 yr, a timescale probably too short to be resolved. Hence, a westward drift similar to that observed in the last few hundred years may well have existed further in the past, even though it is absent in CALS3K.2.

6 CONCLUSIONS

The model of time-dependent azimuthal flows at the surface of the core that we obtained from inversion suggests that there is a correlation between the angular momentum carried by these flows and the millennial variations in the LOD determined from the historical eclipse record. This correlation hinges upon the presence of flows inside the core which contain an important shear in the direction of the rotation axis. Such flows are expected on theoretical grounds from variations in the 3-D structure of the magnetic field and temperature within the core, which produce changes in magnetic and thermal winds.

We must keep in mind that these conclusions are not absolutely robust: we are not able to determine unequivocally whether the azimuthal flows on cylindrical surfaces inside the core are rigid or whether they also contain an important axial shear, as both types of flow can readily explain the LOD observations and a part of the ASV. This is either because our method for inverting the core flows is flawed, or because precise information on core flows cannot be obtained from the current archaeomagnetic field models. In any case, our inverted core flows may not be very well determined.

However, if our results are correct, they suggest that exchanges of angular momentum between the core and the mantle are responsible for the millennial variations in LOD. We note that the very existence of the 1500 yr oscillations in the LOD has been questioned (Dalmau 1997). Here, we have shown that changes in zonal core flows of the order of 4 km yr^{-1} can account for such a change, and also provide the torque required on the mantle.

Our results also suggest that the angular momentum dynamics inside the core at millennial timescale is different than at decade timescales. In the latter case, rigid flows dominate and owe their existence to a normal mode of oscillation—torsional oscillations—and angular momentum is transmitted inside the core in the direction perpendicular to the rotation axis. At millennial timescale, the presence of non-rigid flows allows transmission of angular momentum to occur also in the direction parallel to the rotation axis.

One may wonder whether the timescale of 1500 yr underlies a dynamic component of the geodynamo with a periodic nature inside the core. This specific period cannot be firmly established by the eclipse data since they only cover the last 2700 yr. For certain regions of the globe, the archaeomagnetic data suggests that a periodic signal may extend further back in time (for example in lakes Fangshan (China) and Lama (Russia)), although this is not globally observed (Korte *et al.* 2005). The chaotic evolution of buoyancy driven instabilities and diffusion that controls the dynamics in the core is expected to scan many timescales and the millennial oscillations in azimuthal flows may simply reflect the recent vagaries of the geodynamo. However, another possibility is that this is the timescale at which a resonant excitation of a natural mode of vibration occurs. This normal mode would most likely consist of MAC waves, albeit a special case involving axisymmetric azimuthal flows that have a shear in z. The dynamics governing the oscillation would likely include growth and decay induced by instabilities as opposed to a purely restoring force, for example as it is the case in vacillating magnetoconvection (Zhang 1999).

Just as the development of the theory of torsional oscillations was motivated precisely by the idea of jointly explaining the decade LOD changes and secular variation, it is our hope that the results presented here will stimulate further theoretical development of zonal flows in the core at millennial timescales. Since the nature of these flows ultimately depend on both the magnetic field and the temperature perturbation, important information about these fields inside the core and about the geodynamo could be extracted. In addition, such investigations may shed light on the relation between the surface flow and the rigid flow to calculate the core angular momentum, and may also permit a more rigorous assessment of the nature of the torque.

ACKNOWLEDGMENTS

The authors wish to thank Leslie Morrison for the model of the LOD variations based on the eclipse data and Monika Korte for the archaeomagnetic field models. We also acknowledge Richard Holme for a thorough review which improved this paper significantly. The majority of this work was accomplished during MD's PhD at Harvard University, which was partly supported by scholarships from NSERC/CRSNG of Canada and FCAR of Québec. Additional support was provided by NSF/CSEDI Award EAR-0112469 and NSF Award EAR-0337579. MD is currently supported by a NERC Postdoctoral Fellowship of the UK.

REFERENCES

Abarca del Rio, R., 1999. The influence of global warming in Earth rotation speed, *Ann. Geophysicae,* **17,** 806–811.

