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Is Logic all in our Heads? From Naturalism to Psychologism

Abstract. Psychologism in logic is the doctrine that the semantic content of logical terms is in some way a feature of human psychology. We consider the historically influential version of the doctrine, Psychological Individualism, and the many counter-arguments to it. We then propose and assess various modifications to the doctrine that might allow it to avoid the classical objections. We call these Psychological Descriptivism, Teleological Cognitive Architecture, and Ideal Cognizers. These characterizations give some order to the wide range of modern views that are seen as psychologistic because of one or another feature. Although these can avoid some of the classic objections to psychologism, some still hold.

Keywords: psychologism, cognitive science, artificial intelligence, naturalism, Mill, Frege, Husserl, Quine.

“In the old anti-psychologistic days . . .” W.V. Quine [79]

1. Psychologism and Logic: An Introduction

We want to revisit ‘the old anti-psychologistic days’ in order to discern what psychologism was thought to be at that time and why it was so roundly rejected. Our investigations are not only interesting from the point of view of our understanding the development of modern logic but can also be of use in evaluating modern attempts to revive psychologism in logic. Most moderns feel that the classic psychologistic position — which we below call ‘psychological individualism’ — is not a viable position for logic. We describe this position in enough detail to understand why the classic arguments against it have been seen to carry the day. We then propose ways to be psychologistic about logic in addition to the classic psychological individualism, by aligning them with some positions in modern philosophy and cognitive science. One of these ways is the ‘Ideal Cognizer’ mentioned in the call for papers for this special issue, but we will describe other psychologistic positions also, by proposing a structure within which to characterize positions that bear similarities to the classic version. Along the way, we mention various modern

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thinkers who have claimed to be psychologistic about logic, showing how their views fit into our framework. Our goal, at that point, will be to see whether any of the classic objections to psychological individualism carry over to these other sorts of psychologism in logic. By the end we will have proposed and evaluated a range of psychologisms in logic, and will have shown which of the classic objections can be avoided, but which objections seem still to hold against which positions. It seems to us that many of our modern psychologism-in-logic advocates have seen that one or another of the classic criticisms can be avoided by adopting this-or-that theoretical viewpoint. But they have sometimes not noticed that others of the classic objections still apply. It then becomes a question of whether modern adherents to one or another of these psychologisms-in-logic are prepared to accept these consequences.

1.1. What is Psychologism?

Within philosophy, psychologism emerged in the early 19th century as a kind of methodological stance that advocated an empirical or naturalistic approach to questions in epistemology and metaphysics. It has a foundation in the view that the meanings of words are (personal, subjective) ideas — although as we will see, not all the other psychologistic positions we propose are fully committed to this starting point. As Kusch [61] relates, this perspective reached its peak in the mid- to late-1800s with certain British empiricists and German logicians, who argued that the laws of logic and mathematics were suitably generalized accounts of patterns of human thought and reasoning. Note that this is to make logic *emerge from* the (subjective) psychology of people. This is not the same as claiming that people “are logical”, or “follow logical principles” in their beliefs and other aspects of their individual psychology, either consciously or as a description “from the outside” of their beliefs. And it is yet another matter to claim that Psychology as a science or a discipline employs or presupposes logic. Not all of the claims about, criticisms of, and conclusions regarding psychologism in logic keep these matters separate. This is one way in which a consideration of psychologism can become confused.

Most generally, psychologism in any field (such as logic or mathematics) means that some important aspect of the realm of study (e.g., logic or mathematics) relies upon, or is constituted by, facts and issues of human psychology. We will use the terminology ‘*the content of* logical and mathematical terms’. Others might have preferred ‘the meaning of logical and mathematical terms’. The intent is to locate “what we are talking about”

when using the terms, or “what is the ultimate grounding of the significance of the terms.” In the classic form of psychologism, the content of logical terms is the subjective idea that is associated with the term. Different conceptions of psychologism might associate other things with these terms, but to still be a psychologism, they have to somehow involve subjective items in this association. Classical psychologism combines this with a view that judgments involving these subjective items (or sentences constructed from them) are also subjectively mental. Crudely put, this asserts that a field like logic (or some important aspect of it) is whatever a thinker *thinks* it is; we will make and investigate more informative distinctions along this dimension shortly.

An opposed philosophical position (about logic) is that logic is not a subjective matter of the psychology of sentient creatures, but instead is an objective matter of the relations among independently existing propositions, predicates, and terms. According to this opposed position, propositional logic, the syllogism, probability theory, and the like, describe some independent-of-people purpose, such as the preservation of truth when inferences are made (where inference itself is also to be understood as independent of people). Thus, by this view, there are truths in logical matters that are independent of what people think or do: logical truths exist in a ‘world apart’ from humans — apart from both the subjective ideas and beliefs that people have and from the physical world that humans inhabit. (A so-called “third realm” apart from the subjective and physical realms). These normative standards are “out there” and are to be *discovered* by investigation of this third realm. We can call this view Platonism because of its commitment to a realm of truths independent of the psychology of humans and independent of the physical realm. Another category of opponents to psychologism in logic would be those who hold logic (and mathematics) to be independent of people’s psychological states, but not to be in a third realm. Instead, such theorists would hold, logic and mathematics are “in the physical world”. We will call these theorists ‘physicalists’ (about logic and mathematics), and use the term ‘realist’ to designate both the logico-mathematical Platonists and the logico-mathematical physicalists, and ‘anti-Platonists’ to designate both the logico-mathematical physicalists and the logico-mathematical psychologismists.

An important motivation, particularly from a cognitive science perspective, for some modern holders of psychologism in logic comes from the “naturalistic movement” in modern philosophy. This movement asserts that all claims of epistemic access must be given an account according to which it becomes *possible* for humans, living in a natural world, to come to know facts

about the objects of these claims. ([79], from which the epigraph of this paper is taken, is often seen to be a prime instigator of this movement.) In the case of logic or mathematics, naturalism has been seen as ruling out any third realm. However, it is not so clear that it thereby necessarily generates a psychologistic theory as Quine suggests, since one might hold that logical and mathematical objects and truths are denizens of the physical world. But following Quine himself in [79, 80], one might think of a person’s logic as being “read off” of the pattern of sentences accepted by that person. Some person might turn out to be such that whenever he accepts $p \rightarrow q$ and p , then that person also accepts q . Hence modus ponens is a logical truth (for that person).¹ We will revisit this type of psychologism below, in Section 5. In a similar vein, Haack [39, pp. 238-242] uses the term “weak psychologism” for (what she calls) a ‘nominalistic’ theory of logic, wherein the content of logic has to do with the verbal (and written) manifestations of logical vocabulary.

Clearly the term ‘psychologism’ is used in a variety of related ways and can be interpreted in a variety of ways. In our view, the core meaning is that some important aspect of the realm of study relies upon, or is constituted by, facts and issues of human cognition. From this perspective, psychologism accepts that humans reason, perceive, and think about “the external world” in particular ways, and thereby psychologism could be a general doctrine that can be applied (methodologically or otherwise) in *any* realm of study.² We will be proposing a structure to be imposed onto the notion of psychologism so that the reader can understand and engage the various strands that are in play, although we acknowledge that there could be other types that we do not actively consider.

Within this very general characterization of psychologism, there is room for many different conceptions of just how human cognition is to be accommodated in the realm of study. Some conceptions are very wide; so

¹Well, actually one would have to be convinced that the speaker’s ‘ \rightarrow ’ was a conditional.

²For example, [74] investigates the question of whether the field of formal semantics should be a type of psychology, or should eschew any data from the subjective experience of speakers and be instead a branch of mathematics. She is thus investigating the question of whether Formal Semantics should be psychologistic in nature. Even an avowedly non-mental realm, such as physics, could be studied in this manner — taking in the psychology of people as a portion of its reach. One direction such a theory might be developed is Kantian in nature, where the Newtonian ontology is postulated to be a necessary precondition of our thought — or as we might prefer to say, it is part of our human cognitive architecture that we view the world as governed by Newtonian mechanics. (See [32, esp. pp. 167-181].) Another direction this might take is exhibited by “Naïve Physics” ([44, 45]) and its kin “Qualitative Physics” ([26, 22]) in Artificial Intelligence, which can be seen as investigations of our everyday understanding of how the physical world operates.

wide, in fact, that it would be difficult to conceive of some realm of inquiry that wasn't psychologistic in such a sense. We think that such a wide interpretation of 'psychologism' is not very useful. Friedrich Eduard Beneke (1798–1854), for whom the term was coined³, held that since the only way philosophers could arrive at any of their truths was by introspection, then psychology must be the fundamental philosophical discipline: any subfield of philosophy, such as ethics, metaphysics, logic, political theory, etc., is no more than applied psychology, i.e., the application — conscious or not — of the human cognitive machinery.

With all of the concepts of the philosophical disciplines, only what is formed in the human soul according to the laws of its development can be thought; if these laws are understood with certainty and clarity, then a certain and clear knowledge of those disciplines is likewise achieved. [7, p. xv], quoted in [2]

(For 'human soul', read 'human mind'). Beneke intended these views to apply to logic also, as he explains in [8].

A similarly wide reach characterizes Kitcher's [58, p. 13] view. For him, an "approach is appropriately called 'psychologistic' [if] it focuses on processes which produce belief, processes which will always contain, at their latter end, psychological events." From this viewpoint, any science that itself aims to produce a belief, or aims to give an account of how a belief might be produced in one who investigates that field, will be engaged in a psychologistic approach. One might object to Kitcher's use of terminology here, and say that he is describing or defining the entire process of cognition, not psychologism *about* any science. One of the messages of the present paper is that many different conceptions lie behind different theorists' use of the term 'psychologism'.

Another preliminary topic that is often mentioned in the literature is a subtle distinction between "what the science *is*" vs. "what the primary evidence for the science is". For example, one might distinguish between saying "mathematical statements are mere abbreviations for psychological claims" and "while mathematical statements are about the actual world, the primary evidence for their truth comes from (the psychological process of)

³Kusch [61, p. 101] says that 'psychologism' in this sense was first used, in a characterization of Beneke's philosophy, by Johann Erdmann in [20]. We will mostly use the terms 'psychologistic' and 'anti-psychologistic' to characterize the positions, rather than variants of 'psychologistic' or 'psychologistic'.

introspection.”⁴ The idea is that the former is a type of psychologism, while the latter is most naturally seen as a type of physicalism. However, in the literature on psychologism, especially in the early thinkers, the distinction is not always honored. For example, in Beneke’s theory as briefly mentioned above, all evidence for philosophical theories is claimed to come from introspection and therefore to be psychological; and he then makes an immediate jump to the position that all such theories are *about* psychological items. Strictly speaking, this is not a legitimate jump, but most of the early writers on the topic (both the psychological and anti-psychological logicians) do not acknowledge the second possibility, apparently thinking that if the evidence is psychological then the resulting claims and hypotheses must be “about psychology”. As we said, the inference is not correct; and as a counterexample we might consider that neuroanatomical and neurophysiological facts could be the primary evidence for some theory of cognitive psychology. Since that is possible, the reverse should be possible too: psychological facts could be the primary evidence for some non-psychological theory. So, even if all the evidence for a theory of logic were psychological in nature, it wouldn’t follow that logic itself is. We will not emphasize the possibility at this point, but it will arise again later.⁵ We will not consider any further the very wide notion of psychologism.

⁴Brockhaus [11, p. 495] distinguishes ‘weak’ vs. ‘strong’ psychologism. The former is the view that logical principles are inherent in the laws of mind and uncoverable by psychological methods; the latter is the view that logic is literally a branch of psychology. We will not be following up this distinction very closely, since our concern is with where the logical principles are located. Hanna [40] uses the terms ‘weak’ vs. ‘strong’ psychologism to distinguish between (i) Necessarily, every logical proposition is the mental content of some human, and (ii) For every logical proposition, it is possible that there be a human who has it as a mental content. (Presumably, by ‘logical proposition’ Hanna means a statement with no empirical content, and also one that is true, since contradictory logical propositions may not be even the possible content of any person’s mind.) However, it is not so clear to us that (ii) really distinguishes psychologism from anti-psychologism, since most of the latter theorists believe that all of the relevant logical statements *could* be thought — that is, they do not believe in the unthinkable mysteriousness of any logical statements.

⁵One might also wonder whether this whole issue embodies a basic flaw in the first place. If the logical and mathematical claims are “about the actual world” or “about a third realm”, then this is *not* psychologism even if the evidence for the theory is psychological. For, once you get logic and mathematics into the actual or ideal world, then you don’t have psychologism, you have some sort of realism. (Or at least, you don’t have the classical psychologism). Even Frege, the arch-anti-psychologist, could hold that our primary evidence for the truth of logic and mathematics was our psychological “grasping” of statements about these fields. So, the *provenance of the evidence* could just be irrelevant to the issue of psychologism. What matters is the *source of the content* of the logical terms.

Some modern psychological investigators into human reasoning are not psychologistic in our sense. Johnson-Laird [52] and Rips [81] investigate human performance on logical tasks, such as reasoning in propositional logic or the syllogism. Their efforts describe how people do or do not match the types of reasoning sanctioned in the logical realms, and they give mechanisms according to which human mental architectures will make people arrive (or not arrive) at the logically correct results. We see, in these investigations, that there is a commitment to a distinction between how people perform on reasoning tasks and how they are *supposed to* perform. So, this is not psychologistic, unless the way they are supposed to perform is itself psychologistic. (Johnson-Laird does not seem to think this⁶). Other modern psychologists who investigate human reasoning *do* seem to be psychologistic in their outlook, however, for example, Gigerenzer and Goldstein [35], and in a different way Oakesford and Chater [71], and we will discuss them below. Related to these theories in a complicated manner is that of Macnamara [66]. We discuss these theories below under the headings of “Cognitive Architectures” and “A Psychologistic Logic”.

We now propose four core directions to be psychologistic about some target science (‘science’ being taken broadly so as to include ethics, mathematics, physics, . . . , although our ultimate target is logic and mathematics). This proposed taxonomy aims to accommodate the various themes we have discussed above, although we will argue that not all of these ways can, in the final analysis, deal successfully with all the objections leveled against psychologism.

- A) *Psychological Individualism*: Identify the target science with how individual people cognize about that science. Doing this on an individual-by-individual basis would yield possibly differing accounts of the science depending on the individual being examined.
- B) *Psychological Descriptivism*: Identify the target science with some description of the *observable performance* of people’s behavior in the realm. For example we might consider how *all* the individuals cognize about logic, or perhaps how some important subgroup cognizes about it, or perhaps what the publicly observable output of these people is. We include here various ways to bring in the social realm in which people write about or discuss logic and mathematics.

⁶Macnamara [66, Chapter 2] seems to say that Johnson-Laird [51] *does* believe this, but it is difficult to find that position in Johnson-Laird. See for instance his [52]. Rips [81, Ch. 2] gives an account of a possible mechanism behind this performance that we later will consider to be one of our types of psychologism.

- C) *Cognitive Architectures*: Argue that whatever might be discovered about the target science is a function of the human cognitive machinery that is doing the discovering, and that this machinery itself has been shaped by the world in which it must operate. Postulate an architecture that in some way reflects some important aspect of the logical and mathematical realm. Identify the science with whatever is given by some “faculty” or “module” within this architecture that is common across people. A possible requirement of this position invokes a *Competence/Performance Distinction*: The cognitive architecture in some way reflects an important aspect of the logical and mathematical realm, and under this perspective, the science is identified with how people use this faculty “correctly”. In this view, the competence can be seen as a measure of how much “interference” there is between the logical realm and the actual performance in any particular case.
- D) *The Ideal Cognizer*: Identify the science with what an ideal cognizer would cognize about that science. This is on a par with “ideal observers” in other philosophical theories such as ethics (“An act’s moral worth is determined by how an ideal observer would judge it”, [23]). We will later grapple with the notion of an ideal cognizer, be it human or otherwise.

Psychological Individualism is the view that most detractors of psychologism in logic have in mind, and is probably what the classical psychological logicians were advocating anyway. The standard criticism of this view is that it amounts to relativism (about logic), which is seen as a blatant impossibility.

When one tries to generalize across the performance, beliefs, or public manifestations of beliefs, of individuals, we get Psychological Descriptivism. There are different ways to think of this generalization. One is to imagine that people’s beliefs can be directly inspected, and that we can determine commonalities among them. For instance, we could engage in psychological experimentation or testing to find this out. Another is that we could look to the public or social manifestations of these beliefs — such as writings — to find some common content. A standard objection to such views is that there is *no* similarity between individuals along the dimension of interest to the science, and so an “averaging” or generalization of any manner is a hopeless endeavour.⁷ Both Individualism and Descriptivism apparently identify *the science* (e.g., logic) with *what people think* the science is, and

⁷Or perhaps, that while there might be groups of similar-minded people, there is no hope of similarity between groups, and hence again no notion of logical error.

this forms another strand in the criticism of psychologism in logic that we will consider later.

