Russell vs. Frege on Definite Descriptions as Singular Terms[1]

Francis Jeffry Pelletier — Simon Fraser University
Bernard Linsky — University of Alberta

1. Introduction

In "On Denoting" and to some extent in "Review of Meinong and Others, *Untersuchungen zur Gegenstandstheorie und Psychologie*" published in the same issue of *Mind* in 1905, Russell presents not only his famous elimination (or contextual definition) of definite descriptions, but also a series of considerations against understanding definite descriptions as singular terms. At the end of "On Denoting", Russell believes he has shown that all the theories that do treat definite descriptions as singular terms fall logically short: Meinong's, Mally's, his own earlier (1903) theory, and Frege's. (He also believes that at least some of them fall short on other grounds – epistemological and metaphysical – but we will not discuss these criticisms except in passing).

Our aim in the present paper is to discuss whether his criticisms actually refute Frege's theory. We will first attempt to specify just what Frege's theory is, and present the evidence that has moved scholars to attribute one of three different theories to Frege, in this area. We think that each of these theories has some claim to be Fregean, even though they are logically quite different from each other. This raises the issue of determining Frege's attitude towards these three theories. We will consider whether he changed his mind and came to replace one theory by another, or whether he perhaps thought that the different theories applied to different realms, for example to natural language vs. a language for formal logic and arithmetic. We will not come to any hard and fast conclusion here, but instead will just note that all these theories treat definite descriptions as singular terms, and that Russell proceeds as if he has refuted them all.

After a brief look at the formal properties of the Fregean theories (particularly the logical status of various sentences containing non-proper definite descriptions) and comparing them to Russell's theory in this regard, we turn to Russell's actual criticisms in the above-mentioned articles to examine the extent to which the criticisms hold. Our conclusion will be that, even if

---

[1] We thank many people for discussions of the topic of this paper, especially Harry Deutsch, Mike Harnish, Greg Landini, James Levine, Nathan Salmon, an anonymous referee, and the audience at the Russell-Meinong conference at McMaster University, May 2005.

the criticisms hold against *some* definite-descriptions-as-singular-terms theories, they do not hold against Frege, at least not in the form they are given.

2. Three Fregean Theories of Definite Descriptions

We start with three types of theories that have been attributed to Frege, often without acknowledgment of the possibility of the other theories.[2] Frege's views on definite descriptions are contained pretty much exclusively[3] in his 1892 "Über Sinn und Bedeutung" and his 1893 *Grundgesetze der Arithmetik, Vol. 1*.[4] In both these works Frege strove to make definite descriptions be singular terms, by which we mean that they are not only syntactically singular but also that they behave semantically like such paradigmatic proper names as 'Rudolf Carnap' in designating some item of the domain of discourse. Indeed, Frege claims that definite descriptions *are* proper names: "The *Bezeichnung* <indication> of a single object can also consist of several words or other signs. For brevity, let every such *Bezeichnung* be called a proper name" (1892, p. 57). And although this formulation does not explicitly include definite descriptions (as opposed, perhaps, to compound proper names like 'Great Britain' or 'North America'), the examples he feels free to use (e.g., 'the least rapidly convergent series', 'the negative square root of 4') make it clear that he does indeed intend that definite descriptions are to be included among the proper names. In discussing the 'the negative square root of 4', Frege says "We have here the case of a compound proper name constructed from the expression for a concept with the help of the singular definite article" (1892, p.71).

a. A Frege-Strawson Theory

In "Über Sinn und Bedeutung" Frege considered a theory in which names without *Bedeutung* might nonetheless be used so as to give a *Sinn* to sentences employing them. He remarks,

> It may perhaps be granted that every grammatically well-formed expression figuring as a proper name always has a *Sinn*. But this is not to say that to the *Sinn*

[2] But we will not attempt to make careful attributions in this regard.
[3] An exception is a longish aside in Frege's *Grundlagen* (1884) §74 fn.1. The theory he puts forward there is rather different from the ones we will consider in this paper. For further details see our "What is Frege's Theory of Descriptions?"
[4] In our quotations, we leave *Bedeutung*, *Sinn*, and *Bezeichnung* (and cognates) untranslated in order to avoid the confusion that would be brought on by using 'nominatum', 'reference', and 'meaning' for *Bedeutung*. We got this idea from Russell's practice in his reading notes (see Linsky 2004/5). Otherwise we follow Max Black's translation of "Über Sinn und Bedeutung" and Montgomery Furth's translation of *Grundgesetze der Arithmetik*.

> there also corresponds a *Bedeutung*. The words 'the celestial body most distant from the Earth' have a *Sinn*, but it is very doubtful they also have a *Bedeutung*. …In grasping a *Sinn*, one is certainly not assured of a *Bedeutung*. (1892, p.58)

> Is it possible that a sentence as a whole has only a *Sinn*, but no *Bedeutung*? At any rate, one might expect that such sentences occur, just as there are parts of sentences having *Sinn* but no *Bedeutung*. And sentences which contain proper names without *Bedeutung* will be of this kind. The sentence 'Odysseus was set ashore at Ithaca while sound asleep' obviously has a *Sinn*. But since it is doubtful whether the name 'Odysseus,' occurring therein, has a *Bedeutung*, it is also doubtful whether the whole sentence does. Yet it is certain, nevertheless, that anyone who seriously took the sentence to be true or false would ascribe to the name 'Odysseus' a *Bedeutung*, not merely a *Sinn*; for it is of the *Bedeutung* of the name that the predicate is affirmed or denied. Whoever does not admit a *Bedeutung* can neither apply nor withhold the predicate. (1892, p.62)

> The thought loses value for us as soon as we recognize that the *Bedeutung* of one of its parts is missing….But now why do we want every proper name to have not only a *Sinn*, but also a *Bedeutung*? Why is the thought not enough for us? Because, and to the extent that, we are concerned with its truth-value. This is not always the case. In hearing an epic poem, for instance, apart from the euphony of the language we are interested only in the *Sinn* of the sentences and the images and feelings thereby aroused….Hence it is a matter of no concern to us whether the name 'Odysseus,' for instance, has a *Bedeutung*, so long as we accept the poem as a work of art. It is the striving for truth that drives us always to advance from the *Sinn* to *Bedeutung*. (1892, p.63)

It seems pretty clear that Frege here is not really endorsing a theory of language where there might be "empty names", at least not for use in any "scientific situation" where we are inquiring after truth; nonetheless, it could be argued that this is his view of "ordinary language as it is"—there are meaningful singular terms (both atomic singular terms like 'Odysseus' and compound ones like 'the author of *Principia Mathematica*') which do not *bedeuten* an individual. And we can imagine what sort of theory of language is suggested in these remarks: a Frege-Strawson theory[5] in which these empty names are treated as having meaning (having *Sinn*) but designating nothing (having no *Bedeutung*), and sentences containing them are treated as themselves meaningful (have *Sinn*) but having no truth value (no *Bedeutung*) – the sentence is neither true nor false. As Kaplan (1972) remarks, if one already had such a theory for "empty" proper names, it would be natural to extend it to definite descriptions and make improper definite descriptions also be meaningful (have *Sinn*) and sentences containing them be treated as themselves meaningful (have *Sinn*) but as having no truth value (no *Bedeutung*).

