TORSIONAL OSCILLATIONS

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Torsional oscillations in the Earth's fluid core are the azimuthal oscillations of rigid cylindrical surfaces coaxial with the rotation axis (see Fig. 1). The pattern of the time-dependent part of the fluid motion at the surface of the core revealed from geomagnetic data inversion (see core flow) is highly suggestive of the presence of these waves inside the core, with velocity amplitudes on the order of 10 km/yr and typical periods of decades (see Fig. 2). The observation of torsional oscillations has important consequences for theoretical models of the geodynamo and provides a window through which we can observe some aspects of core dynamics.

Rigid azimuthal cylindrical flows of this sort, also called geostrophic flows, can be expected in the fluid core because rapid rotation prevents large variations in velocity along the rotation axis (see Proudman-Taylor theorem), and because purely azimuthal flows are not affected by the solid spherical boundaries of the mantle and the inner core. However, the Lorentz force prevents the cylindrical surfaces at different radii to rotate freely relative to one another. The magnetic field that permeates the core tends to be "frozen" in the fluid (see Alfvén's theorem) and a differential rotation of the cylinders shears the radial component of the field in the azimuthal direction. By Lenz' law, this produces a force that opposes further relative motion between the cylindrical surfaces. The radial component of the magnetic field behaves as if it were elastic strings attached to the cylindrical surfaces, and provides a restoring force for the establishment of waves that propagate in the direction perpendicular to the rotation axis. These are torsional oscillations, or torsional vibrations, as they were originally called in Braginsky's seminal work on the subject (Braginsky, 1970). Since the restoring force is purely magnetic, torsional oscillations are a type of Alfvén wave. Figure 1

Strictly speaking, torsional oscillations are not a geostrophic flow because they result from an azimuthal force balance between the fluid acceleration and Lorentz forces. However, since the force balance in the direction pointing away from the rotation axis remains geostrophic and the form of the flow is identical to that of geostrophic flows, it is convenient to think of torsional oscillations as time-dependent geostrophic flows.

The importance of geostrophic flows in the core and their intimate connection to the dynamics governing the geodynamo was first established by J. B. Taylor (1963). He showed that if one integrates the azimuthal component of the momentum equation over the surface of cylinders coaxial with the rotation axis, the Lorentz force is the only term in the leading order force balance that does not identically vanish. This imposes a morphological constraint on the magnetic field in the core, namely that the axial Lorentz torque must vanish at all times, i.e.

$$\int_{\Sigma} \left((\nabla \times \mathbf{B}) \times \mathbf{B} \right)_{\phi} d\Sigma = 0 \,, \tag{1}$$

where $d\Sigma = s\, d\phi\, dz$ and (s, ϕ, z) are cylindrical coordinates (see Taylor condition). When this constraint is not satisfied, one possibility to balance the Lorentz torque is by an azimuthal acceleration of the cylindrical surface,

$$\frac{\partial \mathcal{V}_{\phi}(s)}{\partial t} = \frac{1}{\rho \mu_o} \int_{\Sigma} \left((\nabla \times \mathbf{B}) \times \mathbf{B} \right)_{\phi} d\Sigma \,. \tag{2}$$

where $\mathcal{V}_{\phi}(s)$ is the geostrophic velocity, ρ is the density and μ_o is the permeability of free space. The class of motion described by the above equation is therefore not directly influenced by the Coriolis force, but only by the magnetic field. The above system allows oscillatory behaviour of geostrophic motion (i.e. torsional oscillations) about an equilibrium position where the Lorentz torque vanishes. The damping of these oscillations by diffusion of the magnetic field naturally brings the system back toward this equilibrium. Torsional oscillations are therefore an essential ingredient of the geodynamo as they always allow the system to relax toward a state where (1) is satisfied everywhere in the core.

Braginsky (1970) was the first to exploit this theoretical concept in an effort to explain geophysical observations. He sought to explain the decade variations in the length of day (LOD) as exchanges of angular momentum between the mantle and the core, with the angular momentum of the core carried by torsional oscillations. This type of fluid motion, he argued, would also be consistent with a part of the observed geomagnetic secular variation. Braginsky established the wave equation for the torsional oscillations predicted by J. B. Taylor. For $\mathcal{V}_\phi(s)$ proportional to $\exp(-i\omega t)$, it is given by

$$-\omega^2 \rho s^2 h \mathcal{V}_\phi(s) = \frac{d}{ds}\left(\frac{s^3 h}{\mu_o} < (B_s)^2 > \frac{d}{ds}\frac{\mathcal{V}_\phi(s)}{s} \right) - i\omega f(s)\,, \qquad (3)$$

where s and h are respectively the radius and height of the cylinder, $f(s)$ is the torque at the ends of the cylinders due to surface forces at the fluid-solid boundaries, B_s is the the s-component of the field which is assumed to remain steady on the timescale of the oscillations, and $<>$ denotes average over the cylinder surface. Note that the restoring force does not involve the steady azimuthal part of the magnetic field, as this component is not sheared by torsional oscillations.