A different way to deflect the charge of relativism would be for there to be some guarantee that all people have the same (logical and mathematical) ideas, so that there is a common content to logical and mathematical language. For example, it may be due to societal pressures, or it might be that evolution provides this. The latter idea naturally leads to the notion of a cognitive architecture, and modules within that architecture, whereby there might be a “logical faculty” that provides the content to logical and mathematical claims. We look at various ways that this notion might be realized and question whether they can avoid the continuing charge of being relativistic. This leads us to a discussion of other types of architectures, such as that suggested by (some interpretations of) Kant.

The “competence” of people in the realm of mathematics and logic is often considered to be a consequence of the cognitive architecture of humans in this area. This is usually introduced as a way to explain how people can “make mistakes” in logic and mathematics while at the same time have the ability (competence) to do the task right. Sometimes this story comes with a developmental flavor, explaining how children acquire these modules in the course of their maturation. In Sections 6.2–6.4, concerning the role of cognitive models, we explain how this viewpoint is not a psychologism in the sense of defining logic by the cognition of people or other agents, but is rather a *use of logic* in psychology, and hence is committed to some non-subjective origin for the logic thus used.

Sometimes ideal cognizers are conceptualized by imagining a person who would always use their “underlying competence” when called upon to think about the science in question. These ideal cognizers would not allow “performance errors” to impede their underlying abilities, but always make use of the competence that (allegedly) any normal person has in the relevant realm. The theorists who advocate this view say that in certain realms one needs to distinguish carefully between the competence (in that realm) that ordinary people have and their performance (in that realm), which might suffer temporary but inessential mistakes. In the field of logic, it might be claimed, although people’s performances in logical tasks might fall short, we can recognize this and correct it, showing thereby that our underlying competence in logic is not impaired and that the errors in performance are due to some other factors, such as short-term memory limitations or lack of sufficient time, etc. So an Ideal Cognizer theory does not suffer the alleged flaw of a descriptivist theory that there are different underlying performances, because it postulates that a science is what an *ideal* thinker would believe.

These characterizations are intended generally. According to them, *any* area of inquiry might be psychologistic if it is pursued in one or another of these manners. But our interest here is with logic (and mathematics, which we will use for supporting examples). A detailed examination of these types is the core of our study here, but we first consider how we understand logic to fit into this discussion.

1.2. What is Logic?

Like the term ‘psychologism’, the term ‘logic’ has in some people’s minds a wide reach. Historically, especially at the heyday of early psychologism in logic, there were two different strands (see [59]). The Art of Logic is the appropriate use of one’s faculty of reasoning in one’s life. It is the ability to “move about appropriately in the world” by using one’s ability to reason. The Science of Logic was the study of the use of syllogisms in constructing arguments and the formal study of the syllogism (how to determine a purported syllogism’s validity, and metatheoretical issues such as how to “reduce” one syllogistic form to another). Those who focused on the Art tended to view logical inferences as just “mixed together” with all the other ways people go from one “idea” to another, sometimes calling the logic subpart “the study of truthful inference.” Those who focused on the Science tended to be interested in the various metatheoretic properties of the syllogism as a system. Since the Art concerns the *appropriate* use of a faculty of reasoning, it imports a prescriptive characteristic, and just where this lay was an issue of dispute. It was a certain group of thinkers (inspired, perhaps, by Whately [90]) who called logic ‘both an Art and a Science’ who tended to urge that all (“proper”) reasoning (in the sense of the Art) could be captured by logic (in the sense of the Science), by recasting it as syllogisms. (Or even that underneath it all, and despite appearances to the contrary, people *really* did reason in syllogisms in their everyday life — when they reasoned correctly, anyway.)

Of course, there were then, as now, those who denied that the scientific aspect of logic carried any normative force at all, thus forming a different current in the river of disputes concerning psychologism in logic. (Present manifestations of this view can be found in [13, 25, 42, 43, 71].) We will not here take such views into account, and instead we will be concerned with logic in the Science sense. Since the Arts notion makes *any* change from one idea to another an inference, it would make any law concerning “association of ideas” be (a part of) logic, and that seem to go beyond our present concerns. (However, in Section 8, “A Psychologistic Logic”, we will reconsider this restriction.)

We think of items in the Science of logic to be (describable as) systems of representation, especially systems that are designed to illustrate *truth-preserving* inferences. It is natural, especially to those in the knowledge representation area of artificial intelligence, to think of the representation language as being “inside the agent”, whether the agent is human or artificial. And this seems to some necessarily to tilt towards a mentalistic approach to logic, as urged in [10]. Part of our efforts will be to investigate whether this way of looking at (the science of) logic’s role in (the art of) reasoning necessarily manifests psychologism in logic.

Furthermore, truth-preserving inferences aren’t the only thing that one might think of when considering the extent of logic: there is also the set of all theorems or logical truths⁸, such as the Law of Excluded Middle ($\varphi \vee \neg\varphi$), the Law of Non-Contradiction $\neg(\varphi \wedge \neg\varphi)$, the Law of Identity $\forall xx = x$, the Indiscernability of Identicals $\forall x, y(x = y \rightarrow \forall F(Fx \leftrightarrow Fy))$, and the like. We believe these also to be part of logic, and so we understand psychologism to hold that these statements are expressions of ideas or of relations between ideas (in the logic area of the mind). Certainly the classical logicians, both the psychologismists and anti-psychologismists, included both inferences and these logical laws as part of what they called logic. (Perhaps sometimes they are taken to “express ideas” in the psychologistic theories and sometimes they are thought to be “relations between ideas”.)

Nowadays the syllogism has receded in importance and is seen as just a part of first-order logic. And we acknowledge that there are “many logics”, in various senses: (i) different formulations of “the same logic” (as when different connectives or rules of inference are chosen as primitive), (ii) logics that become increasingly stronger (or weaker) in the sense of adding theorems to (or subtracting old theorems from) the stock of existing ones (as first-order logic and the propositional modal logics all extend propositional logic, and quantified modal logics extend them both; while many-valued logics, relevance logics, and non-monotonic logics restrict classical logic), and (iii) logics for different purposes (e.g., temporal logics for reasoning about time and the execution of computer programs, default logics for formalizing “everyday reasoning”, counterfactual logics to explain how “what might be” can enter into reasoning, intensional logics for reasoning about meanings understood as functions on possible worlds, probabilistic logics to compute probabilities given evidence, type theory to represent hierarchies of applications of predicates/functions to other predicates and functions, relevant

⁸We will not here keep a careful distinction between syntactic and semantic accounts — theorems vs. logical truths.

logics, many-valued logics (including fuzzy logic), intuitionistic logic). In some advocates' minds these alternative logics are vying to be "the one true logic", or perhaps "the one true logic for each unique area of application".

The question of what is logic is therefore a difficult one to answer. In the context of psychologism, one might choose to be psychologistic about some and not about others. For example, one might think that intuitionistic logic (with its focus on truth implying verifiability/provability) is psychologistic while temporal logic (with its focus on "real time") is not psychologistic.⁹ One might think that classical first order logic "exists in reality" but that modal operators are interpreted psychologistically as differing conceptions of necessity, thereby making first order logic non-psychologistic but its modal extensions psychologistic.

Since it might be tendentious at this point to fix on one particular definition of 'logic', because that may close off one or another route that some theories about psychologism might wish to explore, we take the view that the reader should pick one or another of these logics to be considered against the examination given below. However, in order to engage the thoughts of the originators and the initial detractors of psychologism in logic, one's conception of logic not only has to include the logical truths mentioned above, but also elementary arithmetic. All the early players (both the psychologistic Benke and Mill, as well as the anti-psychologistic Frege and Husserl) employed talk of numbers and elementary arithmetic operations (addition, etc.) in their discussions of the pros and cons of psychologism. Furthermore, most of the anti-psychologistic logicians (as well as some of the psychologistic ones) over the last hundred and forty years were interested in logicism, and presumed that the logic in question was at least strong enough to generate elementary mathematics. Much of the debate, therefore, concerns the plausibility or otherwise of claiming that some elementary mathematical truths or some elementary inferential principles of classical logic are psychologically grounded. So, we need to include this elementary arithmetic in order to engage the participants. We will not dwell on this matter, but much of our study will follow the tradition of investigating classical logic and elementary mathematics, although at the end we survey a few other arenas of logic, because some of the recent views of psychologism seem to find the existence of these logics to be a support for their positions.

Having now set the stage with our proposed taxonomy of psychologism, and by staking out what we mean by logic in these discussions, we focus extensively on the first type of psychologism, what we call Psychological

⁹Or one might take the reverse attitude toward these two.

Individualism. In the next sections, we recount its origins, present the classical objections to it, and then discuss these objections. We then carry the themes of this discussion to a consideration of the remaining three types of psychologism in our taxonomy — Psychological Descriptivism, Cognitive Architectures (and the associated competence/performance distinction), and finally the Ideal Cognizer.

2. Psychological Individualism in Logic

Psychological Individualism holds that logic is a creature of the individual’s mind. Basically it traces logic and mathematics to the mental objects and mental activities of individuals. This attitude in logic follows a more general trend in intellectual thought from some centuries before the specific discussion of psychologism that we have been describing from the 19th century.

2.1. Descartes and Locke

The more general trend in the four earlier centuries was to downplay the importance and usefulness of formal logic (the syllogism), and to upplay other mental abilities. (That is, to follow the Art of logic rather than the Science of logic.) Descartes complained that formal logic was worthless because it never *yielded* certainty but only *transmitted* certainty once you already had it. But this implied to Descartes and his followers that more important principles would be ones that would *generate* certainty — and those were “the natural light of reason” and “the possession of clear and distinct ideas.” Logic was of no use for this.

Locke likewise held that the formal studies were irrelevant, and that what was of more interest was a “native rustic logic” that explained how one’s ideas generated new ideas.

I am apt to think, that he who shall employ all the force of his reason only in brandishing of syllogisms, will discover very little of that mass of knowledge which lies yet concealed in the secret recesses of nature; and which, I am apt to think, native rustic reason (as it formerly has done) is likelier to open a way to, and add to the common stock of mankind, rather than any scholastic proceeding by the strict rules of mood, and figure. [64, IV.xvii.6]

The Cartesian and Lockean traditions thus did not accord any special importance to the mental realm of formal logic, and their “psychologism”

should be seen as according importance to some other type of mental operations instead. Reasoning of the sort that involves elementary mathematics and logic was seen as a side effect of this more basic reasoning.¹⁰ Since the syllogism is not the real method of reasoning that people employ, according to these theorists, what is? Locke suggests that we should “observe what people do when they ordinarily reason”, to find out. So the Science of (rustic) logic would become an empirical study: we observe the public activities of people as they are engaged in reasoning, and we infer simultaneously what processes their minds are engaged in and a general, theoretical account that explains these activities and processes. In this, they are similar to some more modern theorists that we will mention later (e.g., Oakesford & Chater [71]). George [33, p. 43] calls this ‘eliminative psychologism’ — the replacement of logic with empirical investigation of inferential habits.

2.2. Kant

By contrast with Locke and Descartes, Kant affirmed the importance of formal logic. He saw logic as a Science, and the business of the syllogism was to articulate “the purely formal rules of thought as such”, where this formality was taken by Kant to constitutively embody norms for thought that are completely independent of their content. The formality of logic in this prescriptive sense was taken by Kant to follow just from logic’s generality, that is, because its scope is: thought as such.¹¹ The rules of logic in this way acted as a negative constraint on all truth, a coherence requirement one might say, whose satisfaction — while necessary for truth — could not by itself tell us anything material or substantive about the world (cf. [55, B 84]). Although we can say that for Kant logic was necessary, analytic and a priori, beyond this his position resists easy classification. Kant was an avowed anti-Platonist (cf. e.g., [55, B 882]), suggesting that he is either psychologistic or physicalistic. Some recent interpreters of Kant (e.g. [15, 18, 73]) argue

¹⁰[72] might be consulted for a nice survey of this tradition, especially as it relates to Hume [46].

¹¹An important presentation of this aspect of Kant’s views on logic is MacFarlane [65]. MacFarlane argues convincingly that Kant’s conception of formality was motivated by his view that concepts could only legitimately be used to make judgments about objects. There could be no judgments about concepts per se. And since logic must, on Kant’s view, be free of ontological commitment to objects, it must thereby also be free of any conceptual content. Frege later rejects Kant’s restrictive model of concepts in pursuing his logicist program, and thereby, in the process, some would argue, rejects Kant’s psychologism (cf. [65, pp.43ff]). For another excellent source on Kant’s conception of logic, see Tiles [89], which explores its relation to the earlier rationalist tradition.

that Kant’s ‘empirical realism’ about things-as-we-experience-them is a form of direct realism, in the sense that the existence of objects external to us is presupposed by our consciousness of our own existence, and not in need of inferring from our sensory experience. Kant thereby thoroughly rejects Cartesian foundationalism (cf. [55, B 274 ff], “The Refutation of Idealism”). From this perspective the formal constraints embodied in the rules of logic can be taken as an articulation of something about empirical reality, namely its formal structure; and this view can be seen as physicalistic in spirit. A more psychologistic construal suggests itself when we focus on Kant’s trumpeted ‘Copernican Revolution’ in philosophy, his insistence that we cannot coherently consider ‘things-in-themselves’ apart from any cognitive relation they bear to our representations of them — where this is taken by Kant to imply that things-as-we-experience-them must, in a priori determinable respects¹², conform to our representations (cf. [55, B xvi]). The specific nature of this conformity will be knowable a priori in the sense that our cognitive awareness of it will be through reflection alone, and its justification independent of sensory experience. Insofar as Kant took the mind’s representational capacities to be innate, his view can be seen to be akin to our category of cognitive architectures. But a problem faced by both of these construals — physicalistic and psychologistic — is how to accommodate Kant’s insistence on the normative, prescriptive nature of logic. We return, in Section 6.5, as we discuss Cognitive Architectures, to the psychologistic interpretation of Kant, as elaborated by the early anthropologizers of Kant: Jakob Friedrich Fries (1773–1843) and Friedrich Eduard Beneke (1798–1854), and followed in broad outline by some modern authors.

2.3. Mill

We shall be spending some considerable time investigating what has always been the English *locus classicus* of psychologism in logic, John Stuart Mill’s *A System of Logic Ratiocinative and Inductive* [69] and his *An Examination of Sir William Hamilton’s Philosophy* [70].¹³ The attention and length is justified on the grounds that Mill is the clearest advocate of (a form of) psy-

¹²Exclusive of bringing about their very existence, of course.

¹³There is, naturally, a dispute about how “psychologistic” Mill was. Those who hold him to be anti-psychologistic include [5, 87]. A judicious examination of all the evidence can be found in Godden [37]. We will not follow up on the claims about Mill’s non-psychologism but will instead treat the well-known parts of his position — which are psychologistic, even if Mill perhaps didn’t realize all the consequences of this position — that were the focus of attention of the anti-psychologismists Frege and Husserl.

chologism, and it was mostly against him that the initial anti-psychologists, especially Frege and Husserl, were reacting.¹⁴ Setting out his position at length and at the beginning also helps clarify what psychologistic alternatives there are to his position, and just how telling are the standard objections.

Much of Mill's methodological argumentation was against the "intuitionist" philosophy of William Hamilton¹⁵, which held that explanations in logic and mathematics rested on "intuitively compelling principles" rather than on general causal laws. Mill thought this "intuitionism" was not only bad philosophy but also embodied some sort of religious and political conservatism, because (he thought) the search for such intuitively compelling principles rested on understanding the universe as a divine creation governed by principles of a rational diety.¹⁶ Mill defends an empiricist framework of a broadly Humean sort wherein all knowledge (including of logic and mathematics) derives from sense experience of particulars. The formal logic of Mill's time was still the syllogism, and the syllogism is a theory of general propositions. So there will obviously be some accommodation needed for Mill to give an account of syllogistic reasoning. His surprising answer is that general propositions function only as *abbreviations* of groups of particular propositions.¹⁷

in all cases, the general propositions, whether called definitions, axioms or laws of nature, which we lay down at the beginning of our reasonings, are merely abridged statements, in a kind of shorthand, of the particular facts, which, as occasion arises, we either think we may proceed on as proved, or intend to assume. (II, iii, 3)

Reducing all generality to "abbreviation" of particular sensations will naturally have some important impact on how Mill sees logical laws – which after all are general in nature. Mill traces the content of all the logical laws to features of human psychology. For example, in discussing the Law of Non-Contradiction and of Law of Excluded Middle Mill says (the latter is quoting Herbert Spencer with approval)

¹⁴There are many other writers that Husserl cites as psychologistic: [19, 62, 63, 84, 91], among others.

¹⁵William Sterling Hamilton of Edinburgh, not the somewhat more junior mathematician, William Rowan Hamilton of Dublin. Augustus DeMorgan was a friend of the latter and an opponent of the former, as well as a correspondent with both.

¹⁶As we will see below, it is rather ironic that Mill held this position (that an abstract realm for logic would also permit a God), given the justifications employed by some theorists for psychologistic logic. See the definition of "God-Driven Teleology" in Section 6.1 below.

