The free librations of Mercury and the size of its inner core

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[1] The predicted period of the free libration of Mercury's mantle, if fully decoupled from the core, is approximately 12 yr. If the innermost part of the core is solid, this period is altered and a second free mode is possible. The period of these two modes depend on the mean solar torque acting on the elliptical figures of the mantle and inner core, and also on the gravitational torque they exert on one another. We show that one mode is well approximated by the free gravitational oscillation between the mantle and inner core that would occur in the absence of a solar torque; its period is as small as 4 yr depending on inner core size and other model parameters. The second free mode consists of an in-phase libration of the mantle and inner core. When the inner core is small, the period of this mode coincides with that of the decoupled free mantle libration of 12 yr. However, the period lengthens considerably when the inner core radius is larger than 1000 km, and can be as high as approximately 19 yr if Mercury's core is almost fully solidified. These modes should affect Mercury's libration dynamics at decadal timescale. This suggests that, in principle, constraints on the size of Mercury's inner core may be extracted from its long term libration motion, though in practice this may be difficult given data limitations. **Citation:** Dumberry, M. (2011), The free librations of Mercury and the size of its inner core, *Geophys. Res. Lett.*, *38*, L16202, doi:10.1029/2011GL048277.

1. Introduction

[2] Mercury's rotational dynamics is characterized by a 3:2 spin-orbit resonance in which it completes 3 rotations around itself for each 2 revolutions around the Sun. The combination of Mercury's eccentric orbit and the permanent equatorial ellipticity of its figure lead to periodically reversing gravitational solar torques and to small periodic fluctuations in rotation about its mean rotation rate. These are termed forced librations and their dominant period coincides with the orbital period of Mercury of 87.97 days. The amplitude of the forced librations, as determined by Earth-based radar speckle pattern observation, is 35.8 ± 2 arcsec [*Margot et al.*, 2007]. This is approximately twice as large as expected were Mercury's core fully solidified; this indicates that only the mantle participates in the forced libration, and that the outermost part of the core must be fluid.

[3] The fit to Mercury's observed librations is improved if a signal of decadal timescale with an amplitude of a few tens of arcsec is also present [*Margot et al.*, 2007; *Yseboodt et al.*, 2010; *Veasey and Dumberry*, 2011]. However, this cannot be robustly confirmed because of the limited time-span and quality of the data; the improvement may simply reflect that a smaller misfit can be achieved when more degrees of freedom are added to the system. Nevertheless, if indeed present, this long period libration may be the signature of an excited normal mode, or the resonant amplification of a forced libration at a period close to that of a normal mode. If Mercury's core is entirely fluid and decoupled from the mantle, there is only one such mode: the free libration of the mantle about the Mercury-Sun line if one could view Mercury only at times when it passed perihelion [*Peale*, 2005]. Though the restoring force is external, this is a free mode in the sense that its amplitude and phase are arbitrary. If the mass distribution in Mercury's mantle matches that required to explain the amplitude of the forced 88-day libration, the period of this free mantle libration is 12.07 yr (with the 1σ uncertainty covering 11.73 to 12.44 yr). The close proximity of this free mode to the 11.86-yr periodic perturbations that Jupiter exerts on Mercury's orbit is such that a resonant amplification of this latter planetary forcing can occur and produce a forced libration with an amplitude in excess of 40 arcsec [*Peale et al.*, 2009; *Yseboodt et al.*, 2010].

[4] However, we expect the innermost part of Mercury's core to be solid as a result of planetary cooling [*Solomon*, 1976]. The presence of a solid inner core adds one degree of freedom to Mercury's libration dynamics. A second free libration is possible, that of the inner core about the Mercury-Sun line at perihelion [*Peale et al.*, 2002]. Moreover, a longitudinal misalignment between the equatorial figures of the mantle and inner core results in a gravitational torque between the two [*Buffett*, 1996] and this affects the free librations of the coupled inner core-mantle system.

[5] Gravitational coupling with the inner core has a negligible effect on the 88-day libration of the mantle [*Peale et al.*, 2002; *Veasey and Dumberry*, 2011]. However, the larger the inner core, the greater is the gravitational torque with the mantle, and this may affect the libration at timescales of a few years to a decade. The goal of the present study is to investigate the effect of the inner core on the free modes of libration of Mercury. This is important in order to properly interpret the nature of a long period libration that may eventually be confirmed by observations.

