## Phil 428/526, Fall 2013 Take-Home Midterm Exam

Directions: This is a take-home exam. However, you are not allowed to discuss the exam with others, nor - I hope - will you merely find an answer to one of the questions on the web, but you'll work the answer out yourself. You have ONE WEEK to complete this exam, although I believe you could easily finish in less time. Since this exam is being made available on Wednesday evening Nov. $6^{\text {th }}$, you must have it either given to me or placed in my mailbox on Wednesday afternoon, Nov $13^{\text {th }}$ before 5 pm . Please label your answers with the number of the question you are answering, so that partial credit can be given for correctly done scratch work. Try to be reasonably neat. This exam is worth $33 \%$ of your grade if you are in Phil 428, and $25 \%$ of your grade if you are in Phil 526 .

Answer any six of the following seven questions. Each question is weighted equally. For the essay questions, do NOT write a term paper on the question. Keep your answer to a page or (max) two.

1. Give a semantic tableau for these three modal logic arguments in the modal system indicated in the problem. If the inference is invalid, give a counter-model (an assignment of truth-values to atomic letters in each possible world, with an indication of why this makes the modal formulas true or false in the relevant worlds of the system under consideration). You can use signed or unsigned tableaux, so far as the truth-values go. (But possible worlds are a different matter!)
a. System K: $(p \vee \neg p) \supset \diamond(p \vee \neg q) \vDash(\diamond \neg p \vee \diamond \neg q) \vee \diamond(p \vee q)$
b. System $\mathbf{S}_{3}: \vDash(\square \square p \rightarrow(\square q \rightarrow \square \square q))$
c. System KTB: $\vDash \diamond \square p \supset \square \diamond p$
2. Many sentences of natural language contain information that does not get represented in the standard translations into classical propositional logic. Here are five examples. For each one, give its translation into classical propositional logic, using the scheme of abbreviation given, and explain what information has been left out. In each case, explain how a classicalist (in terms of propositional logic) would justify leaving it out. For each of the sentences, do you think it could be possible to explicitly make what's left out be a part of the translation?
a. Using the scheme of abbreviation where $p$ : She is poor; $h$ : She is honest, translate into propositional logic the sentence "Despite the fact that she is poor, she is nevertheless honest".
b. Use the scheme of abbreviation where $w$ : Jason wanted to be a part of the demonstration; $p$ : Jason is a part of the demonstration; b: Jason being in the demonstration bothers his mother. Translate into propositional logic the sentence
"Even if Jason wanted to be a part of the demonstration, he would not do it if it bothered his mother".
c. Make up an appropriate scheme of abbreviation and translate "John has stopped teasing his wife".
d. Using the scheme of abbreviation where $p$ : The pub is open after class; $q$ : You want to have a drink; translate into propositional logic the sentence "The pub is open after class, if you want to have a drink."
e. Using the scheme of abbreviation where $p$ : I put the usb stick into the hub; $q$ : I will be able to look up the data, translate into propositional logic the sentence "If I put the usb stick into the hub, I will be able to look up the data."
3. Let $\otimes\left(\varphi_{1}, \varphi_{2}, \varphi_{3}\right)$ be a new 3-place truth function that has the truth table given in the table below. (a) Write the truth table for the formula:

$$
\otimes(q, \otimes(r, p, p), \otimes(\otimes(p, p, p), q, \otimes(p, q, r)))
$$

and then (b) write a formula that has the same truth table as the formula in part (a) but which uses only our familiar connectives $\wedge, \vee, \neg, \supset, \equiv$.

| $\varphi_{1}$ | $\varphi_{2}$ | $\varphi_{3}$ | $\otimes\left(\varphi_{1}, \varphi, \varphi_{3}\right)$ |
| :---: | :---: | :---: | :---: |
| $T$ | $T$ | $T$ | $F$ |
| $T$ | $T$ | $F$ | $F$ |
| $T$ | $F$ | $T$ | $T$ |
| $T$ | $F$ | $F$ | $F$ |
| $F$ | $T$ | $T$ | $T$ |
| $F$ | $T$ | $F$ | $F$ |
| $F$ | $F$ | $T$ | $F$ |
| $F$ | $F$ | $F$ | $T$ |

4. The truth function $\otimes\left(\varphi_{1}, \varphi_{2}, \varphi_{3}\right)$ given by the table in problem (3) is functionally complete.
a. Show that $\otimes$ obeys the requirements of Post's functional completeness theorem, thus guaranteeing its functional completeness.
b. Assume you know that $\{\wedge, \vee, \neg\},\{\wedge, \neg\},\{\vee, \neg\},\{\uparrow\}$ (NAND), and $\{\downarrow\}$ (NOR) are each a functionally complete set of connectives. Show that $\{\otimes\}$ is functionally complete, by reducing it to one of these already-known functionally complete sets of connectives. (I.e., show how to define the connectives in some functionally complete set using only the new connective).
5. Use the tableaux method for Priest's conditional logic system $\mathbf{C}^{+}$and determine whether the following are valid. If not valid, give counterexamples that are appropriate to $\mathbf{C}^{+}$.
(a) $p>(p>q) \vDash(p>q)$
(b) $p>(q>r) \vDash q>(p>r)$
6. One way that a many-valued logic might be shown to be functionally complete would be if there were "normal" disjunctions and conjunctions (a disjunction is normal if it yields the "most true" of the values of its arguments; a conjunction is normal if it yields the "least true" of the values of its arguments), and there were names (constants) for all the truth values, and there were "parametric operators". (A parametric operator is a unary connective that is "completely true" when its argument has the parametric value and otherwise "completely false". We used the $J_{i}(\varphi)$ as our parametric operators: this takes the "completely true" value if $\varphi$ has the value $i$, and is completely false otherwise.)
a. Consider a three-valued logic (with values $1, \frac{1}{2}, 0$ ) that has normal disjunction and conjunction, constants for the three values (call them $C_{1}, C_{\frac{1}{2}}, C_{0}$ ), and parametric operators $J_{1}(\varphi), J_{\frac{1}{2}}(\varphi), J_{0}(\varphi)$. Write out a formula - using just these connectives, constants, and operators - that has the same truth table as the formula $\Psi$ below:

| $p$ | $q$ | $\Psi$ |
| :---: | :---: | :---: |
| 1 | 1 | 0 |
| 1 | $\frac{1}{2}$ | 0 |
| 1 | 0 | 1 |
| $\frac{1}{2}$ | 1 | $\frac{1}{2}$ |
| $\frac{1}{2}$ | $\frac{1}{2}$ | 1 |
| $\frac{1}{2}$ | 0 | $\frac{1}{2}$ |
| 0 | 1 | 0 |
| 0 | $\frac{1}{2}$ | 0 |
| 0 | 0 | $\frac{1}{2}$ |

b. A way to define the constants $C_{1}, C_{\frac{1}{2}}, C_{0}$ that are used in the part (a) method of proving functional completeness is to add a (Post-style) cyclic negation to the language. Suppose you add this connective (call it $\sim$ ). Show how to define the three constants in the language mentioned in part (a), using only the normal $\vee$ connective plus cyclic negation. (Note: do this without using the normal $\wedge$.)
7. What is a possible world? (There are various conceptions. Explain some [two or three] of them, and for each conception, give a short argument or consideration either
for it or against it.) You could also consider this issue: A person who doesn't like the notion of a possible world might say "By definition, the only world that exists is our own. So, as a matter of logic, there aren't, and can't be, any possible worlds besides ours." What could be said by some of your earlier-mentioned conceptions of possible worlds?