¹⁷Citations of the form Book.chapter.section are from [69].

I consider it [Non-Contradiction] to be, like other axioms, one of our first and most familiar generalisations from experience. The original foundation of it I take to be, that Belief and Disbelief are two different mental states, excluding one another. This we know by the simplest observation of our own minds. [II.vii.5]

“The law of the Excluded Middle, then, is simply a generalisation of the universal experience that some mental states are directly destructive of other states. It formulates a certain absolutely constant law, that the appearance of any positive mode of consciousness cannot occur without excluding a correlative negative mode, and that the negative mode cannot occur without excluding the correlative positive mode . . . Hence it follows that if consciousness is not in one of the two modes it must be in the other.” [II.vii.5]

We see that Mill has interpreted logical principles to be statements of psychological facts, and this is one direct way in which he is committed to psychologism. The principles of logic, that we state symbolically, represent very general psychological laws that govern the operations of the human mind. They are therefore empirical and not necessary. Their apparent universal validity is due to the fact that all experience validates them.

Mill recognizes, and indeed embraces, this “reduction” or “explanation” of logic in terms of psychology. In another work Mill famously claims logic to be psychological:

Logic is not a science, separate from and coordinate to psychology. To the extent that it is a science at all, it is a part or branch of psychology, distinguished from it on one hand as the part is from the whole, and on the other hand as the art is from the science. [70, ch. xx]

Logic in Mill’s view consists of the study of inference, at least those inferences concerned with truth, but not of how evidence is found. As Mill puts it, “Logic neither observes, nor invents, nor discovers; but judges” (Introduction, §5). But although it follows from this that not all psychological goings-on are part of logic, it also follows that logic *is* a part of the psychological goings-on . . . like the fingers are parts of a hand.

Logic, as I conceive it, is the entire theory of the ascertainment of reasoned or inferred truth. Formal Logic [the theory of the syllogism], therefore, which [other authors] have represented as the whole of

Logic properly so called, is really a very subordinate part of it, not being directly concerned with the process of Reasoning or Inference in the sense in which that process is a part of the Investigation of Truth. (II.iii.9)

We see here that Mill is concerned to include *any* type of inference that is concerned with truth into the realm of Logic; but only some of these inferences should be called Formal Logic.

The contents of all thoughts about numbers also come directly from experience; we get our ideas of numbers from our childhood play with piles of pebbles (II.vi.2). The content of numerical statements — what they are “about” — are the subjective mental constructs thus generated. And our belief in any general truth of mathematics, logic, or geometry is ultimately grounded in experience of particular cases, and such a truth refers, as its semantic content, to the mental events inspired by those experiences — they are “about” those mental events. Mill asks “. . . what is the ground for belief in axioms — what is the evidence on which they rest? I answer, they are experimental truths; generalisations from observation” (II.v.4). And generalizations are not self-subsistent entities — truths in their own right — but are only “mental shorthand” that we can, in principle, do without:

All inference is from particulars to particulars: General propositions are merely registers of such inferences already made, and short formulæ for making more (II.iii.4)

If we had sufficiently capacious memories, and a sufficient power of maintaining order among a huge mass of details, the reasoning could go on without any general propositions; they are mere formulæ for inferring particulars from particulars. (II.iv.3)

Mill’s adherence to induction (his flavor of induction from individual case to a prediction about another individual case) also brings out his commitment to psychologism. Although inferences can be “about” the items in reality, the actual principles that formal logic uses are ones that capture how one mental construct leads to another. Mill does not say directly that all these psychological explanations are fundamentally based on an *individual’s* psychology, as opposed to some sort of average or commonality of all people’s psychology, or opposed to some fact of all people’s having the identical psychology. But his presupposing a general empiricist and inductivist outlook has made this individualism be the most common interpretation of his position.

In fact, Mill thought that his version of logic, with its interposition of psychological states, could answer a question that was much in the air at the time: how can there be *real* inferences (information gained in an inference) while at the same time the theory of the syllogism held that the major premise “already included” the conclusion, and thus did not produce any new information? Mill’s innovative new solution is to deny that the syllogism is itself the proof, but that there is another proof that goes along with the syllogism, which is the true proof and for which the syllogism can serve as a representation.

... though there is always a process of reasoning or inference where a syllogism is used, the syllogism is not a correct analysis of that process of reasoning or inference; which is, on the contrary (when not a mere inference from testimony) an inference from particulars to particulars. . . (II.iii.5)

... the major premise is not the proof of the conclusion, but is itself proved, along with the conclusion, from the same evidence. (III.xxi.4, consciously summarizing II.iii)

[N]o reasoning from generals to particulars can, as such, prove anything, since from a general principle we cannot infer any particulars, but those which the principle itself assumes as known. (II.iii.2)

If, from our experience of John, Thomas, &c., who once were living, but are now dead, we are entitled to conclude that all human beings are mortal, we might surely without any logical inconsequence have concluded at once from those instances that the Duke of Wellington is mortal. The mortality of John, Thomas, and others is, after all, the whole evidence we have for the mortality of the Duke of Wellington. Not one iota is added to the proof by interpolating a general proposition. (II.iii.3)

While the “old view” presupposes that a syllogism begs the question, since the conclusion is already contained in the major premise, Mill’s view more radically says that the major premise is merely a shorthand for previous inductions and a formula for making more inductions. It therefore does not “include” the conclusion and does not make the syllogism a *petitio principii*.

... the conclusion is not an inference drawn *from* the [general] formula, but an inference drawn *according* to the formula; the real logical antecedent or premise being the particular facts from which the general proposition was collected by induction. (II.iii.4)

The inference is finished when we have asserted that all men are mortal. What remains to be performed afterwards is merely deciphering our own notes. (II.iii.3)

[The old view makes a mistake] of referring a person to his own notes for the origin of his knowledge. If a person is asked a question, and is at the moment unable to answer it, he may refresh his memory by turning to a memorandum which he carries about with him. But if he were asked, how the fact came to his knowledge, he would scarcely answer, because it was set down in his notebook: unless the book was written, like the Koran, with a quill from the wing of the angel Gabriel. (II.iii.3)

All inference is thus from particulars to particulars (this is the only sort of inferential knowledge that a fully empiricist epistemology allows us to have); the syllogistic process is only an interpretation of our “notes” of previous inferences. Syllogistic reasoning is therefore only a roundabout way to reach a conclusion that could have been, and in fact *is* reached directly. Although we might have some boundedness in our ability to keep all the evidence before our minds, nonetheless we *could* reason always without generalizations. And this is what logic really is, according to Mill.

Not only *may* we reason from particulars to particulars without passing through generals, but we perpetually do so reason. All our earliest inferences are of this nature. (II.iii.3)

Whether, from the attributes in which Socrates resembles those men who have heretofore died, it is allowable to infer that he resembles them also in being mortal, is a question of Induction. (II.iii.7)

This reasoning from particulars to particulars is characteristic of *all* reasoning except for knowledge gained by “intuition”, which is not part of reasoning.

Truths are known to us in two ways: some are known directly, and of themselves; some through the medium of other truths. The former are the subject of Intuition, or Consciousness; the latter, of Inference . . . [The former are] known antecedently to all reasoning. (Introduction, §4)

We think we have made clear what Psychological Individualism is, by looking at its manifestation in Mill. As we remarked above, there is the usual

problem of making every statement made by Mill actually fit our interpretation. However, as Godden [37] notes in response to the issues mentioned above in footnote 13, even if one focuses on some of Mill's claims that the subject matter of logic is the objects in the world that the reasoning is about and not the acts of thinking, it still seems that his principles of logic are dependent on psychological facts. This comes out in his discussions of the various logical laws, as we cited above. The straightforward reading of Mill — the reading that Mill's opponents were thinking of — is Psychological Individualism. And although he makes no direct claim that there is a separate and different logic for each individual human, the attribution of that view to him accounts for much of the backlash in the classical objections to psychologism that we consider next.¹⁸

3. The Classic Objections to Psychologism

The classic objections to psychologism in logic and mathematics have in fact been directed at Psychological Individualism of the sort we have just attributed to Mill, and so it is at this theory that the following objections should be seen as aimed. We will discuss later the extent to which these same objections apply to the other varieties of psychologism in our proposed taxonomy. In this section we present and categorize the classic objections to psychologism in logic, and in the next section we discuss whether or not they hold against Psychological Individualism's understanding of psychologism in logic and mathematics.

3.1. Frege

Throughout his writings — we quote here from [27, 28, 29, 30], but could have drawn on various other places — Frege presents a general view against psychologism in logic, and makes sweeping statements coming from this position, although with only a few pointed arguments.

¹⁸Godden [37] cites an interesting passage from [70, ch. xxi] where Mill seems to retract a total commitment to psychologism with the words

Whether the . . . Fundamental Laws are laws of our thoughts by the native structures of the mind, or merely because we perceive them to be universally true of observed phenomena, I will not positively decide . . . They may or may not be capable of alteration by experience, but the conditions of our existence deny to us the experience which would be required to alter them. Any assertion, therefore, which conflicts with one of these laws. . . is to us unbelievable. The belief in such a proposition is, in the present constitution of nature, impossible as a mental fact.

As we mention below, Frege is arguing not only against the subjectivity of psychologism but also against the view that logical and mathematical entities are “objective” and belong in “the physical realm.” His view is that we require yet a “third realm” beyond the mental and the physical, although it is sometimes difficult to see where his arguments force this upon us. At best they seem designed to show that logic cannot be in the subjective realm; but this leaves open the possibility that they are in the physical realm. Still, this would be an argument against psychologism and that is our interest here — not whether he also establishes their presence in a third realm.¹⁹

Here are three statements relevant to his general position, taken from different periods of his life. One could perhaps generate some detailed arguments from these general claims, but Frege did so only occasionally.

Never let us take a description of the origin of an idea for a definition, or an account of the mental and physical conditions through which we become conscious of a proposition, for a proof of it. A proposition may be thought, and again it may be true; never confuse these two things. We must remind ourselves, it seems, that a proposition no more ceases to be true when I cease to think of it than the sun ceases to exist when I shut my eyes. (1884, p. xviii)

If we want to emerge from subjectivity at all, we must conceive of knowledge as an activity that does not create what is known but grasps what is already there. . . . If I grasp a pencil, many changes take place in my body. . . . But the totality of these changes is neither the pencil nor creates the pencil; the internal changes are not the grasping. In the same way, that which we grasp with the mind exists independently of this activity, independent of the ideas and their alterations that are part of this grasping or accompany it, and it is neither identical with the totality of these events nor created by it. . . . (1893, pp. 23-24)

If every Thought requires an owner and belongs to the contents of his consciousness, then the Thought has this owner alone; and there is no science common to many on which the many could work, but perhaps I have my science, a totality of thoughts whose owner I am,

¹⁹In the reverse direction, we mentioned before that Quine’s [79] emphasis on a naturalistic explanation for logic and mathematics is a desire to deny logic and mathematics a home in the third realm. But again, it is difficult to see how this requires logic to be “psychologistic” in the sense of being subjective, for it seems to leave open the possibility that logic is in the physical realm.

and another person has his. Each of us is concerned with contents of his own consciousness. No contradiction between the two sciences would then be possible, and it would really be idle to dispute about truth; as idle, indeed almost as ludicrous, as for two people to dispute whether a hundred mark note were genuine, where each means the one he himself had in his pocket and understood the word ‘genuine’ in his own particular sense. (1918, p. 69)

(Note that when Frege uses the term ‘Thoughts’ within his own theory, he means items that are independent of any person, existing in the third realm. For Frege, Thoughts are distinguished from subjective ideas, which might go by the name ‘thought’ in other theories.)

Frege mentions the following absurdities that he claims follow from psychologism in logic and mathematics. We will not offer comments on their cogency at this point, but instead will follow them up in the next section.

- (1) In proving the Pythagorean theorem, for example, we would have to allow for the phosphorus content of the human brain. (1884: p. xviii)
- (2) Since we each have different ideas of numbers, there would be different numbers 2 for each person. (1884: p. 37)
- (3) There would also be too few numbers, because a finite human mind couldn’t have an infinity of numbers. (1884: p. 38)
- (4) Since psychology is about the human mind, there would have been a time at which any psychological law first was true. But this is not the case with logic and mathematics. (1884: p. xix)
- (5) If we can grasp only what is within our own selves, then a conflict of opinion would be impossible. (1893, p. 17)
- (6) It would be strange if the most exact science (mathematics) needed laws from the fledgling science of psychology with its inexact laws. (1884: p. 38)

3.2. Husserl

It is often claimed that Frege’s review [29] of Husserl’s early *Philosophie der Arithmetik*, and their subsequent correspondence turned Husserl into an anti-psychologism-in-logic crusader.²⁰ Husserl’s [47] states not only a ge-

²⁰Husserl’s [47, §46,fn1] reads in part “...G. Frege’s stimulating work *Die Grundlagen der Arithmetik* (1884), p. vi f. I need hardly say that I no longer approve of my own fundamental criticisms of Frege’s antipsychologistic position set forth in my *Philosophie*

neral outlook that was very Fregean in nature (as evidenced in the quotes following), but also a set of very precise arguments that sounded the death knell for psychologism for some considerable time.

Here are three quotes from Husserl, giving his general position.²¹ Once again we will not make any comments about the cogency of each of these claims, saving that for the next section.

To refer to [a number] as a mental construct is [thus] an absurdity, an offense against the perfectly clear meaning of arithmetic discourse, which can at any time be perceived as valid, and *precedes* all theories concerning it. If concepts are mental constructs, then such things as pure numbers are not concepts. But if they are concepts, then concepts are not mental constructs. [48, §22]

Anyone can see from my statements up to this point that for me the pure truths of logic are all the ideal laws which have their whole foundation in the ‘sense’, the ‘essence’ or the ‘content’ of the concepts of Truth, Proposition, Object, Property, Relation, Combination, Fact, etc. More generally stated, they have their whole foundation in the sense of the concepts which make up the heritage of all science, which represent the categories of constituents out of which science as such is essentially constituted. Laws of this sort should not be violated by any theoretical assertion, proof or theory, not because such a thing would render the latter false — for, so would any conflict with any truth — but because it would render them inherently absurd. [47, §37]

No attentive reader of [47] could now miss . . . [that] the first step [of his 1913 turn to Phenomenology] is there carried out with unhesitating decision: the right of the eidetic to be its own self as against the attempt to interpret it psychologically is justified in detail . . . [48, §61]

der Arithmetik, 1, pp. 124-32. I may here take the opportunity, in relation to all of the discussions of these Prolegomena [Husserl’s present work], to refer to the preface of Frege’s later work *Die Grundgesetze der Arithmetik* [28]. . .”

²¹Husserl’s philosophy can be divided into three periods: his early psychologism, as presented in his *Philosophie der Arithmetik* (1891), a middle period during which he wrote his *Logical Investigations* (1900), and a mature period starting in 1913 with the publication of *Ideas* [48] and the beginning of his new philosophy of “phenomenology”. We include here some quotes from this later work to show that he did not abandon his middle period anti-psychologism.

[47] also can be seen as giving a number of pointed arguments against psychologism. If logical laws were based on psychological laws, then:

- (7) Since psychology, as a science, presupposes logic, grounding logic in psychology would be circular. §19
- (8) If logical laws were based in psychology, they would imply the empirical existence of some psychological objects or phenomena . . . “but this is palpably false”. §23
- (9) Psychological laws are *causal* and *descriptive*; logical laws are *non-causal* and *normative*. §19
- (10) Psychology is a factual, empirical science, and therefore establishes generalizations with only approximate regularities. But the laws of logic are exact and are not inductive generalizations. §21
- (11) Psychology is known *a posteriori* and psychological laws are established by induction. Logic is known *a priori* and with certainty. §§21, 22
- (12) Psychologism implies the claim that if our mental constitution were different, the laws of thought, and hence of logic, would be different. And that if we didn’t exist, there would be no logical truths at all. This relativism about truth undermines itself. §36, part 5
- (13) Logical laws ought to be, like psychological laws, vague and approximate. But instead they are so exact that they cannot be guaranteed by any empirical truth. §21
- (14) Logical laws ought to be based, like all empirical laws, on induction. But induction leads only to probable truth and not certainty that logical laws manifest. §21
- (15) Logical laws ought to imply the existence of some psychological events like representation and judgments; but in fact they do not concern anything about mentation and are about necessary relations that are independent of empirical facts. §§23, 45

3.3. A Taxonomy of Criticisms

The fundamental complaint that both Frege and Husserl are making concerns “relativism”. The present context is logic and mathematics, so while perhaps both of them might be happy with relativism in some other areas, such as etiquette or beauty perhaps, they see its intrusion into logic and mathematics as being demanded or at least facilitated by psychologism, and

to have unhealthy and implausible consequences. In fact, the section heading of Husserl's [47, §38] is "Psychologism in all its forms is a relativism". Many of the specific arguments we listed can be seen as reasons why psychologism facilitates some of the perceived objectionable aspects of relativism.

