Strict conditionals

Lewis wanted to define a good conditional, symbolized $\strictif$, to replace what he thought was a terrible conditional, $\supset$. He used the paradoxes of material implication to give weight to $\strictif$, because the following were not valid arguments with $\strictif$ (but were with $\supset$ in its place):

$q \models (p \strictif q)$
$\neg p \models (p \strictif q)$
$(p \wedge q) \supset r \models (p \strictif r) \vee (q \strictif r)$
$(p \strictif q) \wedge (r \strictif s) \models (p \strictif s) \wedge (r \strictif q)$
$\neg(p \strictif q) \models p$

To figure out whether $\strictif$ is an appropriate conditional, first note that we need to have a logic with ρ in it, or else we don't have MP: check it out with a tableau. Without ρ, whether a normal or a non-normal logic, $p, p \strictif q \not\models q$. However, even if we do have ρ, there are a number of other problematic cases, sometimes called "paradoxes of strict implication":

$\Box q \vdash p \strictif q$
$\neg\Diamond p \vdash p \strictif q$

(These hold even in N, so they hold in all the normal and non-normal modal logics. As special cases of them, since $\models \Box(q \vee \neg q)$, we have $\models p \strictif (q \vee \neg q)$ and since $\vdash \neg\Diamond(p \wedge \neg p)$, we have $\models (p \wedge \neg p) \strictif q$.

Priest gives a number of natural language arguments/statements which make it seem implausible that $\strictif$ is the natural language 'if–then', focusing on these two paradoxes. He in particular pays attention to this argument-form (which is a variant of the last formula in the previous paragraph above):

$p \wedge \neg p \models q$

(the old "from a contradiction anything follows"–which he had already identified as a shortcoming of $\supset$). Lewis had already predicted that people would find this problematic, so he gave an argument in its favor. Priest recounts the argument using the Gentzen formulation of natural deduction. Here it is using a Fitch-style (note that I'm using the "disjunctive syllogism" version of $\vee$-E, rather than the more expansive version that introduces subproofs for each disjunct):

1	$(p \wedge \neg p)$	
2	p	1, $\wedge$-E
3	$\neg p$	1, $\wedge$-E
4	$p \vee q$	2, $\vee$-I
5	q	3,4 $\vee$-E