Solutions of equation (3) depend on the knowledge of the radial magnetic field everywhere in the core and on the physical properties near the boundaries that enter $f(s)$. Exact solutions are then not obtainable without solving the dynamo problem as a whole. Nevertheless, estimates of the form and periodicity of the natural modes of oscillation can be obtained for simple magnetic field morphologies and simple models for fluid-solid coupling. An order of magnitude estimate for the period of the fundamental mode is given by

$$\tau_{to} \approx c \left(\frac{\rho \mu_o}{< (B_s)^2 >} \right)^{1/2} , \tag{4}$$

where c is the radius of the core. As an example, using a typical value of $B_s = 0.5$ mT, we find $\tau_{to} \sim 25$ years, which corresponds to the timescale of the changes in LOD and of a part of the geomagnetic secular variation.

Braginsky's original idea that torsional oscillations can both explain the LOD changes while being consistent with the secular variations of the magnetic field has since received further support. Jault et al. (1988) have reconstructed the changes in geostrophic velocities between 1969 and 1985 from maps of the flow at the top of the core which best explain the secular variation of the magnetic field. The changes in core angular momentum calculated from these motions correspond roughly to the changes in mantle angular momentum required to explain the LOD variations during that period. Jackson et al. (1993) subsequently showed that this correlation extends back to 1900. In addition, the variations in time of the rigid cylinder flow suggest large exchanges of angular momentum inside the core and relatively little exchange with the mantle, indicating that the motion is likely to be that of waves of the type consistent with torsional oscillations, a fact that was later verified by Zatman and Bloxham (1997). An example of the time-dependent geostrophic flows in the Earth's core which possibly represent torsional oscillations are shown in Fig. 2 for the period between 1900 and 1990. Recently, Bloxham et al. (2002) showed that rigid cylindrical flows with shorter timescale and wavelength also have wave characteristics suggestive of torsional oscillations. This provides additional support for their presence in the Earth's core and indicates that higher modes are also excited. The same study also showed that a time-dependent flow model solely comprised of torsional oscillations can reproduce the part of the secular variation known as geomagnetic jerks. This suggests that the explanation for geomagnetic jerks is connected to torsional oscillations and further underlines their role in the observable part of the dynamics taking place inside the core. Figure 2

Torsional oscillations are expected on theoretical grounds and we have good evidences that they occur in the Earth's core. Therefore, they represent one of the most robust links between geophysical observations on historical timescales and the dynamics responsible for maintaining the field against Ohmic decay on geologic timescales. Theoretical models of the dynamical processes involved in the geodynamo must then be consistent with torsional oscillations. Conversely, they can be used to extract a wealth of physical quantities and dynamics in the core that are otherwise not directly observable. For instance, Zatman and Bloxham (1997) have used the observed torsional oscillations to build a model of the steady r.m.s. value of B_s on cylinder surfaces inside the core. Similarly, Buffett (1998) used equation (3) in order to constrain physical parameters near the core-mantle boundary and the nature of the torque that transfers the angular momentum between the mantle and the core.

While a large part of the secular variation of the magnetic field can be explained by a combination of steady flows and torsional oscillations, a part of the signal remains unaccounted for by this simple model of the dynamics. The unexplained signal may be due to non-axisymmetric and/or meridional flows associated with the propagation of magnetohydrodynamic waves. The connection between these waves and torsional oscillations remains to be established. Another important issue still unresolved is that of the excitation of torsional oscillations. Estimates of the coupling with the mantle suggest that they should be damped after a few periods, which indicates that an efficient excitation mechanism must exist. The behaviour described above, where torsional oscillations occur with respect to a steady equilibrium state, is correct only if the waves are excited by a mechanism which acts suddenly and then plays no further role in the dynamics. A perhaps more likely scenario is one where a forcing term needs to be added to equation (3) and plays an active role in the dynamics at all times. If this latter view is correct, the resulting time-dependent geostrophic flows would consist of a combination of forced and free "torsional oscillations". The nature of this forcing is at present unknown but perhaps it simply consists of the continual changes in the magnetic field, and hence of the Lorentz torque on any cylinder, produced by the convective dynamics in the core. The changes in the Lorentz torque may occur on many different timescales but those that are close to the natural modes of torsional oscillations would provide efficient excitation. In any case, the elucidation of the excitation mechanism will likely lead to an improved understanding of the physical processes involved in the geodynamo.