We can group Frege's and Husserl's complaints about psychologism into the following three broad categories, with their subcategories. Frege and Husserl perhaps thought of all these problems as being failures due to relativism, but we think of relativism as having just the (A) and (B) problems. In what follows we will not have much to say about the (C) objections . . . which seem either too deep or too shallow to discuss adequately and fruitfully.

A) **Interpersonal Incoherence.** This breaks down into the following complaints.

- i) *No objective truth.* The fundamental objection to relativism in this (and other) realms is that by making the content of all claims be the individual's internal psychology, there would be no independently existing field of objects/properties/etc. that we would be describing. Thus, there can be no truths except those of the form "It seems to me that...."
- ii) *No interpersonal necessity.* Even if a statement or judgment is deemed "inconceivable that it could be otherwise" by one person, there is no reason that this could be expected to hold of any other person.
- iii) *Universality is not necessity.* Even if everyone agreed with one another about which statements are necessary, that would be merely a *description* of their agreement and not an *explanation* of the statements' necessity. As is well-known, "what everyone believes" is often false, or at least problematic.
- iv) *No interpersonal normativity.* Since every claim concerning logical truth and validity is relativized to the person making them, there is no force to the claim that some other person *ought* to reason in so-and-so manner. There can be no justification to a praising or blaming another person of a logical insight or error.²²
- v) *Universality is not normativity.* Even if everyone agreed with one another, that would be merely a *description* and not an explanation

²²Empirical statements, such as claims about what is in fact occurring in one's mind, are *responsive* to the facts of the world, and hence cannot explain why they *should* be the way they in fact are. Frege and others sometimes drew an analogy between this sort of normativity and that in ethics: just because events in the world happen the way they in fact do, this is no reason to say that they *should* be that way, from a moral point of view.

of their normativity. A description of the views of everyone cannot justify why we all *ought* to have these views concerning truth and validity. There can be no justification for the claim that anyone *should* reason in the way s/he does — it is merely a description of how everyone reasons. There can be no notion of good or bad reasoning.

- vi) *The individual differences objection.* Logic and mathematics are the same for everyone, but conceptual contents differ from one to another individual. So even ideas of simple items, like the number 3, will be different in different people.
- vii) *No communication.* Since the meaning or content of the crucial logical language would consist of the internal psychological state of an individual, each individual would have separate contents for every term and communication would be impossible.
- viii) *No agreement or disagreement.* As a result, there cannot be any agreement or disagreement between people about these logical topics, because what one person agrees to or objects to could not be what the other person holds.²³

B) **Personal Incoherence.** This breaks down into the following complaints.

- i) *No personal normativity.* If the truths of logic are due just to one's internal psychological states, then all claims concerning logical truth and validity ultimately come from a description of the individual person's psychological makeup. And all that can be concluded about them is that they are a *description* of that person's mental makeup. A description of one's views cannot legitimately be used even by that person to justify why s/he *ought* to have these views concerning truth and validity. There is no justification the person can give to him-/herself for the claim that the person *should* reason in the way s/he does — it is merely a description of how s/he in fact *happens* to reason. There can be no notion of good or bad reasoning, even from a person's own point of view.
- ii) *No personal necessity.* Likewise there can be no notion of necessity recognized by the person. All s/he can say is that s/he "cannot

²³Since we acknowledge that Mill took "ordinary terms" to be about items in reality (even if Frege and Husserl didn't understand Mill that way), we want to restrict the last two items, (Avii) and (Aviii), to be about logical and mathematical matters.

imagine it otherwise”; but this is not a sure guide to logical or mathematical necessity, as experience has shown in the past.

iii) *Contradictory*. Personal relativism is self-contradictory because it attempts to assert that it is true *simpliciter* without any relativism.

C) **Scientific Incoherence**. This breaks down into:

- i) *The incompleteness objection*. Logical laws and logico-mathematical terms have features that are not mirrored in any individual’s mental life.
- ii) *The exactness objection*. Logical and mathematical laws are *exact* and therefore could not be derived from the inexact laws of psychology.
- iii) *The bad existence implications objection*. Psychological laws have implications about the existence of certain mental items that logical and mathematical laws do not have.
- iv) *The circularity objection*. Psychology *presupposes* logic, and therefore logic cannot be defined in terms of psychology.
- v) *Time-boundedness*. Human psychology had a beginning with the first humans. But logic and mathematics do not have a beginning.
- vi) *Descriptiveness*. An empirical theory is *descriptive* and can not explain why the claims of logic and mathematics are true, much less necessarily true.

One can think of some other reasons too. For example, the view seems committed to “logical pluralism”: the view that there can be no logical conflict between people because they are simply using “different logics”. In keeping with his general anti-psychologistic views, Frege said of such a situation

... what if beings were even found whose laws of thought flatly contradicted ours and therefore frequently led to contrary results even in practice? The psychological logician could only acknowledge the fact and say simply: those laws hold for them, these laws hold for us. I should say: we have here a hitherto unknown type of madness.
[28, p. 12]

4. Pure Psychological Individualism in Logic and an Evaluation of the Criticisms Against it

Pure Psychological Individualism is the basic and fundamental notion of Psychologism, both historically and in the minds of all those who object

to psychologism in logic. Recall that, according to our taxonomy, it is the doctrine that *the content of any logical or mathematical statement is strictly dependent upon the mental apparatus of the individual making the statement.* We grouped the Frege/Husserl criticisms relevant to this position into two broad categories that we will be considering : Interpersonal Incoherence and Personal Incoherence. What should we say about these classic criticisms of psychologism, taken as arguments against psychological individualism? Which are valid objections? What possible modifications are there to this position so that it can evade whatever force they have?

Some modern sympathizers with psychologism in logic (e.g., Jacquette [49]) often asseverate that neither Frege nor Husserl (but especially not Frege) provide an *argument for* their anti-psychologism, so that their claims are merely a form of question-begging against psychologism. This seems to us an inaccurate assessment of the situation. While it is true that neither (but especially not Frege) provide any further support for their view (for example) that the content of mathematical statements exists in some “third realm” or has “eidetic existence”, still and nonetheless, their argument is intended by them and is best understood as saying “either psychologism about logic is correct or else the contents of mathematical statements exist in a third realm; but psychologism is wrong for such-and-so reasons; hence, mathematical content exists in a third realm.” And even if the first premise of this argument is suspect (because there are other options available), this does in no way say that the critique of psychologism is wrong. The critique cannot be ignored on the grounds that the alternative position has not been established uniquely. It is not a question-begging polemic on this level.

One possible reaction to the charge of relativism would be for supporters of psychological individualism to claim that *relativism is true.* After all, it might be said, the objections we have grouped into Interpersonal Incoherence and Personal Incoherence stem from relativism. If relativism is true then we should expect all the rest. So this position says that the various claims concerning the consequences of relativism in the realm of logic and mathematics are in fact correct: there is no objectivity, no interpersonal agreement, no necessity, no exactness, and the like. Mill [69, II.iv.3] offers the following thoughts on the topic.

Why are mathematics ... considered to be independent of the evidence of experience and observation, and characterised as systems of Necessary Truth? The answer I conceive to be, that this character of necessity ascribed to the truths of mathematics, and even ... the peculiar certainty attributed to them, is an illusion ... (II.v.1)

So it seems that Mill sometimes embraces (at least some aspects of) relativism — but in a way that may be closer to what we will later detail as a cognitive architecture position for psychologism: that the “outcomes” of a process of investigation and discovery in some realm are not independent of the processes of investigation and discovery themselves. When discussing the view that there is a difference between “factual truths” and “necessary truths” based on the thought that the negations of the former are conceivable while those of the latter are inconceivable, Mill protests

Now I cannot but wonder that so much stress should be laid on the circumstance of inconceivableness, when there is such ample experience to show that our capacity or incapacity of conceiving a thing has very little to do with the possibility of the thing in itself, but is in truth very much an affair of accident, and depends on the past history and habits of our own minds. (II.v.6)

We can see that there is a certain tension or possible misunderstanding here. The opponent (William Whewell) says a psychologistic logician cannot make sense of logical truths as opposed to truths that are supported by all experience (an objection that mirrors the complaint we labelled (Aiii), *universality is not necessity*). And Whewell says that logical truths have the property that their negations are inconceivable, whereas the negations of the others can be imagined. Mill claims we *can* imagine a denial of a logical truth, thereby rejecting the claim that negations of logical truths cannot be imagined. And so the conclusion ought to be that they are on the same epistemic level as non-logical truths that are supported by all evidence. But sometimes Mill seems to think that he has thereby formed a basis that the two can be distinguished, as in the area II.vi.2-3ff. For example he says

What is commonly called mathematical certainty, therefore, which comprises the twofold conception of unconditional truth and perfect accuracy, is not an attribute of all mathematical truths, but of those only which relate to pure Number, as distinguished from Quantity in the more enlarged sense; and only so long as we abstain from supposing that the numbers are a precise index to actual quantities. (II.vi.3)

Here Mill seems to distinguish “pure number truths” from “applied number truths”, for which he gives examples of geometry and mechanics. But it still seems that pure number truths have mathematical certainty.

It seems pretty clear that the Pure Psychological Individualism doctrine (as opposed to whatever actual alterations some specific philosopher, like

Mill perhaps, might have made to the doctrine) does in fact have the feature of (individualistic) relativism. And as we said before, given this notion of relativism, where all the meaning and import of any logical or mathematical item [term, inference, law] is claimed to reside in the minds of different individuals, all the other consequences on our lists of the interpersonal and personal problems will follow ... at least if we are given some seemingly plausible assumptions about individual differences in mental makeup and learning.

We have spent much time on Psychological Individualism as an interpretation of psychologism in logic because that is historically the battleground for the psychologism wars. And it seems fair to say that the anti-psychologists won that war. However, it is far from clear that these anti-psychologism-in-logic objections hold against the other conceptions of psychologism in logic that we have proposed in our taxonomy: the psychological descriptivism position, the cognitive architectures position, and the ideal cognizer position. We now consider each of these remaining types in turn.

5. Psychologism as Psychological Descriptivism

According to our structuring of the landscape of psychologism, Psychological Descriptivism is the position that what counts as the *common content* of the relevant aspects of mentality is some publicly observable performance (e.g., actions) or the output of this performance (e.g., writings). It is still a psychologism because the ultimate source of these performances or outputs is mental. But we determine what the mentality is by looking to the public manifestations. We start with an outlook on the proper method of anthropology, just to give descriptivism a footing. We will mention other variants of this same type of psychologism as we proceed. Dan Sperber is well-known for his advocacy of the naturalistic method in anthropology. (Many of his influential articles on this topic are collected in [88]). Some of his remarks, especially in [88, pp. 23-39], align with the sort of idea that we are putting forward in this subsection. For example,

Cultural things... [should be regarded as] distributions of representations in a human population, ecological patterns of psychological things. To explain cultural phenomena is, then, to develop an epidemiology of representations. For this, representations have to be viewed not as abstract, but as concrete objects which can be of two forms: they are either mental representations inside brains, or public representations in the environment of brains. Just as an epidemiology

of diseases has to be rooted in individual pathology, an epidemiology of representations has to be rooted in cognitive psychology. [88, p. 23]

As can be seen from this, Sperber’s idea is to track “ideas” and other cultural items by means of their public manifestations. One goal of anthropology is to track how these ideas get transmitted throughout a culture and perhaps modified. He urges this as a remedy for the highly introspective methodology that he finds within classical anthropology, arguing that this sort of naturalism is the only way to make anthropology have a respectable scientific basis. This could form the basis of a psychologist’s position towards logic if we were considering how logic is manifested in a society, because (as Sperber holds) the ultimate source of these performances or outputs is mental. And we could use the important, publicly observable commonality as what defines logic. That is, we could investigate the Art of logical reasoning, and define a formal system (the Science of logic) *from* it.

There are different candidates for this public output. One might be to identify it with what is “common to” or “the average of” all individual people’s mentality on those aspects. For instance, we might think that there can be many different contents to people’s ideas of the concept of a dodecahedron. But the “common” or “average” notion across all humanity (or perhaps: across all of humanity that “is relevant”) is what will count as the “real content” of ‘dodecahedron’. With such a notion of content, we could allow that some people are mistaken in their logical and mathematical ideas — something that couldn’t be done in Psychological Individualism. There are variants on this idea, and we will explore some of them.

5.1. Commonalities and “Averages”

One way might be to take seriously the idea that we can publicly find the “commonality” of what people individually have in their minds. For example, perhaps if we were to give sophisticated psychological tests to people we would discover that (pretty much) everyone had a mental rule of modus ponens, a rule of *and*-elimination, and a rule of biconditional modus ponens. So we conclude that these rules in fact are logical features common to the average person. But we might also discover that the rule of modus tollens is only very rarely a part of anyone’s mental arsenal — so rare in fact that we do not admit that it has psychological reality.²⁴ If we define our logic to be exactly those rules and principles that can be discovered in this way, then that would be one way to be psychologist’s about logic. Johnson-Laird and

²⁴See Evans *et al* [21] for the data supporting these claims.

Byrne [53, p. 18] also seem to have this notion in mind, even if they are not advocating it, when they define psychologism as the view that “logic is a generalization of those inferences that people judge to be valid.”

Similarly, in considering the number 2, for example, we acknowledge that everyone has slightly differing concepts of it, perhaps because of their differing environments in which it was learned (and other concomitant factors that differ across people). Yet, we might claim there to be a core to all these concepts that is the same across people. It might be claimed, for example, that because of the educational system in the English-speaking world, the central parts of the content of ‘two’ is the same across English-speakers, and the same holds of ‘dodecahedron’ and various other logical or mathematical expressions. Grzegorzczuk [38, p. 84], who advocates this position, puts it like this: “Logic is a set of rules that preserve the assertions of the majority of the properly educated and unperverted people.”

5.2. Commonalities and Language

Rather than by formal education, one might instead claim that the commonality is established in the course of learning one’s language (or, the relevant part of the language). For example, Philipse [77] proposes that we get logic by learning our native language. The fact that we same-language speakers can successfully communicate shows that we have “enough similarity” in our mental lives to carry on logic and mathematics as a joint activity. Although this conception allows the anti-psychologistic charge that we all have, or could have, different ideas of the relevant realm, it uses the notion of “similar enough” to finesse (some of) the difficulties that ensue. After all, it might be argued, even without formal educational systems, interacting humans would have to converge on some commonly held meaning of any term to have any successful interaction whatsoever. Such a position addresses some of the interpersonal incoherence problems that stem from the basic relativism charges against Psychological Individualism, even though it perhaps does not address the normativity issue.²⁵

Or again, there might be a way to “cancel out” differences between people so as to arrive at some “average notion” that can be taken to be the content of ‘two’ or ‘modus ponens’. Our clever psychological experimentation might reveal closely-knit clusters of mental similarity, and we conclude that the “average” value in this cluster is to be our psychologistic content

²⁵However, Philipse claims that not only does this view avoid the interpersonal incoherence objections, but also it solves the normativity issue — part of learning language just *is* to learn rationality and logicity.

for the term. But there might be no actual person whose mental contents are exactly this value; nonetheless, this would still count as a version of psychologism because the constructed content of ‘two’ or ‘modus ponens’ is ultimately defined in terms of people’s mentality. And everyone’s concepts of these terms are “similar enough” to this “average” value that communication is possible. As with the language-learning version of descriptivism, this conception allows the charge that we all have, or could have, different ideas of the relevant realm; and it also uses the notion of “similar enough” again to finesse (some of) the difficulties that ensue. Once again, this addresses (some of) the interpersonal problems, even though maybe it does not answer the normativity issue to the same extent as the previous conception.

These two positions are related. The former one argues that there may be no “average” or prototype, but that we are all “similar enough” to communicate. The latter one argues that there might or could be an “average” that can be found, and that we are “similar enough” to this average that all is well in our interactions with one another.

Either of these versions of psychologism might be modified by restricting the class of people whose mental activity counts as relevant to finding the “commonality” or “the average”. Society might, for example, agree that it is the mental contents of those who are recognized as being experts in mathematics and logic that are important in describing the content of ‘two’ and ‘modus ponens’.

5.3. Public Performances and Public Objects

Another direction might be to identify the content of logic and mathematical statements with some sort of systematization of our linguistic *outputs* in the area of mathematics and logic, interpreting this as a way to systematize the states of mind of those who employ logical concepts in their speech and writings. So this too becomes a type of psychologism, since the statements are ultimately the product of individuals, and hence of their individual mentalities. We might make the range of statements be broader (all statement that have mathematics and logic as their purpose, no matter who wrote or uttered them) or narrower (only those statements made by people recognized as experts in mathematics and logic).