Alfè, D., Kresse, G. & Gillan, M., 2000. Structure and dynamics of liquid iron under core conditions, *Phys. Rev.,* **B61,** 132–142.

Alley, R.B., Clark, P.U., Huybrechts, P. & Joughin, I., 2005. Ice-sheet and sea-level changes, *Science,* **310,** 456–460.

Anufriyev, A.P. & Braginsky, S.I., 1977. Influence of irregularities of the boundary of the earth's core on fluid velocity and the magnetic field III, *Geomag. Aeron.,* **17,** 492–496.

Aubert, J., 2005. Steady zonal flows in spherical shell fluid dynamos, *J. Fluid Mech.,* **542,** 53–67.

Backus, G.E., 1968. Kinematics of geomagnetic secular variation in a perfectly conducting core, *Phil. Trans. R. Soc. Lond.,* A., **A263,** 239–266.

Bloxham, J., 1988. The dynamical regime of fluid flow at the core surface, *Geophys. Res. Lett.,* **15,** 585–588.

Bloxham, J., 1989. Simple models of fluid flow at the core surface derived from geomagnetic field models, *Geophys. J. Int.,* **99,** 173–182.

Bloxham, J., 1992. The steady part of the secular variation of the Earth's magnetic field, *J. geophys. Res.,* **97,** 19 565–19 579.

Bloxham, J., 1998. Dynamics of angular momentum in the Earth's core, *Ann. Rev. Earth Planet. Sci.,* **26,** 501–517.

Bloxham, J., 2002. Time-dependent and time-independent behavior of high-latitude flux bundles at the core-mantle boundary, *Geophys. Res. Lett.,* **29,** 1854, doi:10.1029/2001GL014543.

Bloxham, J. & Gubbins, D., 1985. The secular variation of the Earth's magnetic field, *Nature,* **317,** 777–781.

Bloxham, J. & Jackson, A., 1991. Fluid flow near the surface of Earth's outer core, *Rev. Geophys.,* **29,** 97–120.

Bloxham, J., Gubbins, D. & Jackson, A., 1989. Geomagnetic secular variation, *Phil. Trans. R. Soc. Lond.,* A, **329,** 415–502.

Bloxham, J., Zatman, S. & Dumberry, M., 2002. The origin of geomagnetic jerks, *Nature,* **420,** 65–68.

Braginsky, S.I., 1967. Magnetic waves in the earth's core, *Geomag. Aeron.,* **7,** 851–859.

Braginsky, S.I., 1970. Torsional magnetohydrodynamic vibrations in the Earth's core and variations in day length, *Geomag. Aeron.,* **10,** 1–10.

Braginsky, S.I., 1972. Analytical description of geomagnetic field of the past and spectral analysis of magnetic waves in the Earth's core, *Geomag. Aeron.,* **12,** 1092–1105.

Braginsky, S.I., 1974. Analytical description of geomagnetic field of the past and spectral analysis of magnetic waves in the Earth's core II, *Geomag. Aeron.,* **14,** 522–529.

Braginsky, S.I. & Burlatskaya, S.P., 1979. Spherical analysis of the geomagnetic field based on archeomagnetic data, *Izv. Akad.Nauk. SSSR, Fiz. zimli,* **15,** 891–895.

Buffett, B.A., 1992. Constraints on magnetic energy and mantle conductivity from the forced nutations of the Earth, *J. geophys. Res.,* **97,** 19 581–19 597.

Buffett, B.A., 1996. Gravitational oscillations in the length of the day, *Geophys. Res. Lett.,* **23,** 2279–2282.

Buffett, B.A., 1997. Geodynamic estimates of the viscosity of the Earth's inner core, *Nature,* **388,** 571–573.

Buffett, B.A., 1998. Free oscillations in the length of day: inferences on physical properties near the core-mantle boundary, in *The core-mantle boundary region*, Vol. 28, pp. 153–165, eds Gurnis, M., Wysession, M.E., Knittle, E. & Buffett, B.A., Geodynamics series, AGU Geophysical Monograph, Washington, D.C.

