

Some Practice Problems to use tableaux in basic modal logic (system K)

The first thing for you to do is read Priest's description of the modal tableaux rules (the rules for system K are in §2.4.4 on p. 25). Use those rules to try tableaux for the following problems in system K. (Recall the material from the classical logic case: if you are testing whether something is a theorem, you start your tableau with that formula marked as false (either by negating it like Priest does, or by marking it with a “: F” as I indicated for signed tableaux). And you also add “, 0” to indicate that you are testing the formula in the actual world (world 0). If on the other hand you are evaluating whether an argument is or isn't valid, you start by listing the premises (and marking them with “: T” if you are doing signed tableaux) and that the conclusion is false (either by negating it or by adding the “: F” sign). And each of the formulas is being evaluated in world 0. The only way to get other worlds is if the formula itself has modal operators in it, which might require other worlds to evaluate.

Evaluate the following in system K. When an argument is invalid (or a formula is not a theorem), give a countermodel—you state which atomic statements are true/false in which worlds.

1. $\models \Box(p \wedge q) \supset (\Box p \wedge \Box q)$
2. $\Box p, \Diamond q \models \Diamond(p \wedge q)$
3. $\models \Box p \supset \Box \Box p$
4. $\Diamond p \models \Diamond \Diamond p$
5. $\models \Diamond(p \vee \neg p)$
6. $\models \Box p \equiv \Box \neg \neg p \supset p)$
7. $p \models \Box p$
8. $\Box p, \Box \neg q \models \Box(p \supset q)$
9. $\Box p \models \Box(q \supset p)$
10. $\Diamond p \models \Box \Diamond p$
11. $\Diamond \Box p, \Box \Box(p \supset q) \models \Diamond \Box q$