A variant on this would be akin to the notion of formalism in mathematics, according to which mathematical (and logical) terms are just “scribblings on paper with no external signification.” In this version of formalism, the practice of mathematics (and logic) is defined by a society (whether the society thought of widely, or a narrower subsociety of mathematicians

and logicians). But since the force of mathematics is here seen as a kind of systematization of this practice, and hence ultimately derived from the psychology of its members, it can be seen as a type of psychologism that attends to the public practices of (some) members of society.²⁶

5.4. Anthropologism and Societalism

Some of these distinctions emerged in Husserl's own thinking and are mirrored in related treatments of relativism. He [47, §36] defines 'individual relativism' to be what we normally think of as relativism: where statements are relativized to an individual and "X is true" becomes "X is true for me" (but perhaps not for you). But Husserl also defines 'specific relativism' to be where the meaning or applicability of a term is relativized to a species. In this usage "X is true" becomes "X is true for species A" (but perhaps not for species B). When the species is limited to humans, the doctrine is called *anthropologism*. Here, "X is true" becomes "X is true for humans, but perhaps not for chimpanzees". This is reminiscent of Protagoras' famous "Man is the measure of all things: of what is, that they are, and of what is not, that they are not."²⁷ Note that this definition of 'anthropologism' does not state any particular mechanism that accounts for how truth must be relativized to humans. One natural reason, that we will explore in detail below, is the notion of cognitive architectures, especially from a situated, evolutionary perspective. Other reasons — which are somewhat more vague in their explanations — include claims that it comes about because we are language-speaking animals (that is, we *actually* learn and use a language but other species don't²⁸, or maybe that humans have a "higher level" of historical memory than other animals.

We can similarly define 'social relativism': statements are relativized to a society and "X is true" becomes "X is true for society A" (but perhaps not for society B). We call this view *societalism*, echoing Bloor [9]. It is natural to think of societalism as a more localized version of anthropologism, for

²⁶Perhaps fictionalism, the view in philosophy of mathematics that replaces the apparent reference of mathematical terms to objects in the third realm with reference to fictional objects, can be seen in this light also, although we will not attempt to follow this train of thought.

²⁷However, in Plato this sentence continues with what seems to be Individual Relativism, rather than Anthropologism: "As each thing appears to me, so it is for me; and as it appears to you, so it is for you." (From Plato's *Theaetetus*, 152A–B).

²⁸*Actually* learn and use language, as opposed to *having the ability* to learn and use language. The latter ability plays out as an architectural view, while the view being described is supposed to trace the anthropologism to the effects of *actually* employing language.

example by thinking of all people as being ultimately living in “the society of our planet”, a view that makes all of humanity be a very large society. In this view, there is a kind of societalism that has all humanity as a society; and this would be then a version of anthropologism. However, anthropologism and this very wide notion of societalism are usually kept separate; for, not only are societies usually seen to be considerably more localized than all of the planet, but also the forces that give rise to the common beliefs within a society are thought (by societalism) to be quite different from those forces that give rise to the common beliefs of people as a whole (by anthropologism). Anthropologism tends to look to physical commonalities that all people have. We all (or: all normal people) have two ears and two eyes, and basically the same brain. We navigate essentially the same region of space/time, with the same basic physical and sensory abilities and the same basic needs for food and relationships. We therefore are likely to have very similar concepts of things in our environment, and to classify or characterize them similarly. We will therefore all have basically the same sort of logical and mathematical concepts — there will be enough similarity between and among people to describe “averages” and “commonalities” that can be used to compare whether one person is *very* different from the others. Again, it is natural to attribute these similarities and commonalities to the cognitive makeup of humans. Once this step has been taken, it is no longer a type of descriptivism in our sense, but becomes a cognitive architecture view of the sort we consider in Section 6.

Anthropologism thus becomes a type of architectural view when it is developed. But societalism remains a type of descriptivism, since unlike anthropologism (which finds similarity between people on account of their physical makeup as it interacts with global features of the environment in which we all exist), societalism emphasizes the social aspects that cause the interpersonal similarity. Societalism focuses on such notions as similarity of schooling, inheritance of the same or similar languages, institutional memories, and the like. As a matter of fact, the societalist’s claim would be, because of these similarities we all have (pretty much) the same content to our logical and mathematical ideas. Or at least, we do if we come from similar societies. Those of us who are *not* inside some particular society are free to say “you all *agree* that such-and-so is true, but I can see that this is merely because of your teachers.” However, if anthropologism is attributed to a humanity-wide cognitive architecture, it becomes difficult or impossible to say “you all *agree* that such-and-so is true, but I can see that this is merely because you are a human” — since I am a human also.

5.5. Behaviorism

It will no doubt have struck readers that many of the variants on Descriptivism resemble the various maneuvers employed in the heyday of behaviorism. Aach [1] says that the anti-psychologism arguments work only because they are aimed against a psychology that made the contents of words be mental items of individuals. In that, Aach is probably right — they are aimed at pure Psychological Individualism. But there is another type of psychology, namely behaviorism *à la* Watson and Skinner, that does not make this identification, he says. Instead, meanings are related to dispositions to behave in some specified way. This too, then, is a type of Psychological Descriptivism: logic and mathematics would then arise as a coherent theory of how this sort of verbal (and written) behavior “fits together”. Given the behaviorist view of psychology, a proponent of the theory we have just described might still call it psychologism in logic; but since there is no trace of subjectivity — not even some sort of mutual subjectivity in virtue of being members of the same social or natural group — it is unlikely that others would call it psychologism. It seems more like a kind of realism.

5.6. Descriptivism and the Relativism Objections

In this section we have identified various types of descriptivism — ways that observable phenomena can be described — with the view that these might be able to define the semantic content of logical and mathematical language. One feature that ties together all these different conceptions of descriptivism, and justifies calling them all by the same term, is their use of commonality. They find something in common among people to serve as the nonsubjective content that Psychological Individualism could not provide. These views all have some claim to be psychological: either they claim to compare mental concepts across people, or they claim to induce truths about the mentation of groups of people by looking at the public manifestations of actions of members of the groups, or they invoke dispositions to behave in certain ways, or they have learned a common language, and so forth.

So, to what extent do the objections of individual relativism apply to these various strands within Psychological Descriptivism? Well, since they all claim to find a common content for logical and mathematical terms among people, it follows that many of the consequences of individual relativism therefore disappear. For example, the no *objective truth objection*, which we put under Interpersonal Incoherence, no longer holds: every descriptivist view finds an objective item that truths and falsehoods can be

asserted about. And the *no communication objection* falls away, since (due to common schooling or other common social activities) people always talk about the same thing, according to descriptivism.

Descriptivism finds something observable to serve as the content of logic and mathematics. In this, descriptivism mirrors a physicalist view of logic and mathematics, which would hold that logical and mathematical objects and truths are objective matters that inhabit the first realm of the physical world. But our definition of Psychological Descriptivism breaks with this physicalism in locating the origin of this observable content in people's mental life rather than in the natural world. (For otherwise, it would not be a kind of psychologism). For example, we view societalism as a form of psychologism because the contents of mathematics and logic are held to be the mental constructs of people, even though the doctrine claimed that these mental constructs would be the same across members of the relevant society. Similar remarks could be made about a descriptivist view that locates the content of mathematics and logic in the subjective notions of "the average person", claiming that there is some way to empirically determine what these are. The determination would be a fact about the natural world but the object of the investigation would be a function of the subjective constructs of individuals and hence psychologistic.²⁹

In this way the commonality is the non-subjective content that Pure Psychological Individualism could not provide. Hence it is not true that there is an (individual) relativism inherent in this type of psychologism. And so we expect that many of the consequences of relativism will disappear. For example, in all of these views it would seem that the *no objective truth* objection, (Ai), that we put under Interpersonal Incoherence no longer holds: every descriptivist view finds an objective item that truths and falsehoods can be asserted about. And the *no communication* objection, (Avii), concerning the impossibility of communicating, seems to fall away, since (due to common schooling or other common social activities) people always talk about the same thing, according to each of these theories.

²⁹Although they are similar in some ways, these descriptivist views are not the same as a typical physicalist view. Such a physicalist might argue that the reason we all have the same subjective notions of logical and mathematical terms is because if we didn't, then some of us would quickly die out when, for instance, we try to survive in the world where "two plus three is five" at the same time that our subjective notions are telling us that "two plus three is seven." Note that such a view takes the content of mathematical terms to be *some item(s) of the physical world* and not the individual's subjective notions. This is not a psychologism view of logic and mathematics, but a physicalist position instead.

Still, there seem to be some objections that still might hold against Psychological Descriptivism. This general position (with its different theoretical instantiations outlined above) postulates that there is an object for logical and mathematical claims to be made about, but it is not so clear that the objects have the right properties. For example, the position finds objects that form the common basis of ‘3’, ‘2’, ‘5’, and ‘6’. And the position has a story to tell why ‘ $3+2=5$ ’ is right and ‘ $3+2=6$ ’ is wrong — thereby providing some explanation of our normativity requirement and addressing the Interpersonal Incoherence objection of *no normativity*, (Aiv). But these descriptivist stories do not *really* explain why the one is *true* and the other *false* — as opposed to giving a reason to *say* or *deny* them (because our society or educational system or acknowledged experts say we should). Although this reason can provide a normative justification for saying and denying (e.g., “everyone does it” or “society approves of this”), we might wonder whether this is the *correct type* of normativity (the *universality is not normality* objection, (Av), had said that this is not a sufficient rationale). Furthermore, the stories do not *really* explain why they are both *necessary* — the one necessarily true and the other necessarily false. The stories do not seem to be able to generate enough force for this: all they do is assert that *everyone* believes or says (etc.) that they are true or false. But as we noted above, objection (Aiii) says that this is not enough to generate (real) necessity. Although one might suppose that for any objective reality, a group will — over time — converge on some elements of what is right about it or at least identify the beliefs that are terribly wrong about it, given that the group has to live successfully in that objective reality, this would be to suppose that the mathematical and logical objects *already exist* independently of people’s behavior. And this is what is denied in Psychological Descriptivism.

So at least these Interpersonal Incoherence objections remain. Although Descriptivism has achieved a certain popularity in its manner of avoiding various of the standard objections to the subjectivism inherent in Psychological Individualism, it seems that the Interpersonal Incoherence objections still hold, and that such theorists are required to adopt “weakened” notions of normativity and necessity, such as “everybody agrees with such-and-so laws”. The fact is that *descriptive* theories have just moved the locus of the description from the individual to a group, or perhaps to humanity as a whole. One might say (as Husserl did) that this is *still* a type of relativism — only now it is relativized to whatever group we are describing. And (he would also claim) one cannot get necessity or normativity out of a description of the way the world in fact is — even if the part of the world being described is how a group of people think or act. Certainly the weakened

notions that might be contemplated by Descriptivism wouldn't satisfy Frege or Husserl; but we wonder if they should satisfy the *rest* of us. However, we can't know this until a clearly-worded-out version of normativity and necessity of logic is put forward by some descriptivists.

6. Teleology and Cognitive Architectures

6.1. Teleological Theories

Our second major distinction for the landscape of psychologism is what we call the Teleological Cognitive Architecture perspective. The cognitive architecture element of this position holds that there is some specifiable mechanism according to which the human mind works. The mechanism has access to certain information and abilities at various times during the execution of a task, and its resulting actions (including its decisions) are to be understood as conditioned by the ways in which the architecture can operate. Despite the fact that the meaning and content of logical and mathematical expressions are due to individual people's mental constructs, this perspective would argue that different people will have the same contents for their logical and mathematical statements and ideas because of some inherent structure of their mental apparatus. The teleological element comes in if we suppose that aspects of this architecture are aimed at the same goal, or are serving the same purpose — for instance, social dominance or mating success or individual survival or species survival. In the theoretical biology literature, such view is called a teleological view, meaning that there is some common end that this mental apparatus serves to further.³⁰

In the current context, a teleological view of the content of logical or mathematical statements would make such contents be strictly dependent upon the mental apparatus of the individual making the statement. The reason we can have inter-subjective agreement of content and a normative force for logic is because the mental apparatus of everyone is the same, and that is so because it serves a common end, or *telos*. There are various ways that this common *telos* might have come about. One such way to ensure that one's ideas about logic and mathematics are “about reality”, have intersubjective agreement, and have normative force, is suggested by earlier theorists such as Descartes and Locke. We call it *God-Driven Teleology*,

³⁰This type of teleological view is different from earlier accounts of teleology in which the goal, or end, is seen as something that the organism or structure is “aiming towards.” In the present view, it is merely a description of how some aspect of the biological object operates. See [6], for example, for further discussion on modern concepts of teleology.

meaning that God created people and their ideas. And being all-good, God wouldn't allow his creations to be in error . . . or at least, not in error that they couldn't correct. So all people's ideas in mathematics and logic have (or at least, potentially have) the same content.

This is still a version of psychologism, since it traces the content of mathematics and logic to one's individual psychological entities. But it cancels the alleged relativistic consequences of this by insisting that everyone's individual psychology is the same, at least in logic and mathematics, by virtue of God's design in creation.

The interposition of God into explanations of psychologism in logic and mathematics is no longer a popular pastime of theorists. However, there are theories which have a structural similarity to this view. Any view that traces the content of logic and mathematical claims to an individual's mental arena — but evades the relativistic consequences on the grounds that there is some “outside” reason for everyone to have the same psychology concerning logic and mathematics — could be in this camp.³¹ The modern ungodly equivalent is *Evolutionary-Driven Teleology*, which means that the content of any logical or mathematical statement is strictly dependent upon the mental apparatus of the individual making the statement, but evolution has guaranteed that we are all psychologically identical in the ways relevant to logic and mathematics, because of the survival value of such a common apparatus. As can be seen, this blurs the distinction between what is “outside the agent” and what is “inside the agent”, by postulating that a cognitive architecture some way reflects or defines an important aspect of the logical and mathematical realm (as given by evolution), and it identifies the science of logic with whatever is given by the faculty that is common across people.

This sort of view finds the required systematicity between individuals in their shared evolutionary history. Like God-Driven Teleology, Evolutionary-Driven Teleology traces the content of logical and mathematical claims to individuals, but guarantees that these will agree with others through the effects of some outside force — the effect of evolution is to make everyone have the same logical and mathematical mentality. The *telos* of “God's Goodness” is replaced by that of “reproductive success”, but otherwise the theories are identical in structure.

³¹Some care must be given to this formulation. Some theorists who are not psychologistic, such as Frege, would agree that we all have the same psychology in this respect: that we can all grasp the content of logic statements. Where he disagrees with the formulation given in the text is in the *tracing of the content* of these statements to an individual's psychology.

Some think this can be made to be an empirical matter. For example, Cummins [17] traces the sort of reasoning that people do naturally and easily (reminiscent of Locke’s ‘native rustic logic’) evolutionarily to other primates, and separates this from a more general notion of logic. She constructs explanations of a teleological nature to the effect that we are obeying aspects of a primate “dominance hierarchy” that allows us to quickly and accurately detect aspects of trying to “cheat” on the more dominant individual. The sort of reasoning that is involved with this becomes the foundation of our more general notion of logic, which is of a different nature from this basic ability. Similar views are put forward in a series of papers by Lida Cosmides and John Tooby (see, e.g., [16]). As [17] suggests, one might think of a module that reasons about social dominance interactions as an identifiable part of the overall cognitive architecture. The specialized module would take in certain types of information and use its internal processing to generate conclusions relevant to that information. Whether there is a separate module or facility for specific types of reasoning is not a key issue for our discussions. Instead, it is the notion that a background common purpose would describe *why* the agent acts or concludes in the way it does, but it is the architecture that actually *does* the computation. This perspective aligns with psychological positions on the matter of human rationality (for example, [3, 14, 34]) and philosophical positions on realism (for instance, [4]) that appeal to a relation between features of the cognitive architecture and environment in which it evolved. We discuss some of these topics in the subsections to follow, after we have a preliminary discussion about relativism.

6.2. Teleology and Relativism in Logic

There may be other forms of Teleology that find their anti-relativism in some outside power that makes all individual people be the same in the relevant logical and mathematical regard. Any such theory holds that individuals of the human species will all understand the same items by use of the same terms; they will all attribute necessity to the same items; and normativity is guaranteed by the fact that it serves a common purpose to have (evolved to have) these beliefs. And in this way a number of the classical objections to psychologism are answered. However, any of these views will still have a trace of the relativism that Frege and Husserl object to. For one thing, although it seems reasonable for these theories to account for necessary truth in terms of a common human mentality, it then becomes difficult to see how these theories could distinguish between the necessary truth of logic and mathematics and the contingent truth of claims in other

areas where humans are also constrained by the very same outside force to always make identical judgments. For example, it seems plausible to think that humans are constrained by their evolutionary ancestry to think of space as Euclidean. Yet it remains a matter of some contention whether Euclidean geometry has the same apodictic status as truths of elementary arithmetic, despite Kant's claims to the contrary.³² Many things that are universally true in the physical world are not *necessarily* true, e.g., the force exerted by Earth's gravitational field on a body is proportional to the body's mass and varies inversely with the square of the distance the body is from the surface. But it *could* have varied inversely with the cube of the distance — it just *happens* to be the square. And we can recognize this difference. Although all our experience is in accord with it, we nonetheless assign it a different epistemological status from that of logical and mathematical truths. This universal truth is not given the status of necessity, in contrast to our assessment of mathematical and logical truths; but the cognitive architecture position seems unable to differentiate the two, and to account for our ascribing them different statuses. Or again, it might be that, because of our common evolutionary background, all (“normal”) people react to a certain wavelength of electromagnetic energy in the same way. But it is difficult to see that a sentence like “This inkspot is red” has the same apodictic force as “ $2+3=5$ ”. (This is an instance of the Interpersonal Incoherence objection concerning *no universal necessity*, (Aiii).)