Buffett, B.A., 2002. Estimates of heat flow in the deep mantle based on the power requirements for the geodynamo, *Geophys. Res. Lett.,* **29,** doi:10.1029/2001GL014649.

Bullard, E.C. & Gellman, H., 1954. Homogeneous dynamos and terrestrial magnetism, *Phil. Trans. R. Soc. Lond.,* A, **247,** 213–278.

Bullard, E.C., Freedman, C., Gellman, H. & Nixon, J., 1950. The westward drift of the Earth's magnetic field, *Phil. Trans. R. Soc. Lond.,* A, **243,** 67–92.

Burlastkaya, S.P. & Braginsky, S.I., 1978. The comparison of archaeomagnetic data with the analytical representation of the geomagnetic field for the last 2000 yr, *Archaeometry,* **20,** 73–83.

Chao, B.F. & O'Connor, W.P., 1988. Effect of a uniform sea-level change on the Earth's rotation and gravitational field, *Geophys. J.,* **93,** 191–193.

Cheng, M.K., Eanes, R.J., Shum, C.K., Schutz, B.E. & Tapley, B.D., 1989. Temporal variations in low degree zonal harmonics from Starlette orbit analysis, *Geophys. Res. Lett.,* **16,** 393–396.

Christodoloudis, D.C., Smith, D.E., Williamson, R.G. & Klosko, S.M., 1988. Observed tidal breaking in the Earth/Moon/Sun system, *J. geophys. Res.,* **93,** 6216–6236.

Constable, C.G., Johnson, C.L. & Lund, S.P., 2000. Global geomagnetic field models for the past 3000 yr: transient or permanent flux lobes?, *Phil. Trans. R. Soc. Lond.,* A, **358,** 991–1008.

Dahl-Jensen, D., Mosegaard, K., Gundestrup, N., Clow, D.G., Johnson, S.J., Hansen, A.W. & Balling, N., 1998. Past temperatures directly from the Greenland ice sheet, *Science,* **282,** 268–271.

Dalmau, W., 1997. Critical remarks on the use of medieval eclipse records for the determination of long-term changes in the Earth's rotation, *Surv. Geophys.,* **18,** 213–223.

Daly, L. & Le Goff, M., 1996. An updated and homogeneous world secular variation database. part 1. smoothing of the archeomagnetic results, *Phys. Earth planet. Inter.,* **93,** 159–190.

Davis, R.G. & Whaler, K.A., 1996. Determination of steady velocity in a rotating frame of reference at the surface of the Earth's core, *Geophys. J. Int.,* **126,** 92–100.

de Viron, O., Dehant, V., Goosse, H. & Crucifix, M., 2002. Effect of global warming on the length-of-day, *Geophys. Res. Lett.,* **29,** doi:10.1029/2001GL013672.

Dobson, D.P., 2002. Self-diffusion in liquid Fe at high pressure, *Phys. Earth planet. Inter.,* **130,** 271–284.

Dumberry, M. & Bloxham, J., 2003. Torque balance, Taylor's constraint and torsional oscillations in a numerical model of the geodynamo, *Phys. Earth planet. Inter.,* **140,** 29–51.

Fearn, D.R., 1997. The geodynamo, in *Earth's deep interior, the Doornbos memorial volume*, pp. 80–114, ed. Crossley, D., Gordon and Breach Science Publishers, Amsterdam, the Netherlands.

Finlay, C.C. & Jackson, A., 2003. Equatorially dominated magnetic field change at the surface of the Earth's core, *Science,* **300,** 2084–2086.

Gubbins, D., 1982. Finding core motions from magnetic observations, *Phil. Trans. R. Soc. Lond.,* A, **306,** 247–254.

Gubbins, D., 1996. A formalism for the inversion of geomagnetic data for core motions with diffusion, *Phys. Earth planet. Inter.,* **98,** 193–206.

Gubbins, D. & Roberts, P.H., 1987. Magnetohydrodynamics of the Earth's core, in *Geomagnetism*, Vol. 2, pp. 1–184, ed. Jacobs, J.A., Academic Press, London.