A promising future avenue to further our understanding of torsional oscillations and of their excitation is through numerical models of the geodynamo (see Geodynamo: numerical simulations). This is because the time span of the simulations can far exceed the historical record, and also because the simulations allow direct access to field variables at every location in the core and thus offer a means to examine the details of the dynamics. Since torsional oscillations are one of our most robust observation of core dynamics, realistic numerical models of the geodynamo should include them. Oscillations of geostrophic flows have been observed in a numerical model (Dumberry and Bloxham, 2003) and their excitation is consistent with the above scenario of torques produced by the convective dynamics. However, these oscillations do not follow equation (3) because the current limits in computation prevent the use of realistic Earth parameters in the model, and the extrapolation of the results to the real Earth remains uncertain. Nevertheless, the ability of the numerical model to produce geostrophic flows is a step in the right direction. With numerical models becoming increasingly closer to Earth-like conditions, there is hope that they will soon encompass realistic torsional oscillations.

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Cross references

Taylor condition, decade variations in lod, geomagnetic secular variation, Alfvén waves, geomagnetic jerks, core motions, time-dependent models of the main magnetic field, geodynamo: numerical simulations.

Figure Captions

Figure 1: Torsional oscillations in the core. The direction of the azimuthal cylindrical flow indicated by the black arrows oscillates in time. The Earth's axis of rotation points in the direction of Ω.

Figure 2: An example of the time-dependent axisymmetric, equatorially symmetric part of the azimuthal velocity at the surface of the core between 1900 and 1990 inverted from magnetic field model *ufm1* (Bloxham and Jackson, 1992). Colatitudes (θ) are given in degrees. If this part of the velocity extends rigidly inside the core, then it represents time-dependent velocities of rigid cylindrical surfaces (geostrophic flows) with radius $s = r_c \sin\theta$, where r_c is the (spherical) radius of the core. The undulations in radius and time are suggestive of a propagating wave consistent with torsional oscillations.

Figure 1

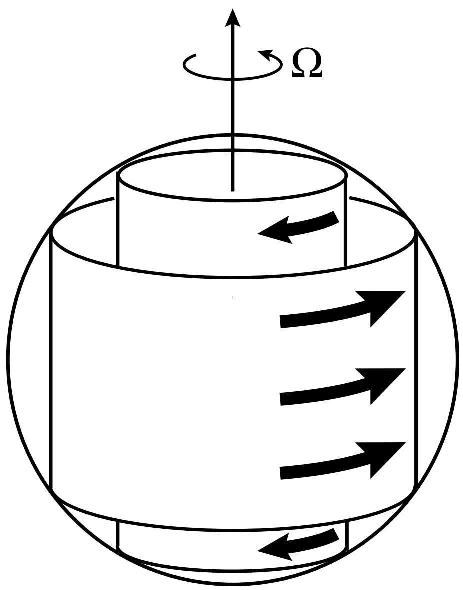
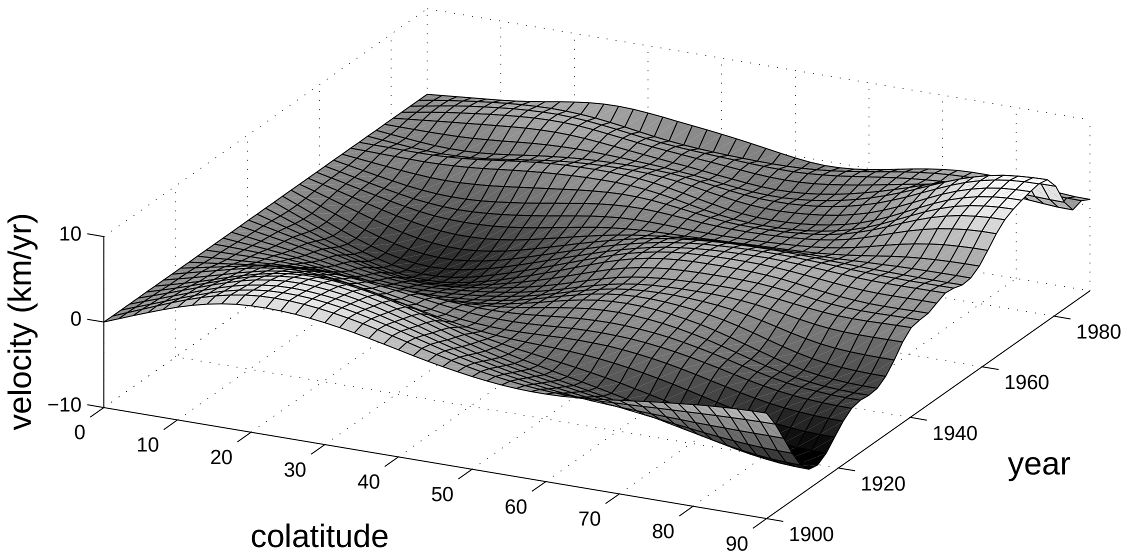


Figure 2



velocity (km/yr)
10
0
−10
0
10
20
30
40
50
60
70
80
90
colatitude
1900
1920
1940
1960
1980
year