

The Basic Modal Rules (for Normal Systems)

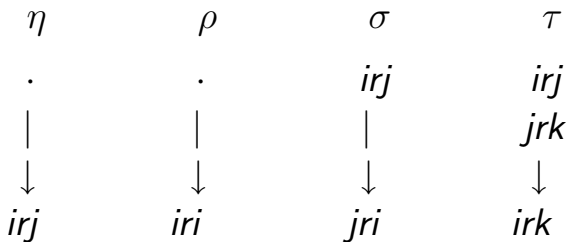
$$\begin{array}{c} \neg \Box \Phi, i \\ \downarrow \\ \Diamond \neg \Phi, i \end{array}$$

$$\begin{array}{c} \neg \Diamond \Phi, i \\ \downarrow \\ \Box \neg \Phi, i \end{array}$$

$$\begin{array}{c} \Box \Phi, i \\ irj \\ \downarrow \\ \Phi, j \end{array}$$

$$\begin{array}{c} \Diamond \Phi, i \\ \downarrow \\ irj \\ \Phi, j \end{array}$$

Rules for (Some) Normal Modal Logics Stronger
 than K ($KD=K\eta$, $KT=K\rho$, $KTB=K\rho\sigma$,
 $S_4=KT4=K\rho\tau$)



Special Rules for S_5 (=KT5 =KTB4 & others, Kv)

$\Diamond\Phi, i$

|

↓

Φ, j

$\Box\Phi, i$

|

↓

Φ, j

(As before, the j that is introduced by the \Diamond -rule must be new; the j used by the \Box -rule typically is not – you infer Φ in the various j -worlds that you already have in the tableaux.)

Rules for the Non-Normal Systems that Priest considers: N , $S_2 (=N\rho)$, $S_3 (=N\rho\tau)$, $S_{3.5} (=N\rho\sigma\tau)$

The basic system is N , which is sort of like K being the basic normal system.

The various normal rules – ρ, σ, τ – remain the same as before. Rules for negated modal formulas ($\neg\Box\Phi, i$ and $\neg\Diamond\Phi, i$) remain the same as in the normal systems, and the $\Box\Phi, i$ rule works the same.

The only difference is in the $\Diamond\Phi, i$ rule, and that difference is reflected *only* with regards to the status of the world i . The world i must be a *normal* world. We tell whether i is a normal world by looking at the branch that contains $\Diamond\Phi, i$. If $i = 0$, then i is normal; furthermore, if on that branch there is a formula $\Box\Psi, i$, then i must be normal. (i.e., if there is *any* \Box -formula that is supposed to be true at i , then i is normal).

Problems in Non-Normal Modal Logics: Which of these are Valid in What Systems?

$$\models (p \rightarrow q) \rightarrow (\neg q \rightarrow \neg p)$$

$$\Box \neg p \models \Box \neg(p \wedge q)$$

$$\models \Box p \supset \Box \Box p$$

$$\models \Box p \rightarrow \Box \Box p$$

$$\models \Box(p \rightarrow p)$$

$$\models \Box \Box(p \vee \neg p)$$

$$\models \Diamond \Diamond p$$

$$\models \Box \Box p \rightarrow (\Box q \rightarrow \Box \Box q)$$