Philosophy 428/526, Fall 2013
Pelletier
Assignment #1

This assignment is due *before class* on Monday, Oct. 7th. You can do this by either (a) submitting them electronically anytime prior to class, or by giving me paper copies at the beginning of class. I will go over some portion of the assignment in class that day. Please do your work independently. (The three assignments are worth 33% of your grade if you are in 428, and 25% of your grade if you are in 526). Assignments can be handwritten if you wish. Try to be reasonably neat. (**Note**: if you do your assignments electronically, and if you use WordPerfect, please save it as pdf. I can't read .wpd documents.)

1. Semantic Tableaux. Construct classical logic tableaux for the following problems. If the argument is invalid, state a counterexample that would make all premises true and the conclusion false. (You can use either unsigned tableaux [like Priest does in Chapter 1] or signed tableaux [like we discussed in class]). In what follows, the symbol $\models$ indicates that the conclusion is coming up. If there is nothing to the left of this symbol, it means that there are no premises and that the conclusion is claimed to be a logical truth.

a. $\models (P \supset Q) \vee (Q \supset P)$

b. $\{(\neg P \wedge Q)\} \models (P \equiv Q)$

c. $\{(P \vee Q), (R \supset P), (Q \supset \neg R)\} \models (\neg(Q \vee R) \supset P)$

2. Equivalent formulas. Give a formula that has the same truth table as ϕ:

P	Q	R	S	ϕ
T	T	T	T	F
T	T	T	F	F
T	T	F	T	T
T	T	F	F	F
T	F	T	T	T
T	F	T	F	T
T	F	F	T	F
T	F	F	F	F
F	T	T	T	T
F	T	T	F	F
F	T	F	T	T
F	T	F	F	F
F	F	T	T	T
F	F	T	F	T
F	F	F	T	F
F	F	F	F	F

3. Question about two-valued truth tables:

The equivalence of $(P\vee(Q\vee R))$ and $((P\vee Q)\vee R)$ gives rise to the notation $(P\vee Q\vee R)$, without the parentheses. This latter formula means "At least one of P, Q, R is true". And we can extend this to any length of disjunction $(P\vee Q\vee R\vee\ldots\vee Z)$, and the result means "at least one of the disjuncts is true".

Letting $\oplus$ stand for "exclusive 'or'", $(P\oplus Q)$ means "Exactly one of P and Q is true".

(a) What does $((P\oplus Q)\oplus R)$ mean?
(b) Is $((P\oplus Q)\oplus R)$ equivalent to $(P\oplus(Q\oplus R))$? (prove that)
(c) If we introduce the notation $(P\oplus Q\oplus R)$, what would it mean?
(d) What would $\neg(P\oplus Q\oplus R)$ mean?
(e) What would $(P\oplus Q\oplus R\oplus\ldots\oplus Z)$ mean? (Imagine that there are some arbitrary number of letters in the formula).
(f) What would $\neg(P\oplus Q\oplus R\oplus\ldots\oplus Z)$ mean? (Again, imagine an arbitrary number of letters).
(g) The preceding discussion of $\oplus$ should remind you of some properties mentioned in our discussion of Post's Functional Completeness Theorem. Explain some relationship between $\oplus$ and those properties. (This is purposefully vague. Think about it and try to come up with some connection.)

4. Showing Functional Completeness. The following questions ask you to prove that the displayed set of (2-valued) propositional connectives are functionally complete. **DO NOT** use Post's functional completeness theorem to show this. You can appeal to the facts that $\{\wedge,\neg\}$, $\{\vee,\neg\}$, $\{\supset,\neg\}$, $\{\downarrow\}$, $\{\uparrow\}$ are each functionally complete (as is any bigger set of connectives that includes one of these functionally complete sets).

(a) $\{\supset,\oplus\}$ ($\oplus$ is "exclusive *or*")
(b) $\{*,T,\perp\}$ (* is "if—then—else", which we've discussed. T and $\perp$ are the 0-ary constants "true" and "false".)

5. Showing Functional Incompleteness. Using Post's Functional completeness theorem, state whether these sets of connectives are or are not functionally complete, and state *why* they are or *why* they aren't.

(a) $\{\wedge,\vee,\supset,T\}$
(b) $\{\wedge,\vee,\oplus\}$
(c) $\{*,\perp,T\}$
(d) $\{\neg,\oplus,\equiv\}$