[5] The name 'Frege-Strawson' for this theory is due to Kaplan, 1972, thinking of Strawson (1950, 1952).

A natural formalization of such a theory is given by “free logics”, which in general allow singular terms not to denote anything in the domain, thereby making sentences containing these be truth-valueless. (See Lambert & van Fraassen 1967, Lehmann 1994, and especially Moscher & Simons 2001 for a survey and recommendation of which free logic Frege should adopt). In these latter theories, there is a restriction on the rules of inference that govern (especially) the quantifiers and the identity sign, so as to make them accord with this semantic characterization of the logic. Even though Frege does not put forward the Frege-Strawson theory in his formalized work on the foundations of mathematics, it has its own interesting formal features, which we will mention below. And some theorists think of this theory as accurately describing Frege’s attitude toward natural language.

b. A Frege-Carnap Theory

Frege also mentions a “chosen object” theory in “Über Sinn und Bedeutung”. In initiating this discussion Frege gives his famous complaint (1892, p.69): “Now, languages have the fault of containing expressions which fail to *bezeichnen* an object (although their grammatical form seems to qualify them for that purpose) because the truth of some sentence is a prerequisite,” giving the example ‘Whoever discovered the elliptic form of the planetary orbits died in misery’, where he is treating ‘whoever discovered the elliptic form of planetary orbits’ as a proper name that depends on the truth of ‘there was someone who discovered the elliptic form of the planetary orbits’. He continues:

> This arises from an imperfection of language, from which even the symbolic language of mathematical analysis is not altogether free; even there combinations of symbols can occur that seem to *bedeuten* something but (at least so far) are without Bedeutung <*bedeutungslos*>, e.g., divergent infinite series. This can be avoided, e.g., by means of the special stipulation that divergent infinite series shall *bedeuten* the number 0. A logically perfect language (*Begriffsschrift*) should satisfy the conditions, that every expression grammatically well constructed as a proper name out of signs already introduced shall in fact *bezeichne* an object, and that no new sign shall be introduced as a proper name without being secured a *Bedeutung*. (1892, p.70)

In discussing the ‘the negative square root of 4’, Frege says

> We have here the case of a compound proper name constructed from the expression for a concept with the help of the singular definite article. This is at any rate permissible if one and only one single object falls under the concept. <footnote> In accordance with what was said above, an expression of the kind in question must actually always be assured of a *Bedeutung*, by means of a special

> stipulation, e.g., by the convention that its *Bedeutung* shall count as 0 when the concept applies to no object or to more than one. (1892, p.71)

Frege is also at pains to claim that it is not part of the "asserted meaning" of these sorts of proper names that there is a *Bedeutung*; for, if it were, then negating such a sentence would not mean what we ordinarily take it to mean. Consider again the example 'Whoever discovered the elliptic form of the planetary orbits died in misery' and the claim that 'whoever discovered the elliptic form of planetary orbits' in this sentence depends on the truth of 'there was a unique person who discovered the elliptic form of the planetary orbits'. If the sense of 'whoever discovered the elliptic form of planetary orbits' included this thought, then the negation of the sentence would be 'Either whoever discovered the elliptic form of the planetary orbits did not die in misery or there was no unique person who discovered the elliptic form of the planetary orbits.' And he takes it as obvious (1892, p.70) that the negation is not formed in this way.[6]

Securing a *Bedeutung* for all proper names is an important requirement, not just in the case of abstract formal languages, but even in ordinary discourse; failure to adhere to it can lead to immeasurable harm.

> The logic books contain warnings against logical mistakes arising from the ambiguity of expressions. I regard as no less pertinent a warning against proper names without any *Bedeutung*. The history of mathematics supplies errors which have arisen in this way. This lends itself to demagogic abuse as easily as ambiguity does–perhaps more easily. 'The will of the people' can serve as an example; for it is easy to establish that there is at any rate no generally accepted *Bedeutung* for this expression. It is therefore by no means unimportant to eliminate the source of these mistakes, at least in science, once and for all. (1892, p.70)

These are the places that Frege puts forward the Frege-Carnap theory.[7] It will be noted that there is no formal development of these ideas (nor of any other ideas) in "Über Sinn und Bedeutung"; but the theory has been developed in Kalish & Montague (1964). It remains, however, a bit of a mystery as to why Frege comes to put both theories into his article without remarking on their differences. Does the Frege-Strawson theory perhaps apply to natural language while the Frege-Carnap theory applies to formal languages? Perhaps, but if so, what are we to make of the different theory proposed in the *Grundgesetze*? We will compare the formal properties of the

[6] Contrary, perhaps, to Russell's opinion as to what is and isn't obvious.
[7] Once again, the name is due to Kaplan (1972), referring to Carnap (1956), especially pp. 32-38.

Frege-Strawson and Frege-Carnap theories below, and also compare both these with the *Grundgesetze* theory.

c. The Frege-Grundgesetze Theory

In the 1893 *Grundgesetze*, where Frege develops his formal system, he also finds room for definite descriptions…although his discussion is disappointingly short. The relevant part of the *Grundgesetze* is divided into two subparts: a rather informal description that explains how all the various pieces of the language are to be understood, and a more formal statement that includes axioms and rules of inference for these linguistic entities.[8]

Frege maintains the central point of the Frege-Carnap theory that he had put forward in "Über Sinn und Bedeutung" by proclaiming (*Grundgesetze*, §28) "the following leading principle: *Correctly-formed names must always bedeuten something*", and (§33) "every name correctly formed from the defined names must have a *Bedeutung*".

