Problem Solving

I declare that the primary purpose of all mathematics is to help us realize ourselves as human beings more completely. When we are honest, thoughtful, sensibly confident, determined, curious and generous, we tend to create good lives for ourselves and for the people around us. I believe that math in general, and mathematical problem solving in particular, can play a role in sensitizing us to careful thought, hopefulness, personal control, perseverance and ingenuity. It seems to me that I get several benefits from trying to solve problems.

-I gain knowledge about the mathematics I use or have to learn in solving problems.
-I get happiness from thinking hard about something.
-I increase my capacity to reason.
-I feel pleasure when I increase my capacity to reason. It makes me feel more powerful.
-I learn to examine my feelings of failure, triumph, frustration, joy and inadequacy, and try to learn how I can be a better person through this examination.

A feeling I have sometimes heard expressed is that a person "must be a genius" to make progress in solving problems, or that "I'm no good at problem-solving". There are at least two problems with this sort of feeling:

1. It isn't true.
2. It portrays the speaker as passive, as not changing. Changing – getting better at stuff – is what people do. It's what students do.

This year, I would like you to see "problem-solving" as a way of developing your thinking skills, your mathematical skills and your intrapersonal skills. You cannot "finish" this development in a year. Nobody can. However, I strongly believe that you can make deep and meaningful progress.

As you engage with problems this year, I would like you to consciously ask this question, (and try to answer it thoughtfully) at every important moment, and again after you have decided you are finished with the problem.

How does my thinking fit in...

a) mathematically?

Is this question a good example of something else? What links can I imagine to other parts of math or other subjects? How could I convince somebody else of these connections? Can this work be extended, or generalized or proven? (And any other questions that create mathematical meaning or further questions...)

b) in my larger world?

Where in my life can I use the knowledge I have gained in the process of working on a problem? Am I too self-critical? Is my perseverance/ creativity/ intelligence a strength in my school life, or in my relationships with others? How do I deal with frustration? Does the pleasure I get from being clever create generosity within me, or a sense of ownership? What is my current mental state? What is the reason for it? What is the result of that state? Do I have control over my emotions, or do they have control over me? Can I enjoy my well-earned success? (And every other question that is important to you...)

STARTING PROBLEMS

Two important processes in striating and developing problems are **specializing** and **generalizing**.

Specializing is the process of simplifying a problem, or testing different examples until you obtain some insight(s). Generalizing is the process of detecting patterns, and of making the specific more wide-ranging, more abstract, more general.

When you are stuck, specializing is often helpful in making progress. Try several specific examples. Ask what happens in a particular case. Use diagrams, physical models, numbers, algebraic symbols, graphs...

As you work through a problem, ideas gained from your specializing may lead you to generalize. These generalizations may then be tested by further specialization, and so on.

Specializing – means choosing examples or simplifications in the problem

a) randomly – to get a feel for the question
b) systematically – to prepare for generalization
c) artfully – to test generalizations

Generalizing – means detecting a pattern, leading to

What seems likely to be true (a conjecture)
Why it is likely to be true (a justification)
Where it is likely to be true (a more general setting of the problem)
We value a rule or algebraic formula to describe these patterns, where appropriate.

Very often, new ideas and conjectures come out of the process, and the use of the written notes will help you develop and record your thinking.

Specializing may involve solving a more limited or simpler version of the problem, In fact,

"If you can't solve your current problem, alter it until you ." – J. Mason

If a question asks about all triangles, can you show it's true for at least equilateral triangles? Or right triangles? If the question is about a three dimensional circumstance, can you solve a two dimensional version?

Generalizing or extending problems is highly valued in mathematics. I encourage you to try to develop or extend any problem you encounter. The process becomes natural, and the intellectual rewards are immense.

RUBRIC WRITING
PREPARING PROBLEMS TO HAND IN

With your solution, you will hand in a process page. This will contain brief notes (jot notes, not necessarily full sentences) of your ideas and feelings during the problem-solving process. These are the headings John Mason suggest, and it's called Rubric writing:

STUCK!
Whenever you realize that you are stuck, Write down STUCK! This will help you to keep going, by encouraging you to write down **why** you are stuck. Without explaining **why or where**, you are stuck, simply writing 'Stuck!' doesn't help. For example, you might include:

I do not understand ...
I do not know what to do about ...
I cannot see how to ...
I cannot see why ...

Being stuck can feel frustrating at first, but it is a very positive sign for you. **Being stuck is an honourable state. All true learning requires a person to be stuck at least once.**

AHA! (or IDEA!)
Whenever an idea comes to you or you think you see something, jot it down. That way you will know later what the idea was. Very often people have a good idea, but lose it later if it is not written down. Follow AHA! with

Try ...
Maybe ...
But why ...

CHECK

Check your calculations or reasoning (Say briefly how.)
Check any idea with examples (specializing)
Check that your solution really answers the question

SUMMARY
When you have done all that you can, take time to write a brief summary of the session's work. Even if you think you didn't get very far, it helps to write up what you have done so when you return you can quickly get started again. Sometimes the act of summarizing actually helps you have an AHA! moment. Include:

Write down key ideas
Write down the key moments that stand out in your memory.
Consider positively what you can learn from this experience.
How does my thinking fit in ...

a) **mathematically?**
b) **in the larger world?**

At first, Rubric writing may seem useless, or even annoying. **You must do it anyway!** After five or six honest attempts, you start to see your problem-solving power increase.

Adapted from: Mason, John. With L. Burton & K. Stacey. (1985). *Thinking Mathematically*. Harlow: Addison-Wesley. (p. 18)