2. Theory

2.1. Libration Dynamics

[6] The libration of the mantle, γ_m, is defined as the longitudinal angle of deviation of the axis of its minimum equatorial moment of inertia from that which would result from uniform rotation. The libration of the inner core, γ_s, is defined similarly. The coupled differential equations that

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govern the combined libration of the mantle and inner core are given by *Peale et al.* [2002]. Here, our focus is on the libration dynamics at a timescale longer than a few orbital periods. In such a situation, the fluctuations of the solar torque in the course of one orbit are averaged out, leaving only a mean solar torque [*Peale et al.*, 2009; *Yseboodt et al.*, 2010]. The dynamical equations controlling the evolution of the long timescale libration of the mantle and inner core, linearized for small amplitudes, are, respectively,

$$\frac{d^2\gamma_m}{dt^2} = -3n^2 G_{201}(e)\Delta I_m \gamma_m - \frac{2\overline{\Gamma}}{C_m}(\gamma_m - \gamma_s), \tag{1}$$

$$\frac{d^2\gamma_s}{dt^2} = -3n^2 G_{201}(e)\alpha\Delta I_s \gamma_s + \frac{2\overline{\Gamma}}{C_s}(\gamma_m - \gamma_s), \tag{2}$$

where G_{201} (e) is defined as [*Kaula*, 1966]

$$G_{201}(e) = \left(\frac{7}{2}e - \frac{123}{16}e^3\right), \tag{3}$$

Mercury's orbital frequency, n, is defined by

$$n^2 = \frac{GM_\odot}{a^3}, \tag{4}$$

a and e are respectively the semi-major axis and eccentricity of Mercury's orbit around the Sun (of mass $M_\odot$) and G is the gravitational constant.

[7] The parameter α represents a normalized measure of the density contrast between the solid (ρ_s) and fluid side (ρ_f) of the inner core boundary (ICB),

$$\alpha = \left(1 - \frac{\rho_f}{\rho_s}\right). \tag{5}$$

The quantities ΔI_m and ΔI_s are moment of inertia ratios characterizing the equatorial ellipticities of the mantle and inner core and are defined as

$$\Delta I_m = \frac{B_m - A_m}{C_m}, \qquad \Delta I_s = \frac{B_s - A_s}{C_s}, \tag{6}$$

where $A_m < B_m < C_m$ are the principal moments of inertia of the mantle with C_m being the polar moment of inertia; $A_s < B_s < C_s$ are the equivalent quantities for the inner core.

[8] The second term on the right-hand side of each of equations (1) and (2) captures the gravitational torque between the mantle and inner core. We assume here that the dominant contribution to this torque comes form longitudinal mass anomalies of harmonic degree 2. The constant $\overline{\Gamma}$ is a measure of the strength of this torque. Its expression in terms of our model parameters is given further below.

[9] Equations (1) and (2) represent a simple libration system where the fluid core is fully decoupled from the inner core and mantle and does not participate in Mercury's libration. The topographic torque from the elliptical solid-fluid boundaries has been shown to affect the libration dynamics only negligibly [*Rambaux et al.*, 2007]. The torque from viscous stresses should also be small and, together with tidal dissipation, act to attenuate the free librations over a timescale of 10^5 yr [*Peale*, 2005]. Neglecting these second order effects allows us to simplify the dynamics and to keep the focus on the gravitational torques. Magnetic coupling, especially at the ICB, may be more important and is discussed further below. No forcing is introduced in equations (1) and (2) because our main goal is to investigate the free libration modes. The linearized form of these equations is justified, and is the relevant case to study, since observations (though limited) suggest that γ_m remains much smaller than one degree.

2.2. Interior Models of Mercury

[10] Many of the quantities entering equations (1) and (2) depend on the internal density structure of Mercury, which is not known. We consider a simple three-layer model of Mercury comprising a mantle, fluid core and solid inner core. Because Mercury is small, compression effects are not significant; for simplicity, we consider the density to be uniform within each region.

[11] The densities of the silicate mantle (ρ_m), fluid core and solid inner core, and the radii of the whole core (r_f) and inner core (r_s) must be chosen such that Mercury's bulk mass and density are matched. We follow here the procedure outlined by *Van Hoolst and Jacobs* [2003] in which ρ_f is determined on the basis of chosen values of ρ_m, ρ_s, the inner core radius r_s and the sulfur concentration of the fluid core prior to the nucleation of the inner core (χ_S^{in}). The numerical values of the different parameters that we use to build our interior density models are identical to those listed in Table 1 of *Veasey and Dumberry* [2011].