In the *Grundgesetze*, Frege uses the symbols ‘ ’εFε’ to indicate the course of values (*Werthverlauf*) of F, that is, the set of things that are F. The *Grundgesetze* §11 introduces the symbol ‘\ξ’, which is called the "substitute for the definite article".[9] He distinguishes two cases:

1. If to the argument there corresponds an object Δ such that the argument is ’ε(Δ=ε), then let the value of the function \ξ be Δ itself;
2. If to the argument there does not correspond an object Δ such that the argument is ’ε(Δ=ε), then let the value of the function be the argument itself.

And he follows this up with the exposition:

> Accordingly \’ε (Δ=ε) = Δ is the True, and "\’εΦ(ε)" *bedeutet* the object falling under the concept Φ(ξ), if Φ(ξ) is a concept under which falls one and only one object; in all other cases "\’εΦ(ε)" *bedeutet* the same as "’εΦ(ε)".

He then gives as examples (a) "the item when increased by 3 equals 5" designates 2, because 2 is the one and only object that falls under the concept *being equal to 5 when increased by 3*, (b) the concept *being a square root of 1* has more than one object falling under it, so "the square root of 1" designates ’ε(ε^2=1)[10], (c) the concept *not identical with itself* has no object falling under it, so

---

[8] Amazingly, in the formal part of the *Grundgesetze* there is no axiom covering the case of improper descriptions! For further discussion of this and of the formal problems with the *Grundgesetze* theory see our "What is Frege's Theory of Descriptions?"

[9] Morscher & Simons (2001: 20) take this turn of phrase to show that Frege did not believe that he was giving an analysis of natural language. To us, however, the matter does not seem so clear: How else would Frege have put the point if in fact he *were* trying to give a logical analysis of the natural language definite article?

[10] That is, it *bedeutet* the course of values of "is a square root of 1", i.e., the set {-1,1}.

it designates $'\varepsilon(\varepsilon\neq\varepsilon)$[11]; and (d) “the *x* plus 3” designates $'\varepsilon(\varepsilon+3)$[12] because *x plus 3* is not a concept at all (it is a function with values other than the True and the False). In the concluding paragraph of this section, Frege says his proposal has the following advantage:

> There is a logical danger. For, if we wanted to form from the words “square root of 2” the proper name “the square root of 2” we should commit a logical error, because this proper name, in the absence of further stipulation, would be ambiguous, hence even without *Bedeutung* <*bedeutungslos*>. … And if we were to give this proper name a *Bedeutung* expressly, this would have no connection with the formation of the name, and we should not be entitled to infer that it was a positive square root of 2, while yet we should be only too inclined to conclude just that. This danger about the definite article is here completely circumvented, since “\’εΦ(ε)” always has a *Bedeutung*, whether the function Φ(ξ) be not a concept, or a concept under which falls no object or more than one, or a concept under which falls exactly one object. (*Grundgesetze* §11, pp. 50-51)

There seem to be two main points being made here. First, there is a criticism of the Frege-Carnap theory on the grounds that in such a theory the stipulated entity assigned to “ambiguous” definite descriptions “would have no connection to the formation of the name.” This would pretty clearly suggest that Frege’s opinion in *Grundgesetze* was *against* the Frege-Carnap view of definite descriptions. And second there is the apparent claim that in his theory, *the* square root of 2 is *a* square root of 2, or more generally that the denotation of improper descriptions, at least in those cases where the description is improper due to there being more than one object that satisfies the predicate, manifests the property mentioned in the description.

At this point there is a mismatch between Frege’s theory and his explanation of the theory. On this theory, in fact the square root of 2 is *not* a square root of 2 – it is a course of values, that is to say, a set. So, on the *Grundgesetze* theory, it looks like we *cannot* “infer that it [the square root of 2] was a positive square root of 2” even though “we should be only too inclined to conclude just that.” [On Frege’s behalf, however, we could point out that everything *in* (= which is a member of) that course of values will be *a* square root of 2; so there is *some* connection between the object that the definite description refers to and the property used in the description. But the course of values itself will not be a square root of 2. Thus, the connection won’t be as close as saying that the *Bedeutung* of ‘the F’ *is* an F.]

[11] The course of values of “is non-self-identical”, i.e., the empty set.
[12] The course of values of the function of adding 3, that is, the set of things to which three has been added.

Is that which we are "only too inclined to conclude" something that we in fact shouldn't? But if so, why is this an objection to the proposal to just stipulate some arbitrary object to be the *Bedeutung*? We don't know what to make of Frege's reason to reject the Frege-Carnap account in this passage, since his apparent reason is equally a reason to reject the account being recommended.

At any rate, this is the general outline of Frege's theory in the *Grundgesetze*. We will compare the logical properties of this theory with those of the Frege-Strawson and the Frege-Carnap theories below.

3. To What do the Different Theories Apply?

It is not clear to us whether Frege intended the Frege-Carnap and Frege-*Grundgesetze* theories to apply to different realms: the "Über Sinn und Bedeutung" theory perhaps to a formalized version of natural language and the *Grundgesetze* theory to a formal account of mathematics maybe? Frege himself never gives an explicit indication of this sort of distinction between realms of applicability for his theories of descriptions, although it is very easy to see him as engaging simultaneously in two different activities: constructing a suitable framework for the foundations of mathematics, and then a more leisurely reflection on how these same considerations might play out in natural language.

Various attitudes are possible here; for example, one who held that the Frege-Strawson theory represented Frege's attitude to natural language semantics would want to say that *both* the Frege-Carnap and the Frege-*Grundgesetze* theories were relevant only to the formal representation of arithmetic. This then raises the issue of how such a view would explain why Frege gave *both* the Carnap and the *Grundgesetze* theories for arithmetic. Possibly, this attitude might maintain, the *Grundgesetze* theory was Frege's "real" account for arithmetic, but in "Über Sinn und Bedeutung" he felt it inappropriate to bring up such a complex theory (with its courses of values and the like) in those places where he was concerned to discuss formal languages—as opposed to those places where he was discussing natural language (and where he put forward the Frege-Strawson account). So instead he merely mentioned a "simplified version" of his theory. In this sort of picture, the Frege-Carnap theory is an inappropriate account of Frege's views, since it is a mere simplified account meant only to give non-formal readers something to fasten on while he was discussing an opposition between natural languages and *Begriffsschriften*. According to this attitude, the real theories are Strawson for natural language and *Grundgesetze* for arithmetic.

