The Basic Modal Rules (for Normal Systems)

$$
\begin{array}{cccc}
\neg\Box\Phi,\ i & \neg\Diamond\Phi,\ i & \Box\Phi,\ i & \Diamond\Phi,\ i \\
| & | & irj & \downarrow \\
\downarrow & \downarrow & \downarrow & irj \\
\Diamond\neg\Phi,\ i & \Box\neg\Phi,\ i & \Phi,\ j & \Phi,\ j
\end{array}
$$

Rules for (Some) Normal Modal Logics Stronger than K (KD=Kη, KT=Kρ, KTB=K$\rho\sigma$, S_4=KT4=K$\rho\tau$)

$$
\begin{array}{cccc}
\eta & \rho & \sigma & \tau \\
\\
\cdot & \cdot & irj & irj \\
| & | & | & jrk \\
\downarrow & \downarrow & \downarrow & \downarrow \\
irj & iri & jri & irk
\end{array}
$$

Special Rules for S_5 (=KT5 =KTB4 & others, Kυ)

$$\begin{array}{ccc} \Diamond\Phi,\ i & \qquad\qquad & \Box\Phi,\ i \\ \vert & & \vert \\ \downarrow & & \downarrow \\ \Phi,\ j & & \Phi,\ j \end{array}$$

(As before, the j that is introduced by the $\Diamond$-rule must be new; the j used by the $\Box$-rule typically is not – you infer Φ in the various j-worlds that you already have in the tableaux.)

Rules for the Non-Normal Systems that Priest considers: N, S_2 (=Nρ), S_3 (=N$\rho\tau$), $\mathsf{S}_{3.5}$ (=N$\rho\sigma\tau$)

The basic system is N, which is sort of like K being the basic normal system.

The various normal rules – ρ, σ, τ – remain the same as before. Rules for negated modal formulas ($\neg\Box\Phi,\ i$ and $\neg\Diamond\Phi,\ i$ remain the same as in the normal systems, and the $\Box\Phi,\ i$ rule works the same.

The only difference is in the $\Diamond\Phi,\ i$ rule, and that difference is reflected *only* with regards to the status of the world *i*. The world *i* must be a *normal* world. We tell whether *i* is a normal world by looking at the branch that contains $\Diamond\Phi,\ i$. If $i = 0$, then *i* is normal; furthermore, if on that branch there is a formula $\Box\Psi,\ i$, then *i* must be normal. (i.e., if there is *any* $\Box$-formula that is supposed to be true at *i*, then *i* is normal).

Problems in Non-Normal Modal Logics: Which of these are Valid in What Systems?

$\models (p \strictif q) \strictif (\neg q \strictif \neg p)$

$\Box\neg p \models \Box\neg(p \wedge q)$

$\models \Box p \supset \Box\Box p$

$\models \Box p \strictif \Box\Box p$

$\models \Box(p \strictif p)$

$\models \Box\Box(p \vee \neg p)$

$\models \Diamond\Diamond p$

$\models \Box\Box p \strictif (\Box q \strictif \Box\Box q)$