Gubbins, D. & Kelly, P., 1996. A difficulty with using the frozen flux hypothesis to find steady core motions, *Geophys. Res. Lett.,* **23,** 1825–1828.

Hide, R., 1966. Free hydromagnetic oscillations of the Earth's core and the theory of geomagnetic secular variation, *Phil. Trans. R. Soc. Lond.,* A, **259,** 615–646.

Hide, R., 1969. Interaction between the Earth's liquid core and solid mantle, *Nature,* **222,** 1055–1056.

Hide, R., 1977. Towards a theory of irregular variations in length of day and core-mantle coupling, *Phil. Trans. R. Soc. Lond.,* A, **A284,** 547–554.

Hide, R., Boggs, D.H. & Dickey, J.O., 2000. Angular momentum fluctuations within the Earth's liquid core and torsional oscillations of the core-mantle system, *Geophys. J. Int.,* **143,** 777–786.

Hills, R.G., 1979. Convection in the Earth's mantle due to viscous shear at the core-mantle interface and due to large-scale buoyancy, *PhD thesis,* N.M. State Univ., Las Cruces, New Mexico.

Hollerbach, R., 2003. The range of timescales on which the geodynamo operates, in *Earth's core: dynamics, structure, rotation*, Vol. 31, pp. 181–192, eds Dehant, V., Creager, K.C., Karato, S.-I. & Zatman, S., Geodynamics series, AGU Geophysical Monograph.

Holme, R., 1998a. Electromagnetic core-mantle coupling—I. Explaining decadal changes in the length of day, *Geophys. J. Int.,* **132,** 167–180.

Holme, R., 1998b. Electromagnetic core-mantle coupling II: probing deep mantle conductance, in *The core-mantle boundary region*, Vol. 28, pp. 139–152, eds Gurnis, M., Wysession, M.E., Knittle, E. & Buffett, B.A., Geodynamics series, AGU Geophysical Monograph, Washington, D.C.

Holme, R. & Whaler, K.A., 2001. Steady core flows in an azimuthally drifting reference frame, *Geophys. J. Int.,* **145,** 560–569.

Hongre, L., Hulot, G. & Khokhlov, A., 1998. An analysis of the geomagnetic field over the past 2000 yr, *Phys. Earth planet. Inter.,* **106,** 311–335.

Jackson, A., 2000. Comment on 'Time evolution of the fluid flow at the top of the core. Geomagnetic jerks' by M. Le Huy, M., Mandea, J.-L. Le Mouël, and A. Pais., *Earth Planets Space,* **52,** 649.

Jackson, A., 2003. Intense equatorial flux spots on the surface of the Earth's core, *Nature,* **424,** 760–763.

Jackson, A., Bloxham, J. & Gubbins, D., 1993. Time-dependent flow at the core surface and conservation of angular momentum in the coupled core-mantle system, in *Dynamics of the Earth's deep interior and Earth rotation*, Vol. 72, pp. 97–107, eds Le Mouël, J.-L., Smylie, D.E. & Herring, T., AGU Geophysical Monograph, Washington, D.C.

Jackson, A., Jonkers, A.R.T. & Walker, M.R., 2000. Four centuries of geomagnetic secular variation from historical records, *Phil. Trans. R. Soc. Lond.,* A, **358,** 957–990.

James, R.W., 1970. Decomposition of geomagnetic secular variation into drifting and non-drifting components, *J. Geomag. Geoelectr.,* **22,** 241–252.

Jault, D., 1995. Model Z by computation and Taylor's condition, *Geophys. astrophys. Fluid Dyn.,* **79,** 99–124.

Jault, D. & Le Mouël, J.-L., 1989. The topographic torque associated with a tangentially geostrophic motion at the core surface and inferences on the flow inside the core, *Geophys. astrophys. Fluid Dyn.,* **48,** 273–296.

Jault, D. & Le Mouël, J.-L., 1991. Exchange of angular momentum between the core and the mantle, *J. Geomag. Geoelectr.,* **43,** 111–129.

Jault, D., Gire, C. & Le Mouël, J.-L., 1988. Westward drift, core motions and exchanges of angular momentum between core and mantle, *Nature,* **333,** 353–356.