Another attitude has Frege being a language reformer, one who wants to replace the bad natural language features of definite descriptions with a more logically tractable one. In this attitude, Frege *never* held the Strawson view of natural language. His talk about Odysseus was just to convince the reader that natural language was in need of reformation. And he then proposed the Frege-Carnap view as preferable in this reformed language. According to one variant of this view, Frege thought that the Carnap view was appropriate for the reformed natural language while the *Grundgesetze* account was appropriate for mathematics. Another variant would have Frege offer the Carnap view in "Über Sinn und Bedeutung" but replace it with a view he discovered later while writing the *Grundgesetze*. As evidence for this latter variant, we note that Frege did seem to reject the Carnap view when writing the *Grundgesetze*, as we discussed above. However, a consideration against this latter variant is that Frege would most likely have written the relevant portion of the *Grundgesetze* before writing "Über Sinn und Bedeutung". And a consideration against the view as a whole in both of its variants is that Frege never seems to suggest that he is in the business of reforming natural language.

One might assume that "Über Sinn und Bedeutung", because it was published in 1892, would be an exploratory essay, and that the hints there of the Frege-Carnap view were superseded by the final, official *Grundgesetze* view. Yet clearly the *Grundgesetze* was the fruit of many years' work, and it is hard to imagine that by 1892 Frege had not even proved his Theorem 1 of *Grundgesetze*, in which the description operator figures.[13]

4. Some Formal Features of the Three Theories

In this section we mention some of the consequences of the different theories, particularly we look at some of the semantically valid truths guaranteed in the different theories, as well as some valid rules of inference.

---

[13] On the other hand, it might be noted that in the Introduction to the *Grundgesetze* (p.6) Frege remarks that "a sign meant to do the work of the definite article in everyday language" is a new primitive sign in the present work. And it is of course well known that Frege says that he had to "discard an almost-completed manuscript" of the *Grundgesetze* because of internal changes brought about by the discovery of the *Bedeutung-Sinn* distinction.

To do this fully we should have a formal development of the three Fregean theories and the Russell theory before us. But we will not try to provide such a careful development and will rely instead on informal considerations of what the semantics will be for a theory that embraces the principles given in the last section for the different views on definite descriptions. With regards to Russell's approach, it is well-known what this theory is: classical first-order logic plus some method of eliminating descriptions (that we will discuss shortly). The Frege-Carnap theory is developed in Chapter 7 of Kalish & Montague (1964)[14], but we needn't know the details in order to semantically evaluate our formulas. All we need to do is focus on the sort of interpretations presumed by the theory: namely, those where every improper description designates the same one thing in the domain and this thing might also be designated in more ordinary ways. The Frege-*Grundgesetze* theory similarly can be conceived semantically as containing objects and courses-of-values of predicates (sets of things that satisfy the predicate). And we can informally evaluate the formulas simply by reflecting on these types of interpretations: improper descriptions designate the set of things that the formula is true of – which will be the empty set in the case of descriptions true of nothing, and will be the set of all instances in those cases where the descriptions are true of more than one item in the domain.[15] There might be many ways to develop a Frege-Strawson theory, but we concentrate on the idea that improper definite descriptions do not designate anything in the domain and that this makes sentences containing such descriptions be neither true nor false. This is the idea developed by (certain kinds of) free logics: atomic sentences containing improper descriptions are neither true nor false because the item designated by the description does not belong to the domain. (It might, for example, designate the domain itself, as in Simon & Morscher 2001, and Lehman 1994[16]). In a Frege-like

[14] And also as Chapter 6 in Kalish, Montague, & Mar (1980).

[15] There will be difficulties in giving a complete and faithful account of the Frege-*Grundgesetze* theory, since its formal development is contradictory, because of Basic Law V. Even trying to set this aside, there will be difficulties in giving an informal account of improper descriptions, because they are supposed to denote a set. And then this set must be in the domain. But we would then want to have principles in place to determine just what sets must be in a domain, given that some other sets are already in the domain. A simple example concerns the pair of descriptions ιxFx and ιx~Fx, when the descriptions are both improper. The former description is supposed to designate the set of things that are F, and the latter description is supposed to designate the complement of this set. But this leads directly to a contradiction, since it presumes the existence of the complement of any set. So we cannot have both descriptions be improper! None of this is discussed by Frege, and he offers no answers other than by his contradictory Basic Law V. Some of our evaluations of particular sentences will run afoul of this problem; but we will try to stick with the informal principles that Frege enunciates for this theory, and give these "intuitive" answers.

[16] Kalish, Montague, and Mar (1980), Chapter 8, have what they call a "Russellian" theory that is formally similar to this in that it takes "improper" terms to designate something outside the domain. But in this theory, all claims

development of this idea, we want the lack of a truth-value of a part to be inherited by larger units. Frege wants the *Bedeutung* of a unit to be a function of the *Bedeutungen* of its parts, and if a part has no *Bedeutung*, then the whole will not have one either. In the case of sentences, the *Bedeutung* of a sentence is its truth value, and so in a complex sentence, if a subsentence lacks a truth value, then so will the complex. In other words, the computation of the truth value of a complex sentence follows Kleene's (1952: 334) "weak 3-valued logic", where being neither true nor false is inherited by any sentence that has a subpart that is neither true nor false.[17]

We start by listing a series of formulas and argument forms to consider because of their differing interactions with the different theories. The formulas and answers given to them by our three different Fregean theories and Russell's theory are summarized in our TABLE. It should be noted that in this TABLE we are always talking about the case where the descriptions mentioned are improper. So, when looking at the formula under consideration, one must always take the description(s) therein to be improper.