And again, it seems to be a special problem for Evolutionary Teleology that the “outside power” that makes all individuals be the same with respect to logic and mathematics does not seem to have any particular way to evade an anthropological version of the *no objective truth* objection, (Ai), since all truths will be of the form “It seems to humans that ...”. Unlike the postulation of an all-good God by God-Driven Teleology, which guarantees not only that all people are the same in their mentality but also that they are all *aimed at the truth* in their logical investigations, there seems to be no reason to suppose that evolution can provide the same guarantee.

³²Presumably this contention arises because of the existence of non-Euclidean geometries, and their interpretation within modern physics. Although this may sound like a geometry version of the ‘many different logics’ claim, most moderns draw different conclusions: Euclidean geometry does *not* contain necessary truths about the world, but classical logic does. We suppose that our moderns might wish to say that Euclidean geometry describes our *mental* space, but that we can recognize that it does not describe *physical* space. Even Frege, who thought of Euclidean geometry a priori wanted it to have a different status from classical logic in terms of its applicability to reality. (Thanks to Hannes Leitgeb for drawing our attention to Frege here; see [31, 68]).

Indeed, many authors have remarked on how evolution is slapdash and only aims at truth if this would further the aim of reproduction. Truth – e.g., theoretical truths about the very large, very small, the distant, or the very abstract – is thought to be only a very weak constraint on reproductive success. And so, it is thought that truth is only in the most general way conducive to success in reproduction. As Rode *et al* put it:

One should not expect the cognitive architectures of evolved organisms to be “rational” when rationality is defined as adherence to a normative theory drawn from mathematics or logic. One should expect their cognitive architecture to be ecologically rational: well designed for solving the adaptive problems their ancestors faced during their evolutionary history. [82, p.302]

A conclusion here seems to be that without the guarantee of “real” normativity and necessity that God-Driven Teleology offers, any of the teleological theories will continue to be a relativism (just as Husserl had claimed). And although *individuals* will have a reason to act and believe as all other people do, and to think that within their species such-and-so statements will be deemed necessary and correct to believe (because “that’s what their purpose is”), nonetheless it will not be true that such a claim can be made without the qualification ‘within their species’. And we members of the relevant species can *recognize* this difference — another thing that seems precluded by the theory. Of course, theorists can adopt a teleological theory in this realm, but then they owe the rest of us an explanation of why such a theory is viable, in light of the fact that such theories fall prey to species-wide versions of the objections we outlined as the (A) and (B) parts of our taxonomy of objections to psychologism in Section 3.3.

6.3. Cognitive Architectures

Although we have stated the various teleological theories in such a way as to focus on the *telos* rather than on *how* we come to realize this end state, it is natural to think of any such theory as invoking the notion of “cognitive architecture” — i.e., it is natural to think that there is some specifiable mechanism according to which the human mind works to instantiate this end state. The mechanism has access to certain information and abilities at various times during the execution of a task, and its resulting actions (including its decisions) are to be understood as conditioned by the ways in which the architecture can operate. One might think of a module within this overall cognitive architecture that reasons about issues of logical reasoning, such as

envisaged by Rips [81, especially chapter 2]: it takes in certain types of information and uses its internal processing to generate conclusions relevant to that information. The *telos* might describe *why* the agent acts or concludes in the way it does, but it is the architecture that actually does the computation. The evolutionary viewpoint suggests some kind of adaptive shaping function between the environment and our cognitive architectures, and hence claims that the very kinds of truths that can be detected about the world are those that are likely to be important to survival. But the same environment cannot shape different individuals in the same way unless there were underlying mental similarities among those individuals in the first place. This apparent circularity is an issue that the Teleological Cognitive Architecture perspective would have to address, or at least acknowledge, in addition to acknowledging the force that the arguments against relativism carry.

As remarked above, a module that reasons about dominance relations can also be seen as the relevant sort of module for logic and mathematics. Gerd Gigerenzer and his colleagues (see, e.g., [35]) argue like this, although about the different field of probabilistic reasoning. Here their conclusion is that we are more adept at understanding and manipulating statements of *frequencies* rather than statements of probabilities. Our probabilistic reasoning module has evolved from a “native rustic ability” to estimate frequencies, and is now being employed to reason about probabilities. The analogous view for the realm of deductive reasoning is what is being advocated by the theories of Cummins and Cosmides & Tooby [16, 17].

A problem facing any of these theories concerns their description of people’s actions. And this is particularly noticeable when thinking about the topic of psychologism in logic, and about whether any of the classic arguments against psychologism touch these views. A natural interpretation of how a “logic module” should work within a cognitive architecture is that the module (and its place within the architecture) determines what is the field under consideration and what should count as correct and incorrect in that realm. Thus, a module that evaluates appearances of dominance and generates actions that are appropriate to those appearances should be self-verifying in the sense that whatever it does is defined to be the correct thing to do (in that realm). If the module says that action A is appropriate to the case at hand, then that is all there is to it. If this disagrees with some other opinion on the matter — say from a different module or a different person — then the other opinion should be changed or ignored. But if one says “well, maybe the module miscalculated and it should have done action B instead”, or “the module said to do A but in reality doing A does not lead to an appropriate outcome in the realm of dominance”, then one no

longer agrees that the field of dominance inference is *defined* by the module. Rather, one has some other, external-to-the-module, view of what reasoning is in the realm of dominance. That is the only way these sorts of comments make any sense; however, this sense entails that there is some “external-to-the-module” standard of correctness by which one can evaluate the dictates of the module.

However, this seems to generate yet another dilemma or circularity for the cognitive architecture perspective. If one chooses the alternative that there is some external-to-the-module standard, then the theory is no longer psychologistic, and becomes realistic instead. But on the other hand, if the field of logic and mathematics is *determined* by the workings of the logic and mathematics module, then the notion of an error becomes quite dicey. For, logic and mathematics would *just be* the way people reason, when they reason in this area. There is no third-realm “real logic and mathematics” for our reasoning to adhere to or violate. In a way, then, the output of this module would be “self-verifying” — whatever it dictates is what is correct. The best one could say in the direction of an error is that someone is not producing the same results that the others do. But this is not necessarily an error, according to the theory, since it could be just a further adaptation to the immediate environment. We wonder if this self-verifying circularity is simply inherent in any cognitive architecture position, at least as we have defined it here.

An alternative response would be to talk about the *reasons* why the output of a person’s module doesn’t agree with classical logic. For example, perhaps there are memory limitations of people, or time limitations, etc., and the module therefore is biased in certain ways and as a matter of course makes use of certain heuristics that are usually but not always correct. (See [54], for an example of this response in the realm of probabilistic reasoning.) We will talk about this response in Section 7, but we might immediately note that it presumes that there is a “correct answer” that is independent of how people actually perform. It therefore avoids the charges of circularity, and as well avoids the charge of relativism; but it is also not clear how it can be made to be a psychologistic theory of the science of probability.

A general problem that is encountered in cognitive architecture theories when we try to view them as a possible modern psychologistic position is that, while such a theory may describe the majority, the fact of the matter is that *not* everyone reasons in the same way. A natural reaction to this worry is that such individual differences are to be expected but that the cognitive architecture (more or less) guarantees that, over time and with evolution (and so forth) the species will identify elements of objective reality that are

relevant (although this is constrained by the very machinery that is applied in the discovery process). Now, however, this view starts departing from a pure psychologistic position, because as soon as we appeal to something like evolution as a power that forces conversion to a correct end-point, we admit to something outside the agents and their architecture that defines aspects of logic. So, such a view is either not psychologistic — in supposing that the cognitive architecture will *discover the appropriate reality* — or is prey to the objections concerning necessity and normativity — if one supposes that logic is *whatever* it happens to fix upon. A truly psychologistic approach to logic and mathematics that employs cognitive architecture will need to not only make people converge in the long term, but must suppose that the “external” reason for this is that people happen upon the independently-existing truths of logic and mathematics. It is difficult, then, to understand how a cognitive architecture could do this while also being self-verifying.

When we are faced with the fact that many or most people do not reason in accord with classical logic and mathematics, we say that there are “reasons” for people’s mistakes. One could take the attitude that “smart people” have applied their cognitive architectures to discover some elements of the objective truths about mathematics and logic, and have systemized their understanding (still highly influenced by the application of their cognitive machinery) but the people with not-so-smart variations of the same machinery may not have discovered these truths and may have trouble reasoning in accordance with them. But of course, this is not psychologism, since it presumes a realist conception of logic and mathematics.

To adopt a cognitive architecture view as a psychologistic position, it is necessary to redefine what “error” amounts to, given that all people are acting in accordance with some module within their cognitive architectures, and that the very field under consideration is defined by this module — as we argued in claiming it was committed to being self-verifying. On the one hand, if we do not allow the module to define what it is to be “right”, then it is not a form of psychologism. But if we do allow for this, then it seems impossible for there to be error, since by hypothesis all people act in accordance with their architectures and the architectures define normativity. How then, can some people “get it right” while others “get it wrong”?³³ We will discuss, in Section 6.6, one natural response to this difficulty, but claim that in the end it is not a form of psychologism.

³³We are not here arguing against the concept of cognitive architectures for logic and mathematics, but rather that they automatically give psychologism in logic a foothold.

6.4. Cognitive Architectures and Relativism

Obviously, the Cognitive Architecture viewpoint is committed to Anthropologism about any area that it claims to be covered by the relevant architecture. Dominance reasoning, for example, is defined to apply only within the *human* (or, possibly, *primate*) realm and it would hold that it makes no sense to talk about whether reasoning by this module would or wouldn't issue forth with correct output in snakes or plants. (For, if it were thought that this might be a possibility, then there would be some external-to-the-module standard to which appeal is being made.)

It would follow, then, that the view is committed to saying that the output of the logic and mathematics module is relativized to humans: “ $2+2=4$ is true for people” (but perhaps not for dogs or aliens). Note (again) that advocates of this position are not allowed any appeal of the form “all humans, as well as the dogs and aliens, will converge on the same set of truths because that is the way the world is”. For, that is not a psychological theory. Once again, it seems incumbent on advocates of this position to acknowledge this consequence (along with the various circularities outlined in Section 6.3). Such theorists ought to be asked what aspects of the physical world are the content of logic and mathematical claims. . . is it Mill's playing with pebbles and gingerbread men? If so, the theory seems to collapse into Psychological Individualism with its attendant problems, despite having a layer of cognitive architecture. Or perhaps these theorists would invest in a third realm?

6.5. Kantian Preconditions for All Experience

Perhaps another version of this picture might seem to hold more promise. This is the interpretation of Kant that was initially offered by the very early “psychological logicians” Jacob Fries and Friedrich Beneke, and more recently put forward by theorists like Brook [12] and Kitcher [57]. In this interpretation, Kant's famous “preconditions for all experience” become “general specifications for a mind capable of performing various cognitive tasks”. Kant's language about mental faculties and the “logical” rules that govern them becomes an essential ingredient in describing how humans work, in this view, and might be assimilated to the notion of “cognitive architecture”.

Perhaps in this sort of view, the *very definition* of ‘logic’, ‘necessity’, and ‘normativity’ is given by our architecture. And then maybe the worry expressed against the Teleological theories (that they become susceptible to an anthropological version of relativism) vanishes. In this picture, since

the *meaning* of ‘logical’ is given by our cognition, when we act in accordance with it, there can’t be any worry that we might not be rational. (We say this, although we acknowledge that a standard criticism of this anthropologistic interpretation of Kant says that since we cannot know the things-in-themselves, but only how they appear to us through our cognitive architecture, a kind of relativism must be entailed because the things-in-themselves could be radically different than we apprehend them, even in regards to their logical and mathematical properties. It is just that we can’t “appreciate” or “understand” or “believe” this.)

A version of this sort of theory has recently been put forward in Hanna [41], who argues that humans form the basic class of cognizers or thinkers, and that given this connection between rationality and logic, the requirement that logic be normative is automatically satisfied. Hanna’s attempt to walk the tightrope between an anthropological-psychologistic version of Kantianism (which postulates a logical-cognitive module in humans) and a more pure version (where the very notion of logic is given antecedent to any psychological facts about humans) is the sort of view that would seem to be required to make psychologism both be a contentful theory and also avoid the objection that it cannot account for necessity and normativity. But one might wonder whether such a theory can really reconcile these two seemingly opposed desiderata. It is not clear to us that Hanna’s so-called “Logic Faculty Thesis” really gives us the relevant “independence from all possible experience”, as opposed merely to another cognitive architecture theory.

Despite the avoidance of some of the relativism objections (especially the normativity-based ones), there seems still to be the same inherent circularity in this Kantian version of the cognitive architecture theory that we discovered and discussed above. Since it is self-verifying, it cannot give a coherent account of error, nor of why there are individual differences. One possible answer to this will be surveyed in the next subsection; but we will claim that the strategy will work only in a non-psychologistic environment.

6.6. Mental Modules and the Competence/Performance Distinction

If we are going to adopt a cognitive architecture view as a psychologism position, then we must explain how to define what “error” amounts to, given that all people are acting in accordance with some module within their cognitive architectures, and that the very field under consideration is *defined* by this module — thereby seeming to make it be self-verifying. If we don’t allow the module to define what it is to be “right”, then it is not

a form of psychologism; but if we do allow for this, then it seems impossible for there to be error, since by hypothesis all people act in accordance with their architectures and the architectures define normativity. How, then can some people “get it right” while others “get it wrong”? A standard answer is to invoke a distinction between one’s *competence* and *performance*, recalling Chomsky’s distinction in *Linguistics*.³⁴

John Macnamara [66] is the main discussant of how a cognitive architecture would employ a competence/performance distinction in the exposition of how logic and psychology are related. Macnamara is also relevant here for his clarification that, to accord people logical competence is in fact *not* to be psychologistic about logic. Macnamara makes appeal to the notion of a psychological capacity that ‘informs logical intuition’ and to the notion of a logical competence as a set of “devices” that perform logical operations. In this sense, logic competence is “in us”. This competence is based on our ability to grasp and use logical intuitions such as logical necessity, the principle of contradiction, and so forth. And the employment of this competence comes out under the right conditions. Among other things, this viewpoint allows for individual differences — and errors — in the realm of logical performance.

Macnamara is clear on where logic will fit into psychology here, and what is presupposed by his picture.

Basic logical competence does not consist of beliefs. . . . I argue that basic logical competence consists of a set of devices that perform certain operations when certain conditions are satisfied. Part of the operations they perform is to invoke unlearned logical symbols. . . . The reader will grasp at once that this commits me deeply to there being a language of thought of the sort argued for by Fodor [24]. [66, p.29]

Throughout I have been using the expression ‘basic logical competence’ to speak about the psychological capacity that informs logical intuition. The force of ‘basic’ is to rule out all theorems that can be proved within a logic and all metalogical theorems about such matters as the completeness, soundness, or decidability of a particular logic. . . . ‘Basic logical competence’ means something that is common to all nonpathological human minds. [66, p.33]

We see that this sort of view presupposes there to be a mental module, which could operate differently in different people — better in some, worse

³⁴We will not engage the substantial literature on the correct formulation of the distinction, nor will we discuss any issues in Chomsky-exegesis, especially his apparently evolving views on this matter.

in others — even though it is the same module. But how can we know that it works better in some than in others? Only, thinks Macnamara, if we admit that there exists an “external” realm of logic.

... I want to dispel any suspicion that my position is psychologistic. I hope to be more persuasive by showing how it agrees closely with the deepest theoretical roots in those arch-antipsychologistic logicians, Frege and Husserl. ... It might at first appear that my position comes dangerously close to psychologism with its emphasis on psychological interpreters and implicators. It might appear that, although I escape Kantianism by locating the implicators in the evaluation of inferences, not in their generation, I am nonetheless grounding logic in psychological properties and events. To be clear of the charge of psychologism, however, it is enough to note that my claims have to do with access to logical principles, not with justifying them.

[66, p. 42]

The last sentence is particularly telling, and shows that the invocation of modules, and some sort of associated competence/performance distinction does not by itself involve the theory in any classical sort of psychologism. Indeed, for Macnamara, it forces what he calls arch-antipsychologistic tendencies; but unlike Macnamara, we might allow that these tendencies could find a home in the first realm — at least, we haven’t here discussed any reasons that would preclude this.

As we consider the role of psychology in logic, it is useful to dispel any confusion with the role of logic in psychology. Macnamara is an example of how logic is employed in explaining psychological phenomena; but as he clearly states, this in no way means that psychology has a role to play in defining or explaining logic.