Jault, D., Hulot, G. & Le Mouël, J.-L., 1996. Mechanical core-mantle coupling and dynamo modelling, *Phys. Earth planet. Inter.,* **98,** 187–191.

Kearney, M.S., 1996. Sea-level change during the last thousand years in Chesapeake Bay, *J. Coastal Res.,* **12,** 977–983.

Kolomiytseva, G.I. & Pushkov, A.N., 1975. Analysis of space-time structure of the geomagnetic field over a 2000-yr period, in *Analysis of space-time structure of the geomagnetic field (in Russian)*, pp. 72–81, Nauka, Moscow.

Korte, M. & Constable, C., 2003. Continuous global geomagnetic field models for the past 3000 yr, *Phys. Earth planet. Inter.,* **140,** 73–89.

Korte, M. & Constable, C., 2005. Continuous geomagnetic field models for the past 7 millennia II: CALS7K, *Geochem. Geophys. Geosyst.,* **6,** Q02H16, doi:10/1004GC000801.

Korte, M., Genevey, A., Constable, C., Frank, U. & Schnepp, E., 2005. Continuous geomagnetic field models for the past 7 millennia I: a new global data compilation, *Geochem. Geophys. Geosyst.,* **6,** Q02H15, doi:10.1029/2004GC000800.

Kuang, W. & Bloxham, J., 1997. On the dynamics of topographical core-mantle coupling, *Phys. Earth planet. Inter.,* **99,** 289–294.

Kuang, W. & Chao, B.F., 2001. Topographic core-mantle coupling in geodynamo modeling, *Geophys. Res. Lett.,* **28,** 1871–1874.

Lambeck, K., 1980. *The Earth's variable rotation: geophysical causes and consequences*, Cambridge University Press, Cambridge.

Le Mouël, J.-L., 1984. Outer core geostrophic flow and secular variation of Earth's magnetic field, *Nature,* **311,** 734–735.

Love, J.J., 1999. A critique of frozen-flux inverse modelling of a nearly steady geodynamo, *Geophys. J. Int.,* **138,** 353–365.

Marton, P., 1970. Secular variation of the geomagnetic virtual dipole field during the last 2000 yr as inferred from the spherical harmonic analysis of the available archeomagnetic data, *Pure appl. Geophys.,* **81,** 163–176.

Mitrovica, J.X. & Peltier, W.R., 1993. Present-day variations in the zonal harmonics of the Earth's geopotential, *J. geophys. Res.,* **98,** 4509–4526.

Morrison, L.V. & Stephenson, F.R., 2001. Historical eclipses and the variability of the Earth's rotation, *J. Geodyn.,* **32,** 247–265.

Mound, J.E. & Buffett, B.A., 2003. Interannual oscillations in the length of day: implications for the structure of mantle and core, *J. geophys. Res.,* **108**(B7), 2334, doi:10.1029/2002JB002054.

Munk, W.H. & MacDonald, G.J.F., 1960. *The rotation of the Earth*, Cambridge University Press, Cambridge.

Nunn, P.D., 1998. Sea-level changes over the past 1 000 yr in the Pacific, *J. Coastal Res.,* **14,** 23–30.

O'Connell, R.J., 1971. Pleistocene glaciation and the viscosity of the lower mantle, *Geophys. J. R. astr. Soc.,* **23,** 299–327.

Olson, P. & Christensen, U.R., 2002. The time-averaged magnetic field in numerical dynamos with non-uniform boundary heat flow, *Geophys. J. Int.,* **151,** 809–823.

Pais, A. & Hulot, G., 2000. Length of day decade variations, torsional oscillations and inner core superrotation: evidence from recovered core surface zonal flows, *Phys. Earth planet. Inter.,* **118,** 291–316.

Poirier, J.-P., 1988. Transport properties of liquid metals and viscosity of the Earth's core, *Geophys. J.,* **92,** 99–105.

Proudman, J., 1916. On the motions of solids in a liquid possessing vorticity, *Proc. R. Soc. Lond.,* A, **92,** 408–424.