Since we are always looking at the cases where the descriptions are improper, every interpretation we consider will be called an *i-interpretation* (for "improper description interpretation"). In an i-interpretation for a particular formula, all definite descriptions mentioned in the formula are improper. If it should turn out that the formula under consideration is false in every i-interpretation (of the sort relevant to the theory under consideration), then we will call it *i-false*, i.e., false in every interpretation for the theory where the descriptions mentioned are improper. Similarly, we call some formulas *i-true* if they are true in every i-interpretation that is relevant to the theory. Of course, if a formula is true (or false) in *every* interpretation (not restricted to i-interpretations) for the theory, then it will also be i-true (or i-false); in these cases we say that the formula is "logically true" or "logically false" in the theory. If a formula is true in some i-interpretations and false in other i-interpretations, then it is called *i-contingent*. Of course, an i-contingent formula is also simply contingent (without the restriction to i-interpretations). Similar considerations hold about the notion of *i-validity* and *i-invalidity*. An argument form is i-valid if and only if all i-interpretations where the premises are true also

---

involving such terms are taken to be false, rather than "neither true nor false". (It seems wrong to call this a "Russellian" theory, since singular terms are not eliminated. It might be more accurate to say that it is a theory that whose sentences have the same truth value as the Russellian sentences when the descriptions are eliminated.)

[17] There are certainly other 3-valued logics, but Frege's requirement that the *Bedeutung* of a whole be a function of the *Bedeutungen* of the parts requires the Kleene "weak" interpretation.

make the conclusion true. If an argument form is valid (with no restriction to i-interpretations), then of course it is also i-valid.

In the case of the Frege-Strawson theory, sentences containing an improper description are neither true nor false in an i-interpretation. We therefore call these *i-neither*. When we say that an argument form is *i-invalid** (with the *), we mean that in an i-interpretation the premise can be true while the conclusion is neither true nor false (hence, not true).

Things are not so simple in Russell's theory. For one thing, the formulas with definite descriptions have to be considered "informal abbreviations" of some primitive sentence of the underlying formal theory. And there can be more than one way to generate this primitive sentence from the given "informal abbreviation", depending on how the scope of the description is generated. If the scope is "widest", so that the existential quantifier corresponding to the description becomes the main connective of the sentence, then generally speaking[18], formulas with improper descriptions will be i-false because they amount to asserting the unique existence of a satisfier of the description and by hypothesis this is not satisfied. But often they will be contingent because there will be non-i-interpretations in which there is such an item and others in which there is not. But it could be that the description itself is contradictory and therefore the sentence is logically false, and hence i-false. Furthermore, there might be definite descriptions that are true of a unique object as a matter of logic, such as 'the object identical to *a*'; and in these latter cases, if the remainder of the sentence is "tautologous" then the sentence could be logically true…for example, "Either the object identical with Adam is a dog or the object identical with Adam is not a dog", whose wide scope representation would be (approximately) "There exists a unique object identical with Adam which either is a dog or is not a dog."[19]

We will therefore take all descriptions in Russell's theory to have narrow scope, and so our claims in the TABLE about i-truth, i-falsity, i-contingency, i-validity, and i-invalidity should be seen as discussing the disambiguation of the "informal abbreviation" with narrowest scope for all the descriptions involved, and then assuming that these descriptions are improper.

---

[18] But not always; see for example #4 in our TABLE.

[19] *Principia Mathematica* had no individual constants, so this description could not be formed. It is not clear to us whether there is any formula that can express the claim that it is logically true that exactly one individual satisfies a formula, if there are no constants. Since Russell elsewhere thinks that proper names of natural languages are disguised descriptions, it is also not clear what Russell's views about forming these 'logically singular' descriptions in English might be.

One final remark should be made about the interpretation of the TABLE. It was our intent that the various Fs and Gs that occur in the formulas should be taken as variables or schema, so that *any* sentence of the form specified would receive the same judgment. But this won't work for some of our theories, since predicate substitution does not preserve logical truth in them. For example, in Frege-Strawson, if we substitute a complex predicate containing a non-denoting definite description for F in a logical truth, a logical falsehood, or a contingent formula, then the result becomes neither true nor false. So the Frege-Strawson theory does not preserve logical truth under predicate substitution. In Russell's theory, substitution of arbitrary predicates for the Fs and Gs can introduce complexity that interacts with our decision to eliminate all descriptions using the narrowest scope. For example, #3, when F stands for 'is a round square', generates Russell's paradigm instance of a false sentence: 'The round square is a round square'. And this is the judgment generated when we eliminate the definite description in Russell's way, and is what we have in our TABLE. But were we to uniformly substitute '~F' for 'F' in formula #3, we generate $\sim F\iota x{\sim}Fx$; and now eliminating the description by narrowest scope yields

$$\sim\exists x(\sim Fx \ \&\ \forall y(\sim Fy \supset x=y) \ \&\ Fx)$$

It can be seen that the formula inside the main parentheses is logically false (regardless of whether the description is or isn't proper), and so there can be no such *x*. And therefore the negation of this is logically true. Yet, we followed Russell's rule in decreeing that the original formula #3 is false when the description is eliminated with narrowest scope. This example shows that Russell's theory allows one to pass from logical falsehood to logical truth; adding another negation to the premise and conclusion will show that it does not preserve logical truth under predicate substitution, unless one is allowed to alter scope of description-elimination. Therefore, we are going to restrict our attention to the case where the Fs and Gs are atomic predicates in the theories under consideration here. And so we will not be able to substitute ~F for F in #3, with this restriction.

Table: How Four Theories of Descriptions View Some Arguments and Formulas

	Formula or Rule	**F-S**	**F-C**	**F-*Gz***	**Russell**
1	i) $\forall x Fx \therefore F\iota xGx$ ii) $F\iota xGx \therefore \exists xFx$ iii) $\forall xFx \supset F\iota xGx$ iv) $F\iota xGx \supset \exists xFx$	i-invalid* valid i-neither i-neither	valid valid logically true logically true	valid valid logically true logically true	i-invalid valid i-false log. true
2	$\exists y\ y = \iota xFx$	i-neither	logically true	logically true	i-false
3	$F\iota xFx$	i-neither	i-contingent	i-false	i-false
4	$(P \vee {\sim}P) \vee G\iota xFx$	i-neither	logically true	logically true	logically true
5	$F\iota xFx \vee {\sim}F\iota x{\sim}Fx$	i-neither	logically true	i-false	logically true
6	$(\exists y\forall x(Fx\equiv x=y) \supset F\iota xFx)$	i-neither	logically true	logically true	logically true
7	$G\iota xFx \vee {\sim}G\iota xFx$	i-neither	logically true	logically true	logically true
8	$\iota xFx = \iota xFx$	i-neither	logically true	logically true	i-false
9	$\iota x\ x\neq x = \iota x\ x\neq x$	i-neither	logically true	logically true	logically false
10	$\iota x\ x=x = \iota x\ x\neq x$	i-neither	logically true	i-false	logically false
11	$(\iota xFx=\iota xx\neq x)\vee(\iota x{\sim}Fx=\iota xx\neq x)$	i-neither	logically true	i-contingent	logically false
12	$(G\iota xFx\ \&\ G\iota x{\sim}Fx)\supset G\iota xGx$	i-neither	logically true	i-contingent	logically true
13	$(\iota xFx = \iota xGx) \supset G\iota xFx$	i-neither	i-contingent	i-false	logically true
14	$\forall x(Fx\equiv Gx) \therefore \iota xFx = \iota xGx$	i-invalid*	valid	valid	i-invalid
15	i) $\forall x(Sxa \equiv x=b)$ ii) $b = \iota xSxa$	Interderiv-able. Not i-equivalent	Not Interder-ivable; so not i-equivalent[20]	Not Interder-ivable; so, not i-equivalent[20]	Log. equiv-alent; hence interderivable