[12] The ratio ΔI_m must be consistent with the observed amplitude of the forced libration and this requires ΔI_m = (2.03 ± 0.12) × 10^{-4} [*Margot et al.*, 2007]. Further, the combination ($B_m - A_m$) and ($B_s - A_s$) must be consistent with the longitudinal gravity variation of degree 2, order 2, at Mercury's surface which is expressed by the harmonic coefficient C_{22}; observations from MESSENGER's first two flyby's suggest C_{22} = (0.81 ± 0.08) × 10^{-5} [*Smith et al.*, 2010]. For our three-layer model with homogeneous densities, longitudinal variations in mass are modeled in terms of the equatorial ellipticities of the exterior surface (ϵ_m), the solid side of the core-mantle boundary (CMB) (ϵ_f) and further assuming that the shape of the equatorial ellipticity at the ICB (ϵ_s) corresponds to that of an equipotential surface. These ellipticities can be determined entirely in terms of C_{22} and ΔI_m once a specific choice of r_s and χ_s^{in} is made. The exact relations and the procedure are detailed by *Veasey and Dumberry* [2011]. With this approach, the constant of gravitational coupling between the inner core and mantle is expressed by

$$\overline{\Gamma} = \frac{4\pi G}{5}\alpha C_s \epsilon_s \left[(\rho_f - \rho_m)\epsilon_f + \rho_m \epsilon_m\right]. \tag{7}$$

Incorporating the small radial density variations within each layer would not alter the amplitude of $\overline{\Gamma}$ by much because the torque primarily depends on the bulk values of densities. Thus, although our interior model of Mercury is simple, we believe that it captures the first order effects of internal gravitational coupling and offers easy tractability of the results.

2.3. Free Libration Modes

[13] In absence of gravitational coupling between them, the frequencies of the free librations of the mantle (ω_m) and

inner core (ω_s) about the Mercury-Sun line are, respectively [*Peale et al.*, 2002; *Peale*, 2005]

$$\omega_m = \sqrt{3\ n^2\ G_{201}(e)\frac{(B_m - A_m)}{C_m}}, \tag{8}$$

$$\omega_s = \sqrt{3\ \alpha\ n^2\ G_{201}(e)\frac{(B_s - A_s)}{C_s}}. \tag{9}$$

If a longitudinal misalignment is created between the mantle and inner core, and if no other force but their gravitational coupling subsequently acts on them, they will freely oscillate about their position of gravitational alignment. The frequency of this mantle-inner core gravitational (MICG) free mode of oscillation is [*Van Hoolst et al.*, 2008; *Veasey and Dumberry*, 2011]

$$\omega_g = \sqrt{\frac{2\overline{\Gamma}(C_m + C_s)}{C_s C_m}}. \tag{10}$$

[14] The free modes of oscillation of the linear system of equations (1) and (2) can be expressed in terms of a combination of these modes taken in isolation. Assuming a time-dependency of the form exp[$-i\omega t$] for γ_m and γ_s, where ω is the frequency and i is the imaginary number, we find two modes with natural frequencies given by

$$\omega_1^2 = \frac{\omega_\sigma^2}{2} + \frac{\omega_\beta^2}{2}, \qquad \omega_2^2 = \frac{\omega_\sigma^2}{2} - \frac{\omega_\beta^2}{2}, \tag{11}$$

where

$$\omega_\sigma^2 = \omega_m^2 + \omega_s^2 + \omega_g^2, \tag{12}$$

$$\omega_\beta^2 = \sqrt{\omega_\sigma^4 - 4\omega_m^2\omega_s^2 - 4\omega_g^2\left(\frac{C_s\omega_s^2 + C_m\omega_m^2}{C_m + C_s}\right)}. \tag{13}$$

In mode 1, the mantle and inner core librate out of phase by π, while in mode 2 they librate in phase.