Roberts, P.H. & Scott, S., 1965. On the analysis of secular variation, *J. Geomag. Geoelectr.,* **17,** 137–151.

Rochester, M.G., 1960. Geomagnetic westward drift and irregularities in the Earth's rotation, *Phil. Trans. R. Soc. Lond.,* A, **252,** 531–555.

Rosen, R.D., 1993. The axial momentum balance of Earth and its fluid envelope, *Surv. Geophys.,* **14,** 1–29.

Rubincam, D.P., 1984. Postglacial rebound observed by LAGEOS and the effective viscosity of the lower mantle, *J. geophys. Res.,* **89,** 1077–1087.

Rubincam, D.P., 2003. Gravitational core-mantle coupling and the acceleration of the Earth, *J. geophys. Res.,* **108**(B7), 2338, doi:10.1029/2002JB002132.

Sakai, H., 1980. Spherical harmonic analysis of the geomagnetic field during the last 2000 yr, *Rock Mag. Paleogeophys.,* **7,** 8–15.

Stephenson, F.R. & Morrison, L.V., 1984. Long-term changes in the rotation of the Earth: 700 BC to AD 1980, *Phil. Trans. R. Soc. Lond.,* A, **313,** 47–70.

Stephenson, F.R. & Morrison, L.V., 1995. Long-term fluctuations in the Earth's rotation: 700 BC to AD 1990, *Phil. Trans. R. Soc. Lond.,* A, **351,** 165–202.

Stern, D.P., 2002. A millennium of geomagnetism, *Rev. Geophys.,* **40**(3), 1007, doi:10.1029/2000RG000097.

Tanner, W.F., 1992. 3000 yr of sea-level change, *B. Am. Meteorol. Soc.,* **73,** 297–303.

Taylor, G.I., 1917. Motions of solids in fluid when the flow is not irrotational, *Proc. R. Soc. Lond.,* A, **93,** 99–113.

Taylor, J.B., 1963. The magneto-hydrodynamics of a rotating fluid and the Earth's dynamo problem, *Proc. R. Soc. Lond.,* A, **274,** 274–283.

Van Orman, J.A., 2004. On the viscosity and creep mechanism of Earth's inner core, *Geophys. Res. Lett.,* **31,** L20606, doi:10.1029/2004GL021209.

Voorhies, C.V., 1986. Steady flows at the top of Earth's core derived from geomagnetic field models, *J. geophys. Res.,* **91,** 12 444–12 466.

Voorhies, C.V. & Backus, G.E., 1985. Steady flows at the top of the core from geomagnetic-field models—the steady motions theorem, *Geophys. astrophys. Fluid Dyn.,* **32,** 163–173.

Whaler, K.A., 1980. Does the whole of the Earth's core convect, *Nature,* **287,** 528–530.

Wu, P. & Peltier, D.R., 1982. Viscous gravitational relaxation, *Geophys. J. R. astr. Soc.,* **70,** 435–485.

Yoder, C.F., Williams, J.G., Dickey, J.O., Schutz, B.E., Eanes, R.J. & Tapley, B.D., 1983. Secular variation of the Earth's gravitational harmonic j_2 coefficient from LAGEOS and nontidal acceleration of Earth rotation, *Nature,* **303,** 757–762.

Yoshida, S., Sumita, I. & Kumazawa, M., 1996. Growth model of the inner core coupled with the outer core dynamics and the resulting elastic anisotropy, *J. geophys. Res.,* **101,** 28 085–28 103.

Zatman, S. & Bloxham, J., 1997. Torsional oscillations and the magnetic field within the Earth's core, *Nature,* **388,** 760–763.

Zatman, S. & Bloxham, J., 1998. A one-dimensional map of B_s from torsional oscillations of the Earth's core, in *The core-mantle boundary region*, Vol. 28, pp. 183–196, eds Gurnis, M., Wysession, M.E., Knittle, E. & Buffett, B.A., Geodynamics series, AGU Geophysical Monograph, Washington, D.C.

Zhang, K., 1999. Nonlinear magnetohydrodynamic convective flows in the Earth's fluid core, *Phys. Earth planet. Inter.,* **111,** 93–103.