Tableaux for Normal Propositional Modal Logics

There are many different ways to give tableaux for modal propositional logics. Priest's is one of the more straightforward ones. (He uses $\neg$ in place of the sign ": F", but he uses numbers as signs of worlds. By convention, he starts with 0 as the sign for the actual world.) Here are the rules for the basic normal logic K:

$$
\begin{array}{cccc}
\neg\Box\Phi,\ i & \neg\Diamond\Phi,\ i & \Box\Phi,\ i & \Diamond\Phi,\ i \\
| & | & irj & \downarrow \\
\downarrow & \downarrow & \downarrow & irj \\
\Diamond\neg\Phi,\ i & \Box\neg\Phi,\ i & \Phi,\ j & \Phi,\ j
\end{array}
$$

Practice using these rules with the examples in Priest Chapter 2, or the ones in another document on this webpage.

Most the other normal modal logics that Priest considers: KD, KT, KT4, and KTB, are governed by the following rules:

$$
\begin{array}{cccc}
\eta & \rho & \sigma & \tau \\
 & & & \\
\cdot & \cdot & irj & irj \\
| & | & | & jrk \\
\downarrow & \downarrow & \downarrow & \downarrow \\
irj & iri & jri & irk
\end{array}
$$

The rule for S_5 (aka KT5, KTB4, and others) is different. In S_5, every world is accessible from and to every world, so we don't really have to keep track of the interrelations so carefully. Priest calls this sort of accessibility relation υ, for "universal". The two rules he gives for S_5 are:

$$
\begin{array}{cc}
\Diamond\Phi,\ i & \Box\Phi,\ i \\
| & | \\
\downarrow & \downarrow \\
\Phi,\ j & \Phi,\ j
\end{array}
$$

Like with the $\Diamond\Phi,\ i$ rule for all these systems, and like the η-rule, the j-world introduced by $\Diamond\Phi,\ i$ in the υ-system is required to be new to the branch(es) that it is introduced on. On the other hand, the $\Box\Phi,\ i$-rule introduces $\Phi,\ j$ for all the worlds that are already in the branch(es). The way to look at the difference is that $\Diamond\Phi,\ i$ says "Φ is true in some world that is accessible from i, whereas $\Box\Phi,\ i$ says Φ is true at every world that is accessible from i. (And since in S_5, every world is accessible from i, we need to enter $\Phi,\ j$ for every one of the j's that are in the branch.)

Let's try using these rules for some normal modal logics besides K. Examples on next page.

What (if any) systems do the following hold in?

1. $\vDash (\Box p \vee \Box q) \supset \Box(\Box p \vee \Box q)$
2. $\Box(p \equiv q) \vDash \Box(\Box p \equiv \Box q)$
3. $(\Diamond(p \supset (q \vee r)) \vDash ((\Box p \supset \Diamond q) \wedge (\Box p \supset \Diamond r))$
4. $\Box(p \supset q), \Diamond(p \wedge r) \vDash \Diamond(q \wedge r)$
5. $\vDash \Box(\Box p \supset \Box q) \vee \Box(\Box q \supset \Box p)$
6. $\vDash \Diamond \neg p \vee \Diamond \neg q) \vee \Diamond(p \vee q)$
7. $\Box p, \Box q \vDash \Box(p \equiv q)$
8. $\vDash \Box(\Box p \supset q) \vee \Box(\Box q \supset p)$