[15] For all interior models considered in our study, ω_g is of the same order and often higher than ω_m, while ω_s tends to be smaller than ω_m by a factor 4 or more. Thus, $\omega_s^2 \ll \omega_m^2$, ω_g^2, and to a good approximation, ω_σ^2 and ω_β^2 in equation (11) can be replaced by

$$\tilde{\omega}_\sigma^2 = \omega_m^2 + \omega_g^2, \tag{14}$$

$$\tilde{\omega}_\beta^2 = \sqrt{\tilde{\omega}_\sigma^4 - \frac{4C_m\omega_m^2\omega_g^2}{C_m + C_s}}. \tag{15}$$

Further, when the inner core is small, $C_s \ll C_m$, and we retrieve

$$\omega_1 = \omega_g, \qquad \omega_2 = \omega_m. \tag{16}$$

Thus, in the case of a small inner core, the two free modes of libration coincide with the free mantle libration and the MICG mode taken in isolation.

3. Results

3.1. Gravitational Modes

[16] In Figure 1 we show how the period of the two free modes of our system vary as a function of inner core size for three different choices of mantle density: ρ_m = 3100 kg m^{-3} (red lines) ρ_m = 3300 kg m^{-3} (green) and ρ_m = 3500 kg m^{-3} (blue). These bracket the range of plausible mantle densities in the study of *Rivoldini et al.* [2009]. The periods are expressed in Earth years. Figures 1a–1d correspond to different choices of initial core sulfur composition: (a) χ_s^{in} = 0.02 (i.e., 2% by weight), (b) χ_s^{in} = 0.06, (c) χ_s^{in} = 0.10; (d) χ_s^{in} = 0.14. In Figures 1a–1d, we also show how the periods of the free mantle libration predicted by equation (8) (dashed black line) and of the MICG mode predicted by equation (10) (dashed grey line) vary as a function of inner core size for the case of ρ_m = 3300 kg m^{-3}. Note that ω_m does not vary with inner core size; all our models are constrained to keep the same ratio ΔI_m, and this fixes the free mantle libration to a period equal to 12.07 yr. The kink in the curves at large inner core radii marks the location of the eutectic radius, the point at which inner core growth proceeds by incorporating sulfur compounds as opposed to pure iron. Beyond this point, $\overline{\Gamma}$ remains constant while C_s keeps increasing and thereby decreasing the frequency of the MICG mode [*Veasey and Dumberry*, 2011].

[17] Figure 1 shows that when the inner core is smaller than approximately 1000 km, the two natural modes of the system are indeed well approximated by ω_m and ω_g. In fact, when $\omega_g > \omega_m$ (Figures 1b–1d), one of the mode is well approximated by ω_g even when $r_s > 1000$ km. (There is a range of r_s where this is not the case in Figure 1a because of the transition from $\omega_g < \omega_m$ to $\omega_g > \omega_m$ at $r_s \approx 1200$ km.) The MICG mode has a period that ranges between approximately 4 and 16 yr for the different model parameters in Figure 1. For most model parameters, the period is shorter than 12 yr; the transition at which ω_g is greater than ω_m at all values of r_s occurs at $\chi_s^{in} \geq 0.028$.

[18] The period of the second mode, however, departs significantly from 12 yr when $r_s > 1000$ km. The precise value of the period depends on the choice of r_s, ρ_m and χ_s^{in}; it can be as high as 19.26 yr for our most extreme case (Figure 1d). In this mode the mantle and inner core librate in phase (the amplitude of the inner core is slightly larger than that of the mantle). The radius of 1000 km marks the transition at which the moment of inertia of the inner core is no longer negligible compared to that of the mantle. The increase in the moment of inertia of the combined mantle and inner core is responsible for the lengthening of the period of the free mode.

3.2. Magnetic Coupling

[19] We now asses the importance of magnetic stresses at the ICB on the libration dynamics. Assuming a stationary fluid core, the axial torque on the inner core Γ_{icb} resulting from the shear of the radial magnetic field threading the ICB (of RMS amplitude B_r) by an axial oscillation of the inner

