Conditional Logics

Priest presents his favourite account of how natural language conditionals should be represented (there are a few more things in later chapters that we won't consider... Chapter 5 is the main thing.

Priest considers

Strengthen Antecedent: $p \supset r \vDash (p \wedge q) \supset r$
Transitivity: $(p \supset q), (q \supset r) \vDash (p \supset r)$
Contraposition: $p \supset q \vDash \neg q \supset \neg p$

Note that these are all still valid if $\supset$ is replaced by $\prec$3, in both normal and non-normal modal logics.

Priest's rationale for his natural language "counterexamples" is that conditionals are *enthymematic*, that is, they suppress certain information. In the case of a conditional, for example "If this plane lands in Rome, then it lands in Italy" suppresses the ("obvious") fact that Rome is in Italy.

Similarly, his natural language counterexamples suppress certain "obvious" facts–for example, that he is not killed in a car accident between the time he says 'If it doesn't rain tomorrow, then we'll go on a picnic' and tomorrow.

However, the problem is that there are an open-ended number of possible "obvious" facts, such as that Martians don't invade the planet. You just can't list them all.

However, there is the phrase "all other things being equal", or its Latin counterpart *ceteris paribus*.

Priest's proposal is that *every* conditional has a *ceteris paribus* clause in it. So, a conditional 'if p then q' really is represented as 'if p and C_p, then q', where C_p means "all other things being equal, given that p".

It is an open-ended conjunction, and depends on what the antecedent is.

Priest wants to use a new symbol $>$ for this conditional:

"if p then q" is symbolized $p > q$

$p > q$ is true at w if q is true at every accessible world at which $(p \wedge C_p)$ is true.

In working the details of this out, Priest proposes to stay with the modality of S_5, where every world is accessible to every other world.

But he is going to "relativize" modalities to formulas: For every formula ϕ, we introduce the accessibility relation $w_1 R_\phi w_2$, which means "w_2 is, *ceteris paribus*, the same as w_1 and ϕ is true at w_2.

$$[\![\phi > \psi]\!]^w = 1 \text{ iff for all } w' \text{ such that } wR_\phi w', [\![\psi]\!]^{w'} = 1$$

Priest introduces the notation $f_\phi(w)$, which means "the set of *ceteris paribus*-related ϕ-worlds, accessible from w". So we could say

$$[\![\phi > \psi]\!]^w = 1 \text{ iff } f_\phi(w) \subseteq [\psi]$$

([p] = the set of worlds where p is true)

Tableaux rules are just as they are in Kυ, except that we can have $ir_\phi j$ accessibilities.

The rules for $>$ are:

$$\begin{array}{c} \phi > \psi,\ i \\ ir_\phi j \\ \downarrow \\ \psi,\ j \end{array} \qquad\qquad \begin{array}{c} \neg(\phi > \psi),\ i \\ \downarrow \\ ir_\phi j \\ \neg\psi,\ j \end{array}$$

You read off countermodels just as in Kυ, except when you encounter $ir_\phi j$, this is "world j is, *ceteris paribus*, an i-world".

It is natural to want ϕ to be true at w_2, if $w_1 R_\phi w_2$.

It is also natural to say that, if ϕ is already true at w_1, then the set of worlds that are the same as w_1 except that ϕ is true there must already include w_1.

These two conditions are, more formally:

(1) $f_\phi(w) \subseteq [\phi]$

(2) If $w \in [\phi]$, then $w \in f_\phi(w)$

This is the logic Priest calls C^+

These two conditions are dealt with like this:

Condition (1) calls for an alteration to the $\neg(p > q),\ i$-rule; condition (2) calls for a new type of rule:

$$
\begin{array}{l}
\neg(\phi > \psi),\ i \\
\quad\downarrow \\
ir_\phi j \\
\phi,\ j \\
\neg\psi,\ j
\end{array}
\qquad\qquad
\begin{array}{ccc}
 & . & \\
\swarrow & & \searrow \\
\neg\phi,\ i & & \phi,\ i \\
 & & ir_\phi i
\end{array}
$$