7. Ideal Cognizers

Against pure Psychological Individualism, the *individual differences* objection, (Avi), in the Interpersonal Incoherence category asserts that different people would (or at least, could) have different ideas of mathematical and logical objects and employ different methods of reasoning from one another. This in turn leads to the other Interpersonal Incoherence objections of miscommunication and agreement/disagreement, because the meanings of these terms were different for the different cognizers. The Ideal Cognizer theory addresses these objections by postulating an agent whose mental life is acknowledged to be “the only one that counts”. Of course, a psychologistic

logician could not legitimately postulate an agent whose mental life was *correct*, for this would not be psychologism — it would instead be postulating a notion of correctness external to the ideal agent.

This conception of psychologism is not very clear, despite its apparent appeal to some.³⁵ We might start by asking for the sense in which the agent is “ideal”. As we just mentioned, the theory cannot properly refer to some standard *outside* the mental life of this agent, under threat of the theory not being psychologistic. (And anyway, if these standards were to exist independently of the ideal agent, then it is very unclear why one would wish to add an *agent* to a theory that already has all the standards). So whatever considerations are used, they must refer to features of this Ideal Cognizer and its products.

One fundamental division within the notion of ‘ideal’ is whether it is meant (a) that the agent optimally behaves in any situation that it finds itself, or (b) that various impediments which we recognize as existing in “normal” agents are to be abstracted away from. In the former conception, we would wish an agent to perform as well as can be done, given the properties of the environment in which this is to be done. Such an agent would be subject to the necessity of performing its actions quickly enough for them to be relevant in whatever situation it finds itself, to be concerned for its safety even at the expense of logical completeness or correctness, and the like. In short, this would be a situated and bounded agent. And the notion of logic that would emanate from it would be likewise situated and bounded. Of course, from within the theory this could not be said, since after all the notion of what is logically correct is *defined by* what this agent does. The fact that we *can* make this judgment — recognizing that the boundedness of its performance means that its performance does not match our conception of logic — shows that this notion of an ideal cognizer cannot form the basis for any reasonable notion of psychologism in logic.

There are artificial intelligence researchers who take seriously the notion that all aspects of rationality (including logic, mathematical reasoning, and decision making) are situated and resource-bounded — for example, Pollock’s Oscar ([78] and other works of his), and the more theoretical position advocated in Russell [83] that is widely followed by both practitioners and theoreticians in AI. But because of this situated resource-boundedness, none of these authors wish to *define* the content of logic to be what such

³⁵For instance, one hears of Bayesian probabilistic logic being defined as a theory that accounts for the betting behavior of an ideal agent who always computes the correct odds that accord with its degree of belief in different propositions. And in the Introduction above we mentioned the use of an Ideal (Moral) Agent within ethical theory.

an agent does. Indeed, in these agents, there is reasoning employing “ideal logic”, but it can get “trumped” by the requirements of memory, or time, or urgency. If such a situated agent is what is meant by ‘ideal cognizer’, then the correct conclusion is that a psychologistic account of logic that is based on this agent’s mentality would *not* generate logic or mathematics. And these theorists, who are concerned with constructing useful artificial agents, would not likely argue so. Rather, the notion of the ideal cognizer for current AI theorists embraces the notion of situated and bounded rationality, and reframes the concept of “ideal cognizer” as the cognizer (some particular cognitive machinery) that generates the best performance, given a particular task to be performed within a particular world defined by, say, a rate of change, (in)accessibility of all relevant information, or indeterminism of the actions the cognizer might take. Such an agent is situated and has resources that are bounded either by the agent’s own properties, or indirectly by the demands of the world in which it finds itself. So an account of this type of agent would be an account of “what should be concluded or acted upon in this or that situation, given the various conditions that affect its reasoning”, as Russell puts it [83].

The second conception of an ideal cognizer is one that is independent of any context or situation. One can see that such a conception of an ideal agent has many similarities to the notion of competence as discussed by Macnamara [66] (and recounted in the last section). And both views reject psychologism in logic for similar reasons. Russell [83] calls this the ‘optimal rationality perspective’, which he rejects as limited in usefulness for either understanding or developing artificial agents that operate, as we must, in the real world. It postulates a cognizer that has unlimited time, unlimited memory, unflinching attention to detail, appropriate grasping of the entire context and where each part fits in, and so forth. This version of psychologism says that logic and mathematics is what such an agent *would* have in its mental repertory if it *were* to cognize on such-and-so problem or on this-and-that mathematical item. As in the first conception, since there is but one relevant cognizer about logic and mathematics, there is no worry about any disagreement among cognizers; but in this conception, since the ideal cognizer is always correct, there is no fear, as with the other conception of ideal cognizer, that an incorrect theory of logic and mathematics might be in use due to the exigencies of the situation or agent.

But isn’t this notion circular? Doesn’t it define logic as what a perfect logician would cognize about logic? And a perfect logician as one who employs logic correctly all the time and in every circumstance? But if one tries to evade the charge of circularity by defining logic as “whatever this ideal

agent does in whatever circumstances we find it”, doesn’t this make the whole concept vacuous? Without some independent view of what logic and mathematics are, we are left with the self-verifying circularity that *everything* this agent does is logic and mathematics. This seems better, maybe, as a definition of ‘rationality as a whole’, rather than as a definition of some *part* of rationality such as “action in accordance with logic”. But even here there can be questions. Does the notion of investigating what an agent would do when it has no restrictions whatsoever have much utility, or even make sense? As we remarked, Russell [83] would likely argue no. Isn’t rationality — or at least the largest part of it — a matter of determining what would be best to do in such-and-so actual situation? And all of these have factors that call for a “bounded rationality”, to use Herb Simon’s term ([85, 86]).³⁶

Even if these objections were somehow overcome, it seems to us that this can’t possibly be an adequate notion of psychologism in logic. One of the initial points of naturalism’s appeal to psychologism was to move away from entities that were not available to ordinary experience. And a large push for current neo-psychologism comes from the distaste many have to the notion of a “third realm”, with its non-subjective but also non-physical inhabitants that somehow manage to cause us to believe and do things by means of our “grasping” them. But on the current view, doesn’t the ideal cognizer dwell in the third realm (or perhaps the imagination? but if in the imagination, then we ask: in each person’s imagination? wouldn’t this just be Pure Psychological Individualism?) just as much as those items most objectionable to modern philosophers who want to naturalize logic and mathematics? So this theoretical construct seems to have no use in any case, even if it could be made coherent.

8. Logic that *is* Psychologistic

Pelletier & Elio [75] reviewed the justificatory remarks made by the initiators and developers of logics for default reasoning (‘non-monotonic logic’) in Artificial Intelligence. It was discovered that all of them employed one of two attitudes in their justification, with the second being by far the most common: (a) there is some objective result that can (apparently) not be achieved easily when reasoning with classical logic but which can be achieved by using some non-monotonic logic (e.g., accuracy in a medical diagnosis), (b) in their

³⁶The notion of bounded rationality that is usually attributed to the early writings of Simon is: the property of an agent that behaves in a manner that is as nearly optimal with respect to its goals as its resources will allow.

ordinary lives people are remarkably able to quickly and successfully reason in the absence of total information — and even when the information is total they can quickly employ only a portion and reach the correct state, unlike programs that employ classical logic. An example quotation illustrating this second attitude is:

A key property of intelligence—whether exhibited by man or by machine—is flexibility. This flexibility is intimately connected with the defeasible nature of commonsense inference. . . we are all capable of drawing conclusions, acting on them, and then retracting them if necessary in the face of new evidence. If our computer programs are to act intelligently, they will need to be similarly flexible. A large portion of the work in artificial intelligence on reasoning or deduction involves the development of formal systems that describe this process. (Ginzberg, [36])

In [75] it was argued that the (b) attitude dictated that AI researchers needed therefore to find out how “ordinary people” reasoned in the types of circumstances, and construct their formal systems in accordance to this. It can be seen that this invokes Locke’s notion of a “native rustic logic” but with a desire to formally describe (in the sense of an abstract system of rules) the manner in which people reason in ordinary circumstances. It is rather like making a Science out of the Art of reasoning, a view rather in the spirit of Oaksford & Chater [71] and Harman [42, 43] but without the aversion to formal systems.

In subsequent work (e.g., [76]), Pelletier & Elio described various psychological experiments that attempted to uncover the patterns of reasoning that would be employed by native rustic logic in certain circumstances. The circumstances investigated in these works were inspired by some rules of inference that the AI researchers had employed in their formal systems, but which they had justified only by their own introspection or by how they formally interacted with other features (which in turn were also justified only by their own introspection).

The outlook adopted by Pelletier & Elio here amounts to a type of Psychological Descriptivism, in particular the type invoking commonalities or “averages” described in Section 5.1: One investigates how people would actually reason in such-and-so situation, then one infers the sort of mental states that would give rise to these sorts of performances, and the construction of a systematic formal theory of native rustic logic is the result.

And that is (one type of) psychologism in logic. Many of the classic objections do not apply to it — we already mentioned in Section 5.6 the ones

that descriptivism in general avoids. But further, the objections we mentioned in that section as still applying to such descriptivist theories — such as whether it provides the right type of normativity for the rules or why this should mean that theorems of the logic are necessary — are *accepted* by the proponents of this type of psychologistic logic. Unlike the case in mathematics and classical logic, no one thinks that forming beliefs (or acting) in the ways described by such a logic should be necessary, or even contain any sort of normativity beyond “they usually work”. Thus, nonmonotonic logic or commonsense reasoning allows some room for psychologismists to maneuver, because it does not require the sort of necessity and normativity that classical logic and mathematics do. We remind readers of a remark we made in our early description of logic and psychologism: one could be psychologistic about some logics and not about others. In that earlier passage we suggested that one could be a realist about classical logic but psychologistic about modal logic, by thinking that the notions of *necessary* and *possibly* were determined in some psychologistic manner. Here, we instead point to the idea that nonmonotonic reasoning gets its justification psychologically.

9. Concluding Remarks

We have argued that the historically early version of psychologism — which we called “Pure Psychological Individualism” — is implausible in logic and mathematics. Our reasons are pretty much the initial objections that were raised by Frege and Husserl. But there are some versions of psychologism that can avoid at least some of these objections.

We proceeded to consider three other views that either have been or could, with some plausibility, be called psychologism in logic and mathematics. The first of these, Psychological Descriptivism, focuses on publicly-observable manifestations of people’s underlying logical and mathematical abilities. One variant would try to find either “the common content” or “the average content” that individuals have for logical concepts, and use that as the true, across-people meaning of the terms. Others would look to the written or spoken manifestations of logic and mathematics, and construct a general account of what is being asserted in these manifestations considered as one general theory. Both of these views can be framed in a wide manner (where all people equally contribute to the data) and a narrow manner (where only some acknowledged experts contribute to the data to be described). We described forms of societalism that seem to capture these new views of what psychologism in logic might be. We also found that, while some of the initial criticisms put forward against psychologism

seem to be avoided, others remain and proponents of these theories need to address them.

The second of these three views was that cluster we called ‘Teleological Cognitive Architectures’. In this group we included theories that tried to find a purpose, end, or *telos* to explain why it might be that all people converge upon the same logical and mathematical concepts. Here we discussed God-Driven Psychological Individualism and Evolutionary Psychological Individualism. We argued that, while these views might avoid many of the objections to Pure Individualism (e.g., might avoid some of the “relativistic” problems), at least the Evolutionary version still faced some of the remaining objections, such as being unable to explain where necessity and normativity would come from, leaving therefore a kind of anthropologism-relativism as a residue. (The God-driven version would attribute it to God’s nature, but this does not seem plausible to attribute to evolution). If a theorist wished to adopt one of these (or similar) views, these remaining difficulties need to be addressed (possibly by saying that they *incorrectly* presume that logic is necessary and normative), rather than just pointing to the problems that have been avoided. We then turned to other types of cognitive architecture views, and argued that they too have difficulties. One understanding of a psychologistic-oriented cognitive architecture theory *defines* logic in terms of *whatever* the relevant module does. Like the teleological theories, this theory finds a common content to everyone’s logical and mathematical ideas; but again, like the teleological theories, it seems to fail at finding necessity and normativity. A different understanding of a cognitive architecture theory makes this module responsible for *finding* and *interpreting* the logical and mathematical parts of the world. But here we note that this type of “psychologism” does *not* think of the content of logic to be “in the mind” but rather that the mind is somehow designed or able to discover logic-in-the-world. Such an account (as given in [66]) is not at all psychologism in any of the senses in which it has been argued about. In fact, it is consistent with the Fregean notion that our minds can “grasp” logical, mathematical, and semantic facts that exist in a third realm.

Underneath it all, there is a tension in psychologistic cognitive architectural theories of logic which stems from trying to simultaneously *define* logic and mathematics as “whatever the relevant module/faculty does” and also trying to find some *justification* of the necessity and normativity that logic and mathematics possesses. Anti-psychologismists-in-logic will always accuse their opponents’ theories of being non-psychologistic if they do not define logic in this way; but they will accuse them of being unable to account

for the crucial necessity and normativity of logic if they do define logic in this manner.

Finally, we considered the view mentioned in the call for papers of this special issue, that logic and mathematics are defined as the mental abilities of an Ideal Cognizer in these realms. Since there does not exist such a being, it is difficult to evaluate whether this will answer all the objections that were initially brought forward. Certainly if it is defined in the right way it will avoid the objections, since its definition could merely be “whatever avoids the objections.” But it is much less clear whether there can be any independent definition that is given. And to whatever definition is put forward, we can always ask “would this theoretical construct ever be of use in giving an account of logic and mathematics? would this theoretical construct ever explain how people come to know logic? could this theoretical construct ever be given a more naturalistic explanation than Frege’s third realm — or is it simply equivalent to Frege’s postulation?”

Looking at the critiques of Frege and Husserl, it appears that they find the main problems with psychologism in logic to be its perceived relativistic implications. And the main issue that they concentrated on was the consequence that different people would generate different logico-mathematical items, with each of them being “right — for the person” but having no interpersonal objectivity. Modern psychologisms-in-logic focus on this consequence and try either to show that in one way or another people will converge on the same items or else to assume that there is but one agent whose content in logic and mathematics matters. But a problem with the former class of approaches is that they provide no account of the necessity and normativity that logic and mathematics (seem to) embody, while a problem with the latter type of theory is that it is circular unless some outside-of-the-ideal-cognizer conception of logic is postulated.

Attempts by the former sort of theory to “build in” the normativity and necessity by appealing to the definition of the terms by means of human cognition, seem to open themselves to variants of these objections stated in terms of the species as a whole: “X is necessary and a normative standard” becomes “X is necessary and a normative standard for humans” (but maybe not for other sentient creatures and hence maybe not in reality). Attempts by the latter sort of theory need to ask what added value there is to postulating a non-physical (non-existent?) *cognizer*, as opposed to simply postulating a non-physical field of logic itself. These possible consequences are ones that should be faced up to by advocates of these positions.

In a nutshell, our conclusions are that

1. Psychological Individualism implies “full-blown relativism”, so that if one wishes to embrace this sort of psychologism in logic, s/he would have to concede all the Frege-Husserl consequences and say that they just didn’t matter. (E.g., say “since relativism is correct, these are the consequences for logic and mathematics”). This could be combined with a more subtle version of relativism, such as that offered by Kölbel [60].
2. Psychological Descriptivism thwarts some aspects of relativism — e.g., by finding a common-to-all-people content for logic and mathematics. But such objects cannot contribute (what anti-psychologism-in-logic theorists would call) the essential properties of necessity and normativity to logic, but only the ersatz property of “everyone in fact does this”. Such descriptivists might wish to say that this is correct for some forms of logic (e.g., formal accounts of “native rustic logic”, as we described in our Section 8 concerning default reasoning), and the anti-psychologismists could happily agree with this. But it is another matter to explain away these properties for classical logic and mathematics. (Although perhaps they would wish to follow Mill in this regard, calling it an illusion).
3. Cognitive Architectures also thwart some aspects of relativism by presuming that all people *by nature of their mental structures* have the same content to their mental concepts of logic and mathematics. Again, though, it seems that such theories must either presume that the cognitive architecture is what guarantees that everyone will ultimately *find* the same contents or else guarantees that they *generate* the same contents because of their identical cognitive architectures. But the former is *not* what one would like to call a psychologism, since it presumes that there exists such contents independent of whether people actually discover them, and therefore is compatible with Frege. (It is also compatible with non-Fregean physicalism where these architectures are evolved to discern physical truths in the evolutionary long run). And the latter alternative once again seems unable to account for the presumed necessity and normativity of logic. (Here evolutionary theories would have to say that they evolved similarly in order to succeed reproductively, but not because any such item as logic exists in reality. So, such theorists would have to give some other explanation for the apparent existence of necessity and normativity when in fact they do not exist. Perhaps Mill’s “illusion”?)
4. Ideal Cognizer theory also finds a unique content for logical and mathematical language in the concepts of an ideal cognizer. Since this cognizer

is ideal, we are also afforded an explanation of the necessity and normativity of these contents. However, such a theory is circular, we argued. For, without an independent notion of the contents of the language of logic and mathematics, one cannot describe the relevant ideal cognizer. But it was supposed to be the ideal cognizer that gives us the contents.

