Superconductivity (Camerino)

Problem Set 2 Due Mar. 28, 2013 Marsiglio

1. (a) Set up the Cooper pair problem (as in class) with an attractive potential between the two pairs of magnitude $-V_0$ (negative since it is attractive). Assume a constant density of states over the entire range of integration ($\equiv g(0)$), and assume that the attractive potential is effective only when the electron kinetic energy lies within a shell of width ω_D around the Fermi level, i.e.

$$V_{kk'} = \begin{cases} -V_0 & \text{for } \epsilon_F - \omega_D < \epsilon_k, \epsilon_{k'} < \epsilon_F + \omega_D \\ 0 & \text{otherwise.} \end{cases}$$

Define $\lambda \equiv g(0)V_0$, and calculate the binding energy $\Delta \equiv 2\epsilon_F - E$, where E is the eigenvalue of the two electron scattering problem. Use the weak coupling approximation, $\epsilon_F >> \omega_D >> \Delta$. Calculate the pair wave function, $\psi(\rho)$, where ρ is the relative coordinate (you may need to do a numerical integral here, and you have to normalize the wave function). Use realistic values for the various required parameters: $k_F = 10^{10}\text{m}^{-1}$, $k_D = 1.001k_F$, where k_D is the Debye wavevector, corresponding to the Debye frequency, and $\lambda = 0.28$.
(b) Repeat (a) for a slightly more realistic model, which accounts for the fact that the attractive potential works only near the Fermi surface, whereas far away from the Fermi surface a repulsive potential is present between the electrons. Then the model is:

$$V_{kk'} = \begin{cases} -V_1 & \text{for } \epsilon_F - \omega_D < \epsilon_k, \epsilon_{k'} < \epsilon_F + \omega_D \\ V_2 & \text{for all } \epsilon_k, \epsilon_{k'} \end{cases}$$

Take for simplicity, an upper cutoff E_{max} for ϵ_k as $E_{max} = 2\epsilon_F$. Then the Fourier components of the pair wavefunction will take on two different functional forms, one for below $\epsilon_F + \omega_D$ and one above this. Solve the resulting two equations, and determine the remaining unknown coefficient by normalizing the wave function, as in (a). The formulas simplify if you define a pseudopotential,

$$\mu^* = \frac{\mu}{1 + \frac{\mu}{2}\ln(\frac{\epsilon_F}{\omega_D})}$$

where $\mu \equiv g(0)V_2$. As before, $\lambda \equiv g(0)V_1$. Use parameters as before except select $\lambda = 0.5$ and now choose μ to maintain the same overall effective attractive potential at the Fermi surface (hint: the gap equation can be written exactly as in (a) but with λ replaced by $\lambda - \mu^*$. Thus the gap and hence binding remains the same as in (a). Calculate the pair wavefunction as in (a). Plot the two results (from (a) and (b)) on the same figure. What is the most significant difference, and why does it occur ? What do you expect to happen to the RMS pair radius (you can calculate it if you like) ?