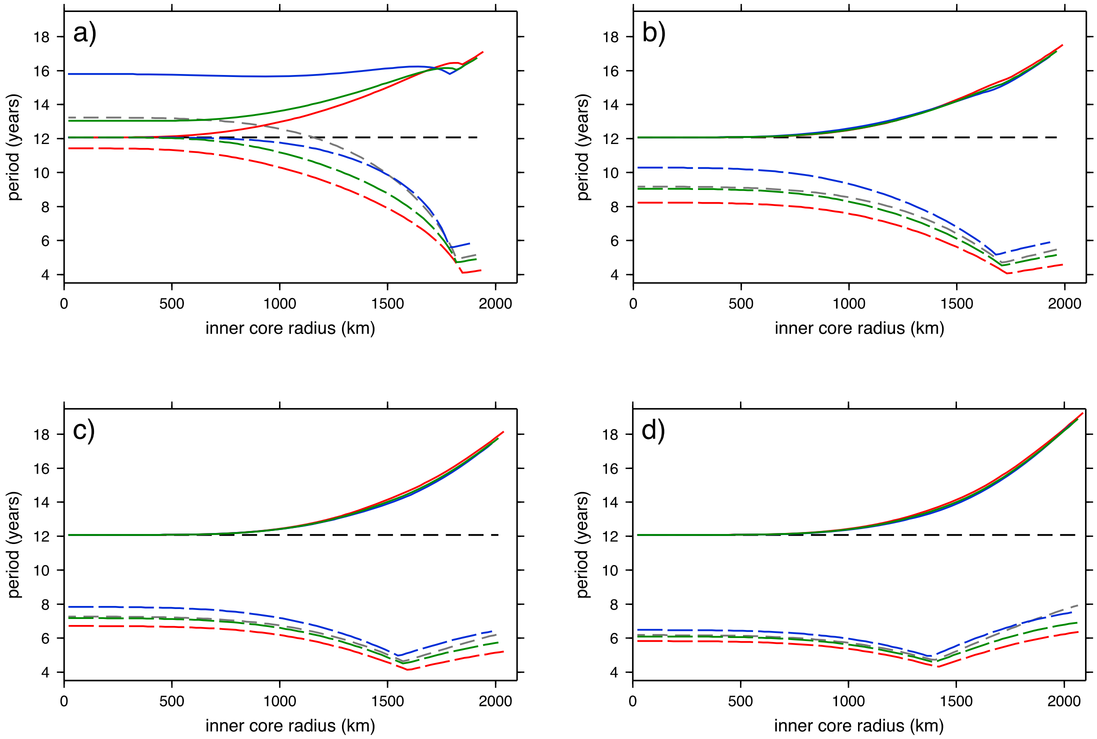

Figure 1. Period of the free modes as a function of inner core radius. The mode with frequency ω_1 is shown by the long-dashed colored lines; that with frequency ω_2 is shown by the solid colored lines. Different colors correspond to different choices of mantle density: ρ_m = 3100 kg m^{-3} (red), ρ_m = 3300 kg m^{-3} (green), ρ_m = 3500 kg m^{-3} (blue). Each panel represents a different value of initial sulfur composition of the fluid core: (a) χ_s^{in} = 0.02; (b) χ_s^{in} = 0.06; (c) χ_s^{in} = 0.10; (d) χ_s^{in} = 0.14. The short-dashed black and grey lines show the period $2\pi/\omega_m$ (predicted from equation (8)) and $2\pi/\omega_g$ (equation (10)), respectively.

core at frequency ω and angular velocity Ω_s is [*Dumberry and Mound*, 2010]

$$\Gamma_{icb} = -\frac{(1+i)}{\sqrt{\omega}}\frac{2}{3}\sqrt{\frac{\sigma}{2\mu}}\pi r_s^4 B_r^2 \Omega_s \tag{17}$$

where σ is the electrical conductivity (assumed equal in both the fluid and solid core) and μ is the magnetic permeability of free space. The frequency of a freely oscillating inner core under the influence of this sole magnetic torque is obtained by setting $\Gamma_{icb} = -i\omega C_s \Omega_s$, and isolating ω. We find

$$\omega_B = \left[(1-i)\frac{2}{3}\sqrt{\frac{\sigma}{2\mu}}\frac{\pi r_s^4 B_r^2}{C_s}\right]^{2/3}. \tag{18}$$

Using $\sigma = 5 \times 10^5$ S m^{-1} [*Stacey and Anderson*, 2001], as typically used for the Earth's core, and B_r = 0.01 mT (roughly 1% that of Earth's core [e.g., *Anderson et al.*, 2010]) gives a period of approximately 500 yr (and a similar damping time) for an inner core of 1000 km. This is obviously a very rough estimate, but it suggests that magnetic coupling should not affect the decadal timescale dynamics of the modes calculated above (but note that the magnetic damping time is much shorter than the estimated viscous damping time of 10^5 yr). However, if the RMS radial magnetic field close to the ICB is much larger, of the order of 0.1 mT, as is perhaps possible under specific dynamo scenarios [*Christensen*, 2006; *Vilim et al.*, 2010], the period decreases to approximately 20 yr and magnetic coupling at the ICB may play a more important role.