We do not intend to prove all the entries in the TABLE, but instead merely to indicate why a few chosen ones have the values they do and thereby give a method by which the reader can verify the others.

[20] In both Frege-Carnap and Frege-*Grundgesetze*, (15i) entails (15ii), but not conversely.

We start with the Frege-Strawson theory. The idea is that improper descriptions have *no* designation (at least, not in the universe of objects), and sentences containing such descriptions have no truth value. As stated, this principle would decree that, when ιxFx is improper, no formula containing such a description is either true or false. Hence all the entries of individual formulas in the TABLE will be i-neither. This is because of his insistence that the *Bedeutung* of the whole is determined by the *Bedeutungen* of the parts; and so if the parts are missing some *Bedeutungen* then so will the whole be missing its *Bedeutung*, even in formulas like #4.

Entries #1a and #14 are arguments, and in every i-interpretation where the premise is true the conclusion will be neither true nor false, and so we call it i-invalid*. Entry #1b is valid, because if the premise is true then the description must be proper and therefore designate some item in the domain. But in that case the conclusion must be true, making the argument valid. Note therefore that since #1d is not a theorem, it follows that the Frege-Strawson system does not have a deduction theorem. #15 is interesting because if #15i is true, then that is precisely the information required to show that the description is proper and so #15ii would follow; and if #15ii is true then the description is presupposed to be proper and hence #15i would follow. So #15i and #15ii are interderivable. But the biconditional formula between the two contains a definite description, and any description might be improper. So the two formulas cannot be i-equivalent. (Again a case where the deduction theorem fails).

A crucial feature of the Frege-Carnap theory is that there is always a referent for any description (#2), even ιx x≠x, and so this means that rules of universal instantiation and existential generalization can be stated in full generality; thus #1i and #1ii are valid, and furthermore the theory has a deduction theorem so that #1ii and #1iii are logically true. It follows from #2, which is just a restatement of the fundamental intuition, that formulas like #7 must be logically true. It also follows that a law of self-identity can be stated in full generality, and thus #8 and #9 are true. Furthermore, in Frege-Carnap, all improper descriptions designate the same item of the domain in any interpretation (namely, whatever the guaranteed-to-be-improper ιx x≠x does), and so formulas like #10 must be true and arguments like #14 must be valid.[21] We note that the crucial #3 is i-contingent. Note also that if the description is proper then #3 must be true;

[21] Morscher & Simons 2001: 21 call #14 “the identity of coextensionals” and say it is an “obvious truth” that should be honored by any theory of descriptions.

therefore #6 will turn out to be not just i-true, but logically true (since the antecedent guarantees the propriety of the description). For, in the improper case the description designates some element of the domain, and in some interpretations this element will be an F while in other interpretations it won't. The status of the other formulas can be established by arguments like the following for #5. #5 is certainly i-true in Frege-Carnap, because if both descriptions are improper, then they both designate the same item of the domain, and this item is either F or ~F. But furthermore, it is even true when at least one of the descriptions is proper, because then #3 will be in play for whichever description is proper and hence that disjunct will be true. And hence the entire disjunction will be true.

There are some difficulties of representing natural language in the Frege-Carnap theory. Consider #15i and ii, under the interpretation "Betty is Alfred's one and only spouse" and "Betty is the spouse of Alfred", represented as

#15i $\forall x(Sxa \equiv x=b)$

#15ii $b = \iota xSxa$.

While the two English sentences seem equivalent, the symbolized sentences are not, in Frege-Carnap: consider Alfred unmarried and Betty being the designated object. Then the first sentence is false but the second is true. Since they are not i-interderivable, they are therefore not i-equivalent, contrary to our intuitions about the natural language situation they are intended to represent.

Many of the logical features of the Frege-Carnap theory hold also in the Frege-*Grundgesetze* theory, since there is always a *Bedeutung* for every definite description: #2 holds, and so #7 holds. And once again, #6 will be not just i-true but logically true. Because the *Bedeutung* of a description is a function of what F is true of (as in the Frege-Carnap theory), #8 and #9 are i-true and #14 is valid. In Frege-Carnap the crucial #3 is i-contingent, while in Frege-*Grundgesetze* it is i-false. In Frege-Carnap, the improper description designates some "arbitrarily chosen" element of the domain, and in some interpretations this element will be F while in some other interpretations it will be ~F. But surprisingly, in the Frege-*Grundgesetze* theory, since the improper description designates the set consisting of all the elements that are F, this set cannot be F. For, to be F in the *Grundgesetze* theory is to be an element of the set $\{x:Fx\}$. If the improper ιxFx were to be F, then $\{x:Fx\}\in\{x:Fx\}$, which contradicts the axiom of foundation. So #6 has to be false for all improper descriptions. #13 will be i-false for much the same reason: the consequent cannot be true, but the antecedent can be, as for example when predicates F and G are co-extensive but not true of a unique individual.