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References

- [1] AACH, JOHN, ‘Psychologism reconsidered: A re-evaluation of the arguments of Frege and Husserl’, *Synthèse*, 85 (1990), 315–338.
- [2] ABBAGNANO, NICOLA, ‘Psychologism’, in Paul Edwards, (ed.), *Encyclopedia of Philosophy*, Macmillan, New York, 1967.
- [3] ANDERSON, JOHN, ‘Is human cognition adaptive?’, *Behavioral and Brain Sciences*, 14 (1991), 471–484.
- [4] BAC, MURAT, and RENÉE ELIO, ‘Schema-based alethic realism: Agency, the environment, and truthmaking’, *Minds and Machines*, 14 (2004), 173–196.
- [5] BAKER, GORDON, *Wittgenstein, Frege and the Vienna Circle*, Blackwell, Oxford, 1988.
- [6] BEKOFF, MARC, and COLIN ALLEN, ‘Teleology, function, design, and the evolution of animal behavior’, *Trends in Ecology and Evolution*, 10 (1995), 253–255.
- [7] BENEKE, FRIEDRICH EDUARD, *Die Philosophie in ihrem Verhaeltnis zur Erfahrung, zur Spekulation, und zum Leben*, Mittler, Berlin, 1833.
- [8] BENEKE, FRIEDRICH EDUARD, *System der Logik als Kunstlehre des Denkens*, F. Dümmler, Berlin, 1842.
- [9] BLOOR, DAVID, ‘What can the sociologist of knowledge say about $2+2=4$?’, in P. Ernest, (ed.), *Mathematics, Education and Philosophy*, Falmer, London, 1994, pp. 21–32.
- [10] BRINGSJORD, SELMER, and YINGRUI YANG, ‘Cognitive illusions and the welcome psychologism of logicist artificial intelligence’, in Dale Jacquette, (ed.), *Philosophy, Psychology, and Psychologism*, Kluwer Academic Press, Dordrecht, 2003, pp. 289–312.
- [11] BROCKHAUS, RICHARD, ‘Realism and psychologism in 19th century logic’, *Philosophy and Phenomenological Research*, 51 (1991), 493–524.
- [12] BROOK, ANDREW, *Kant and the Mind*, Cambridge UP, Cambridge, 1994.
- [13] CHERNIAK, CHRISTOPHER, *Minimal Rationality*, MIT Press, Cambridge, MA, 1986.

- [14] COHEN, L. JONATHAN, 'Can human rationality be demonstrated experimentally?', *Behavioral and Brain Science*, 4 (1981), 317–370.
- [15] COLLINS, ARTHUR, *Possible Experience*, Univ. California Press, Berkeley, 1992.
- [16] COSMIDES, LIDA, and JOHN TOOBY, 'Cognitive adaptations for social exchange', in J. Barkow, L. Cosmides, and J. Tooby, (eds.), *The Adapted Mind: Evolutionary Psychology and the Generation of Culture*, Oxford UP, NY, 1992, pp. 163–228.
- [17] CUMMINS, DENISE, 'The evolutionary roots of intelligence and rationality', in Renée Elio, (ed.), *Common Sense, Reasoning, and Rationality*, Oxford UP, Oxford, 2002, pp. 132–147.
- [18] DE PIERRIS, GRACIELA, 'The constitutive a priori', in Philip Hanson, and Bruce Hunter, (eds.), *Return of the A Priori, Canadian Journal of Philosophy Supplementary Volume 18*, 1993, pp. 179–214.
- [19] ERDMANN, BENNO, *Logik, Band 1: Logische Elementarlehre*, M. Niemeyer, Halle, 1892.
- [20] ERDMANN, JOHANN, *Grundriß der Geschichte der Philosophie*, Hertz, Berlin, 1866.
- [21] EVANS, JONATHAN ST. B. T., STEPHEN NEWSTEAD, and RUTH BYRNE, *Human Reasoning: The Psychology of Deduction*, Lawrence Erlbaum and Associates, Hillsdale, NJ, 1993.
- [22] FALTINGS, BOI, 'A theory of qualitative kinematics in mechanisms', in *IJCAI*, 1987, pp. 436–442.
- [23] FIRTH, RODERICK, 'Ethical absolutism and the ideal observer', in W. Sellars, and J. Hospers, (eds.), *Readings in Ethical Theory*, Prentice-Hall, Englewood Cliffs, NJ, 1970, pp. 200–221.
- [24] FODOR, JERRY, *The Language of Thought*, Crowell, Scranton, 1975.
- [25] FOLEY, RICHARD, *The Theory of Epistemic Rationality*, Harvard UP, Cambridge, MA, 1987.
- [26] FORBUS, KENNETH, 'Qualitative reasoning about space and motion', in Dedre Gentner, and Albert Stevens, (eds.), *Mental Models*, Lawrence Erlbaum Associates, 1983, pp. 53–73.
- [27] FREGE, GOTTLOB, *Die Grundlagen der Arithmetik*, Koebner, Breslau, 1884. Translation by John Austin as *The Foundations of Arithmetic* published 1953 by Oxford UP, Oxford. Page references to the 1960 Harper Torchbook reprint.
- [28] FREGE, GOTTLOB, *Grundgesetze der Arithmetik, Band I*, Pohle, Jena, 1893. Translated by M. Furth as *The Basic Laws of Arithmetic*, 1967, University of California Press, Berkeley.
- [29] FREGE, GOTTLOB, 'Review of E.G. Husserl *Philosophie der Arithmetik*, I', *Zeitschrift für Philosophie und philosophische Kritik*, 103 (1894), 313–332. Translation in [67, 195–209].
- [30] FREGE, GOTTLOB, 'Der Gedanke: Eine logische Untersuchung', *Beiträge zur Philosophie des deutschen Idealismus*, (1918), 58–77. Translated by P. Geach and R. Stoothoff as 'Thoughts' in [67, 351–372].
- [31] FREGE, GOTTLOB, 'On Euclidean geometry', in M. Beaney, (ed.), *The Frege Reader*, Blackwell, Oxford, 1997, pp. 251–252.
- [32] FRIEDMAN, MICHAEL, *Kant and the Exact Sciences*, Harvard UP, Cambridge, 1992.

- [33] GEORGE, ROLF, 'Psychologism in logic: Bacon to Bolzano', in Dale Jacquette, (ed.), *Philosophy, Psychology, and Psychologism*, Kluwer Academic Press, Dordrecht, 2003, pp. 21–49.
- [34] GIGERENZER, GERD, 'How to make cognitive illusions disappear: Beyond "heuristics and biases"', *European Review of Social Psychology*, 2 (1991), 83–115.
- [35] GIGERENZER, GERD, and DANIEL GOLDSTEIN, 'Reasoning the fast and frugal way: Models of bounded rationality', *Psychological Review*, 106 (1996), 650–669.
- [36] GINZBERG, MATTHEW, 'AI and nonmonotonic reasoning', in D. Gabbay, C. Hogger, and J.A. Robinson, (eds.), *Handbook of Logic in Artificial Intelligence and Logic Programming: Vol. 3: Nonmonotonic Reasoning and Uncertain Reasoning*, Oxford UP, Oxford, 1993, pp. 1–33.
- [37] GODDEN, DAVID, 'Psychologism in the logic of John Stuart Mill: Mill on the subject matter and foundations of ratiocinative logic', *Journal of the History and Philosophy of Logic*, 26 (2005), 115–143.
- [38] GRZEGORCZYK, ANDRZEJ, *Logic — A Human Affair*, Scholar Press, Warsaw, 1997.
- [39] HAACK, SUSAN, *Philosophy of Logics*, Cambridge UP, Cambridge, 1978.
- [40] HANNA, ROBERT, 'Logical cognition: Husserl's *Prolegomena* and the truth in psychologism', *Philosophy and Phenomenological Research*, 53 (1993), 251–276.
- [41] HANNA, ROBERT, *Rationality and Logic*, MIT Press, Cambridge, 2006.
- [42] HARMAN, GIL, 'Internal critique: A logic is not a theory of reasoning and a theory of reasoning is not a logic', in D. Gabbay, R. Johnson, H-J. Ohlbach, and J. Woods, (eds.), *Handbook of the Logic of Argument and Inference: The Turn Towards the Practical*, Elsevier Science, Amsterdam, 2002, pp. 171–186.
- [43] HARMAN, GIL, 'The logic of ordinary language', in Renée Elio, (ed.), *Common Sense, Reasoning, and Rationality*, Oxford UP, Oxford, 2002, pp. 93–103.
- [44] HAYES, PATRICK, 'The naïve physics manifesto', in D. Mitchie, (ed.), *Expert Systems in the Micro-Electronic Age*, Edinburgh UP, Edinburgh, 1978, pp. 242–270.
- [45] HAYES, PATRICK, 'The second naïve physics manifesto', in *Formal Theories of the Commonsense World*, Ablex, Norwood, NJ, 1985, pp. 1–36.
- [46] HUME, DAVID, *A Treatise of Human Nature*, Oxford UP, Oxford, 1740. Edited by P.H. Nidditch, 1978.
- [47] HUSSERL, EDMUND, *Logische Untersuchungen, Band 1*, Halle, Berlin, 1900. Section references to the translation (of the 1913 2nd Edition) by J.M. Findlay, 1970, as *Logical Investigations, Vol. 1*, Routledge and Kegan Paul, London.
- [48] HUSSERL, EDMUND, *Ideen zu einer reinen Phänomenologie and phänomenologischen Philosophie, Vol. 1*, Halle, Berlin, 1913. Translated by Boyce Gibson as *Ideas: General Introduction to Pure Phenomenology*, London: George Allen & Unwin, 1931.
- [49] JACQUETTE, DALE, 'Psychologism the philosophical shibboleth', *Philosophy and Rhetoric*, 30 (1997), 312–331. Reprinted in [50, 1-19]; page references to this reprint.
- [50] JACQUETTE, DALE, (ed.) *Philosophy, Psychology, and Psychologism*, Kluwer Academic Press, Dordrecht, 2003.
- [51] JOHNSON-LAIRD, PHILIP, *Mental Models*, Harvard UP, Cambridge, 1983.
- [52] JOHNSON-LAIRD, PHILIP, 'Mental models and deduction', *Trends in Cognitive Science*, 5 (2001), 434–442.

- [53] JOHNSON-LAIRD, PHILIP, and RUTH BYRNE, *Deduction*, Lawrence Erlbaum Assoc., East Sussex, 1991.
- [54] KAHNEMAN, DAVID, PETER SLOVIC, and AMOS TVERSKY, *Judgment Under Uncertainty: Heuristics and Biases*, Cambridge UP, Cambridge, 1982.
- [55] KANT, IMMANUEL, *Critique of Pure Reason*, Hackett Pub. Co., Indianapolis, 1996, 1787. Translated by Werner Pluhar.
- [56] KANT, IMMANUEL, *Logic*, Bobbs-Merrill, New York, 1996, 1800. Translated by R.S. Hartman and W. Swartz.
- [57] KITCHER, PATRICIA, *Kant's Transcendental Psychology*, Oxford UP, Oxford, 1990.
- [58] KITCHER, PHILIP, *The Nature of Mathematical Knowledge*, Oxford UP, New York, 1984.
- [59] KNEALE, WILLIAM, and MARY KNEALE, *The Development of Logic*, Oxford UP, Oxford, 1962.
- [60] KÖLBEL, MAX, *Truth Without Objectivity*, Routledge, New York, 2002.
- [61] KUSCH, MARTIN, *Psychologism*, Routledge, London, 1995.
- [62] LANGE, FRIEDRICH, *Logische Studien*, Baedeker, Leipzig, 1877.
- [63] LIPPS, THEODOR, *Grundzüge der Logik*, Voss, Hamburg, 1893.
- [64] LOCKE, JOHN, *An Essay Concerning Human Understanding*, Clarendon Press, Oxford, 1690. Edited by P.H. Nidditch, 1975.
- [65] MACFARLANE, JOHN, 'Frege, Kant, and the logic in logicism', *Philosophical Review*, 111 (2002), 25–65.
- [66] MACNAMARA, JOHN, *A Border Dispute*, MIT Press, Cambridge, 1986.
- [67] MCGUINNESS, BRIAN, (ed.) *Gottlob Frege: Collected Papers on Mathematics, Logic, and Philosophy*, Blackwell, Oxford, 1984.
- [68] MERRICK, TERI, 'What Frege meant when he said: Kant is right about geometry', *Philosophia Mathematica*, 14 (2006), 44–75.
- [69] MILL, JOHN STUART, *A System of Logic Ratiocinative and Inductive*, Parker, London, 1843. Section references to 8th Edition, 1891, reprinted 1967.
- [70] MILL, JOHN STUART, *An Examination of Sir William Hamilton's Philosophy*, Longmans, Green and Co., London, 1865. Chapter references to 6th Edition, 1889.
- [71] OAKSFORD, MICHAEL, and NICK CHATER, 'Commonsense reasoning, logic, and human rationality', in Renée Elio, (ed.), *Common Sense, Reasoning, and Rationality*, Oxford UP, Oxford, 2002, pp. 174–214.
- [72] OWEN, DAVID, *Hume's Reason*, Oxford UP, Oxford, 2002.
- [73] PARSONS, CHARLES, 'The transcendental aesthetic', in Paul Guyer, (ed.), *The Cambridge Companion to Kant*, Cambridge UP, Cambridge, 1992, pp. 62–100.
- [74] PARTEE, BARBARA, 'Semantics – mathematics or psychology?', in R. Baeuerle, U. Egli, and A. von Stechow, (eds.), *Semantics from Different Points of View*, Springer-Verlag, Berlin, 1979, pp. 1–14.
- [75] PELLETIER, FRANCIS JEFFRY, and RENÉE ELIO, 'What should default reasoning be, by default?', *Computational Intelligence*, 17 (1997), 165–187.
- [76] PELLETIER, FRANCIS JEFFRY, and RENÉE ELIO, 'The case for psychologism in default and inheritance reasoning', *Synthese*, 146 (2005), 7–35.
- [77] PHILIPSE, HERMAN, 'Psychologism and the prescriptive function of logic', in M.A. Notturmo, (ed.), *Perspectives on Psychologism*, E.J. Brill, Leiden, 1989, pp. 58–73.

- [78] POLLOCK, JOHN, *How to Build a Person*, MIT Press, Cambridge, 1989.
- [79] QUINE, WILLARD V., *Ontological Relativity and Other Essays*, chap. “Epistemology Naturalized”, Columbia UP, New York, 1969, pp. 69–90.
- [80] QUINE, WILLARD V., *Philosophy of Logic*, Prentice Hall, Englewood Cliffs, NJ, 1970.
- [81] RIPS, LANCE, *The Psychology of Proof: Deduction in Human Thinking*, MIT Press, Cambridge, 1994.
- [82] RODE, CATRIN, LIDA COSMIDES, WOLFGANG HELL, and JOHN TOOBY, ‘When and why do people avoid unknown probabilities in decisions under uncertainty? Testing some predictions from optimal foraging theory’, *Cognition*, 72 (1999), 269–304.
- [83] RUSSELL, STUART, ‘Rationality and intelligence’, *Artificial Intelligence*, 94 (1997), 57–77. Reprinted in Renée Elio (ed.) *Common Sense, Reasoning, and Rationality*, Oxford: Oxford UP, 2002, pp. 37–59.
- [84] SIGWART, CHRISTOPH, *Logik, Band 1: Die Lehre vom Urteil, vom Begriff und vom Schluss*, H. Laupp, Tübingen, 1878.
- [85] SIMON, HERB, ‘A behavioral model of rational choice’, *Quarterly Journal of Economics*, 69 (1955), 99–118.
- [86] SIMON, HERB, *Models of Bounded Rationality*, MIT Press, Cambridge, 1982.
- [87] SKORUPSKI, JOHN, ‘Mill on language and logic’, in John Skorupski, (ed.), *The Cambridge Companion to Mill*, Cambridge UP, Cambridge, 1998, pp. 35–56.
- [88] SPERBER, DAN, *Explaining Culture: A Naturalistic Approach*, Blackwell, Oxford, 1996.
- [89] TILES, MARY, ‘Kant: From general to transcendental logic’, in Dov Gabbay, and John Woods, (eds.), *Handbook of the History of Logic, Vol. 3. The Rise of Modern Logic: From Leibniz to Frege*, Elsevier, Amsterdam, 2004, pp. 85–130.
- [90] WHATELY, RICHARD, *The Elements of Logic*, J. Mawman, London, 1826.
- [91] WUNDT, WILHELM, *Logik, Band 1: Allgemeine Logik und Erkenntnistheorie*, F. Enke, Stuttgart, 1880.

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