## Cupcake Factory

Let's imagine that we are opening a cupcake factory. I don't really want to give up my teaching job, so let's try out the business in small steps. First, I found a neighbour who is willing to rent us her kitchen for $\$ 100$ per month. This will include utilities and allow us access to the kitchen from 5:00 pm until 8:00 pm weekdays and 9:00 am until 6:00 pm on weekends. After securing the kitchen space, I looked at costs for supplies and possible selling prices for the cupcakes. I am confident that we can sell each cupcake for $\$ 0.80$ more than it costs us to make it.
"Thinking to yourself about the mathematics is embedded in this problem."
"Write down one question that you would want to answer if you were opening this business with me."
"Write down a question that you think a teacher might ask."

We share the questions and I write them on an overhead. I try to guide the discussion so that the solutions will offer us a good introduction to functions (some vocabulary, the notion of large solution set, and the notion of multiple representations.)

Comment [S1]: I have used a paragraph like this on an overhead to start my functions unit. I would choose not to write the next ideas on the overhead and, instead, I would just say them to the students. My goal in this is to emphasize that the mathematics and this problem are part of an emerging conversation that we will have together. Not publishing the prompting questions (keeping them in my head or on a cue card) helps me stay responsive to the ideas of the students.
Comment [S2]: I came to this idea of silent thinking time through Catherine Humphries. I may have used it before, but her lessons showed me that I could make silent thinking time an explicit part of my lessons and that I could make it a norm in my classroom.

Here is a list of sample student questions which I would write on an overhead slide as students offer them up:

1. How many cupcakes need to be sold in order to cover the cost of rent and utilities in the factory?
2. How many cupcakes would we need to sell to have a profit of, say, $\$ 160$ (enough to buy a nice calculator for our classroom)?
3. How much would we profit if we sold 750 (or 0,1,2,3,100...) cupcakes?
4. Arrange the data we just generated into a table of values.

| Cupcakes | Profit |
| :--- | :--- |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |
| 100 | 160 |
|  |  |
| 750 |  |

5. Graph your table of values by hand.
6. What is the maximum number of cupcakes that we could make? Minimum? What are the maximum and minimum profits that are possible?
7. How can you calculate profit if you know cupcakes?
a. In words?
b. In an equation?
8. Use your calculator to verify your graph. List the similarities that you see between your calculator response and all of the above answers.

Comment [S3]: This gets at $\mathbf{x}$-intercept and its contextual meaning.

Comment [S5]: What is $\mathbf{y}$ if we know x ? I encourage students to "just name a number", and explain that this is the difference between functions work and their previous mathematics: functions describe a large set of possibilities.

Comment [S6]: Tables as tools to think with. This question evolved with guidance from me and I have liked the thinking that it gets at. By treating the table as a random sample to data, I managed to challenge some student preconceptions: 1) that the right hand column must go up by a set quantity; 2) that graphs and tables tell the whole story. I deliberately gave them 4 consecutive points and the $y$-intercept. This gives students a good feel for how the pattern is working. But, then I simply record what we already know. My students struggle to know that they can in fact select a scale (or not) and select a set of data to tell the story. This kind of table emphasizes the many choices we have in studying this problems solutions.

Comment [S7]: Here again, students have trouble reconciling the random sample with what they need to build a graph. I ask for this graph as homework and provide comments that emphasise the decisions that we make in creating a graph and the large solution set for this problem.

Comment [S8]: This gets at estimation and domain and range.

Comment [S9]: There is a possibility here to observe the importance of the 0.80 as rate here and the 100 as rent. I allow my students to take the lead in these early discussion so that whatever I bring in from the curriculum is helping them with their thoughts not providing a list of new words.

Comment [S10]: I modify or omit this question depending on students' experience with their calculators.

Comment [S11]: When students offer up these questions, I look to make sure that we will encounter some of the ideas that I think are important in the study of linear functions. I have an entire unit to let these ideas become clear, so this is the day for students to hear the functions words describe ideas that they already have.