The Frege-Carnap and Frege-*Grundgesetze* theories differ on #5, #10, and #11. This is because in Frege-Carnap all improper descriptions *bedeuten* the same items in the domain of any one interpretation, while in Frege-*Grundgesetze* different descriptions *bedeuten* different entities if the predicates are true of different classes of things in the domain, even of the same interpretation. So for example, in #10 on the Frege-Carnap theory, $\iota x\ x{\neq}x$ is necessarily improper and therefore *bedeutet* the chosen individual. But unless there is just one entity in the domain, $\iota x\ x{=}x$ is also improper and *bedeutet* that very same entity. On the other hand, if the domain *does* contain only one element, so that $\iota x\ x{=}x$ *bedeutet* it, then that element is forced to serve as the chosen object also and again #10 is true. However, on the Frege-*Grundgesetze* theory, the description $\iota x\ x{=}x$ will *bedeuten* the set of all things in the domain[22] while the description $\iota x\ x{\neq}x$ will *bedeuten* the empty set. So there are i-models where #10 fails. (It does not fail in every interpretation, since the domain that contains just the empty set will make $\iota x\ x{=}x$ be proper and *bedeuten* the empty set, while $\iota x\ x{\neq}x$ will also *bedeuten* the empty set[23]).

On the whole, the Russell answers for our formulas and arguments in the TABLE are quite easily computed (so long as we focus on the narrow scope for the description). As with the other theories, many of the formulas follow from the basic #2. It is a hallmark of Russell's theory that if there is no F in the domain then the formula #3 will be false. Argument #1i is i-invalid because if there is no G in the domain then it is certainly possible for everything in the domain to be F and yet not have "the G" be one of them. #8 will be i-false because when we eliminate the descriptions we are left with a sentence saying that "there is an *x* which is an F…", and since the descriptions are improper, this will always be false in an i-model. Formulas such as #5 and #7 will come out as i-true because of the negated disjunct. #9, #10, and #11 will be logically false because the descriptions are impossible and so their elimination will amount to asserting the

---

[22] This illustrates a problem in the Frege-*Grundgesetze* theory: Since the *Bedeutung* of any description has to be in the domain, it follows that, in the case of a "universally applicable" description such as $\iota x\ x{=}x$, the set of all things in the domain has to be in the domain. But then a set is an element of itself, contrary to the principle of well-foundedness.

[23] It is not at all clear that there can be a domain that contains only the empty set, in the *Grundgesetze* framework. But once again, we will not go into the details of this difficult issue.

existence of something that has an impossible collection of properties. When the description embedded in this disjunct is eliminated with narrow scope, we will have a negation of something that is i-false. Hence that disjunct will be i-true and the entire formula will thereby be i-true. #15i and #15ii will be equivalent because they are interderivable and the deduction theorem holds for Russell (unlike the Frege-Strawson case).

5. Russellian Criticisms of Singular Term Analyses

Russell criticizes Frege as follows (where Russell says 'denotation' understand 'Bedeutung'; where he says 'meaning' understand 'Sinn'):

> If we say, 'the King of England is bald', that is, it would seem, not a statement about the complex *meaning* of 'the King of England', but about the actual item denoted by the meaning. But now consider 'the King of France is bald'. By parity of form, this also ought to be about the denotation of the phrase 'the King of France'. But this phrase, though it has a *meaning*, provided 'the King of England' has a meaning, has certainly no denotation, at least in any obvious sense. Hence one would suppose that 'the king of France is bald' ought to be nonsense; but it is not nonsense, since it is plainly false. (1905a, p.419)

This sort of criticism misses the mark. It is not part of the Frege-*Grundgesetze* Theory, nor of the Frege-Carnap theory, that 'the King of France is bald' is *nonsense*. It is, of course, a feature of the Frege-Strawson account that it lacks a truth value, which is still some way from nonsense, for although it lacks a *Bedeutung* it still has a *Sinn*. A further criticism of the Frege-Strawson view is contained in the sentences just following:

> Or again consider such a proposition as the following: 'If *u* is a class which has only one member, then that one member is a member of *u'*, or, as we may state it, 'If *u* is a unit class, *the u* is a *u'*. This proposition ought to be *always* true, since the conclusion is true whenever the hypothesis is true. ... Now if *u* is not a unit class, 'the *u*' seems to denote nothing; hence our proposition would seem to become nonsense as soon a *u* is not a unit class.
>
> Now it is plain that such propositions do *not* become nonsense merely because their hypotheses are false. The King in *The Tempest* might say, 'If Ferdinand is not drowned, Ferdinand is my only son'. Now 'my only son' is a denoting phrase, which, on the face of it, has a denotation when, and only when, I have exactly one son. But the above statement would nevertheless have remained true if Ferdinand had been in fact drowned. Thus we must either provide a denotation in cases which it is at first absent, or we must abandon the view that the denotation is what is concerned in propositions which contain denoting phrases. (1905, p. 419)

Russell here is arguing against the Frege-Strawson view on which sentences with non-denoting descriptions come out neither true nor false (Russell's "meaningless"?), because if the antecedent of a conditional hypothesizes that it is proper then the sentence should be true. (Our #6 captures this). But as Russell says, one needn't abandon all singular term analyses in order to obey this intuition. So it is strange that he should think he has successfully argued against Frege, unless it is the Frege-Strawson view that Russell is here attributing to Frege. And yet Russell had read the relevant passages in "Über Sinn und Bedeutung" as well as *Grundgesetze* in 1902, making notes on them for his Appendix A on "The Logical Doctrines of Frege" for his *Principles of Mathematics*. Indeed just later in "On Denoting" he does in fact attribute the *Grundgesetze* theory to Frege:

> Another way of taking the same course <a singular term analysis that is an alternative to Meinong's way of giving the description a denotation> (so far as our present alternative is concerned) is adopted by Frege, who provides by definition some purely conventional denotation for the cases in which otherwise there would be none. Thus 'the King of France', is to denote the null-class; 'the only son of Mr. So-and-so' (who has a fine family of ten), is to denote the class of all his sons; and so on. But this procedure, though it may not lead to actual logical error, is plainly artificial, and does not give an exact analysis of the matter. (1905, p.420)

Even granting Russell's right to call it "plainly artificial", he does not here find any logical fault with Frege's *Grundgesetze* theory. In any case, he joins the other commentators in not remarking on the different theories of descriptions Frege presented in different texts. Indeed he states two of them without remarking on their obvious difference.