[20] Magnetic coupling at the CMB is likely to be much weaker than at the ICB, because the electrical conductivity of the mantle is certainly much smaller than that of the inner core. Thus it should not affect the period of the free modes, though dissipation at the CMB associated with magnetic coupling may again dominate that from viscous coupling.

4. Discussion and Conclusion

[21] Our results suggest that the size of Mercury's inner core affects its free modes of libration. The modes have periods between approximately 4 yr and 19 yr; thus, the size of the inner core has an influence on libration dynamics at such timescale. This influence can manifest itself in two ways. First, if one or both of the free modes are excited, they take part in the libration motion. Secondly, forced librations caused by the perturbations of Mercury's orbital parameters by other planets occur on timescales of a few months to a couple of decades. If the frequency of an orbital forcing is close to that of a free libration mode, amplification can occur. The amplification and phase of the resulting libration motion depends on the frequency of the free modes, and thus on the size of the inner core.

[22] The presence of free modes in the decadal timescale libration requires a recent or on-going excitation mechanism, otherwise viscous or electromagnetic friction at the ICB and CMB should have led to complete attenuation. No

such mechanism has yet been shown capable to excite a free libration to an observable amplitude of a few arcsec. In contrast, amplification of a planetary forcing can generate a libration of similar amplitude to that of the 88-day forced libration [*Peale et al.*, 2009; *Yseboodt et al.*, 2010]. Thus the influence of the inner core on the decadal libration is likely more important through this effect. If the inner core is as large as 1000 km (as suggested by uniformly distributed lobate scarps on Mercury's surface that are thought to result from a global planetary contraction [*Solomon et al.*, 2008; *Heimpel and Kabin*, 2008]), the period of the free mantle libration is lengthened from its 12-yr period. This would change Mercury's response to a given orbital forcing, most notably its response to the 11.86 yr forcing by Jupiter's orbit. This suggests that including the effect of gravitational coupling between the mantle and inner core on Mercury's response from planetary forcing is worth exploring.

[23] An additional motivation for this concerns the second free mode introduced by the presence of the inner core, the MICG mode. This mode may also amplify Mercury's response at specific orbital forcing frequencies. In this mode, the ratio of inner core to mantle libration amplitude is proportional to C_m/C_s. Thus, for a small inner core, although the inner core libration may be amplified, the associated mantle libration is likely to remain below detection level. However, for a large inner core such that $C_s \approx C_m$, the amplitude of the mantle libration may be more important. Whether the MICG mode (with a period in the range 4-10 yr for a large inner core) can result in a sizable amplification of specific orbital frequencies – notably those associated with Venus' orbit (5.66 yr), Earth's orbit (6.58 yr) and the half-period of Jupiter's orbit (5.93 yr) – needs testing.

[24] One dynamical element not included in our study, but which may have an important effect, is anelastic deformation within the inner core. If the bulk viscosity of the inner core is small enough that significant viscous deformation takes place on a decadal timescale, the period of the free modes would be altered. More importantly, viscous deformation acts as an additional source of dissipation and this may also reduce the resonant amplification of the forced libration from planetary forcing. This too, needs further investigation.

[25] The present size of the inner core is not known and it is an important parameter for Mercury's thermal evolution and its dynamo regime. In principle, a precise determination of the amplitude, phase and periodicity of a decadal timescale libration of Mercury by observations, and comparison with model predictions, can be used to place constraints on the size of the inner core. For instance, if a 11.86-yr libration consisting in the resonant amplification of Jupiter's orbital forcing is confirmed, this would suggest that the inner core radius cannot be larger than approximately 1000 km. Given the limitations on the time-span and quality of the observations, it is currently not possible to determine robustly the characteristics of Mercury's long term librations, and thus to extract information on the inner core. However, it may become possible to do so in the future once we have a sufficiently long record of Earth-based radar data with good precision. Additional observations from MESSENGER and BepiColumbo satellite missions may be particularly useful since they will provide direct information on γ_m and be more sensitive to the decadal timescale libration than radar data (which only give direct information on $d\gamma_m/dt$).

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