On the other hand Russell's presentation of one of his "puzzles" for a theory of descriptions does touch on Frege (cf. our #7):

> (2) By the law of excluded middle, either 'A is B' or 'A is not B' must be true. Hence either 'the present King of France is bald', or 'the present King of France is not bald' must be true. Yet if we enumerated the things that are bald, and then the things that are not bald, we should not find the present King of France in either list. Hegelians, who love a synthesis, will probably conclude that he wears a wig. (1905, p.420)

The empty set, which is the Frege-*Grundgesetze* theory's *Bedeutung* for 'the present King of France', will be in the enumeration of things that are not bald. With the Frege-Carnap view we just don't know which enumeration it will be in (but it will be in one of them). Perhaps Russell is attributing the Frege-Strawson theory to the Hegelians. (It is neither true nor false that the present King of France is bald…so he must be wearing a wig!)

Russell's discussion is unfair to Frege's various accounts. Russell's main arguments are directed against Meinong, and since both Meinong and Frege take definite descriptions to be designating singular terms, Russell tries to paint Frege's theory with the same brush as he uses on Meinong's theory. Although there is dispute on just how to count and individuate the number of different Russell arguments against Meinong in Russell's various writings on the topic, we see there to be basically five logical objections[24] against singular term theories of descriptions raised in Russell's 1905 works: (a) Suppose that there is not a unique F. Still, the sentence 'If there were a unique F, then $F\iota xFx$' would be true. (cf. our #6) (b) The round square is round, and the round square is square. But nothing is both round and square. Hence 'the round square' cannot denote anything. (c) If 'the golden mountain' is a name, then it follows by logic that there is an x identical with the golden mountain, contrary to empirical fact. (d) The existent golden mountain would exist, so one proves existence too easily. (e) The non-existing golden mountain would exist according to consideration (c) but also not exist according to consideration (b).

Russell's conclusion from all these criticisms was that no singular term account of descriptions could be adequate. As we quoted before, he thinks of Frege as "another way of taking the same course" as Meinong. And as for his "quantificational analysis" of descriptions, while he admits that the account is counter-intuitive, he challenges others to come up with a better account that avoids these considerations.[25]

But however much these considerations hold against Meinong, Ameseder, and Mally (who are the people that Russell cites), they do not hold with full force against Frege.[26] Against the first consideration, we should note that #6 is i-true, indeed logically true, in both Frege-Carnap and Frege-*Grundgesetze*. It is only the Frege-Strawson theory that this objection holds against, since it judges the sentence to be neither true nor false when the description is improper. As for the second point, Frege has simply denied that $F\iota xFx$ is i-true (and it might be noted that Russell's method has this effect also, decreeing it to be i-false), and that is required to make the consideration have any force. Against the third consideration, Frege could have said that there

---

[24] There is also a non-logical objection embedded in the "Grey's Elegy" consideration, to the effect that we would need an infinite hierarchy of names of the content of definite descriptions. It is not so clear to us how this is supposed to tell against Frege, who *does* embrace such an infinite hierarchy of indirect *Sinn*, although others, such as Salmon (in this volume), find this objection to be the core of the Grey's Elegy argument. Furthermore, as a second point, arguments that descriptions and logically proper names must differ appear in Russell's work after "On Denoting", but specifically address logically proper names, rather than the arguably more general notion of singular term.

[25] "I…beg the reader not to make up his mind against the view – as he might be tempted to do, on account of its apparently excessive complication – until he has attempted to construct a theory of his own on the subject of denotation."

[26] Perhaps the arguments do not hold against Meinong, Ameseder, and Mally either. But that is a different topic.

was nothing wrong with the golden mountain existing, so long as you don't believe it to be golden or a mountain. Certainly, whatever the phrase designates *does* exist, by definition in the various theories of Frege. And against the fourth consideration, Frege always disbelieved that existence was a predicate, so he would not even countenance the case. Nor would the similar case of (e) give Frege any pause.

So, Russell's considerations do not really provide a conclusive argument against *all* singular term accounts of definite descriptions, and in particular they do not hold against Frege's various theories (except the one objection to the Frege-Strawson theory). And it is somewhat strange that Russell should write as if they did. For as we mentioned earlier, in 1902 he had read both "Über Sinn und Bedeutung" and *Grundgesetze*, making notes on Frege's theories.[27] Yet he betrays no trace here of his familiarity with them, saying only that they do not provide an "exact analysis of the matter", but never saying how they fall short. In fact, a glance at the TABLE reveals that there is one commonality among all the theories of descriptions we have discussed: they never treat

#3 $F\iota xFx$

as logically true, or even i-true, unlike Meinong and his followers. (When the description is improper, the Frege-Carnap theory treats it as i-contingent – sometimes true, sometimes false – and the Frege-*Grundgesetze* theory treats it as false for every atomic predicate, just like Russell's theory.[28] The Frege-Strawson theory also treats it as not always true.) It is this feature of *all* these theories that allows them to avoid the undesirable consequences of a Meinongian view; and it is rather unforthcoming of Russell to suggest that there are really any other features of his own theory that are necessary in this avoidance. For, each of Frege's theories also has this feature.

6. Where Russell's Theory Differs from Frege's

We have seen that there are various ways to present a theory of definite descriptions as singular terms, and that they give rise to assignments of different logical statuses to different formulas. We've also seen that Russell's theory and Frege's theory in fact essentially *agree* on the critical

---

[27] Russell's notes on Frege are in the Bertrand Russell Archives at McMaster University, item RA 230.030420. They are published in Linsky (2004/5).

[28] It is also false for all conjunctions of atomic predicates (as in 'is a golden mountain'). As remarked above, it is not always false when the predicate has certain "inherently negative" constructions, such as explicit negation ('is not a golden mountain') and conditionals ('if a golden mountain, then valuable').

#3, which is all that is required to avoid Russell's explicit objections to the singular-term theory of definite descriptions.

In this section we wish to compare some of our test sentences with the formal account of definite descriptions in *Principia Mathematica* ∗14, so that one can see just what differences there are in the truth values of sentences employing definite descriptions between Russell's theory and the various Frege theories, as summarized in the TABLE. Our idea is that another test of adequacy of a theory of definite descriptions will be the extent to which the theory agrees with intuition on these sentences. And so we are interested in the ways where Russell's theory differs from the Frege-Carnap and Frege-*Grundgesetze* theories. As can be seen from the TABLE, there are many such places. The places where both of these Frege theories agree with one another and disagree with Russell are:

#1 i)	$\forall xFx \therefore F\iota xGx$	Frege: valid	Russell: i-invalid
iii)	$\forall xFx \supset F\iota xGx$	Frege: logically true	Russell: i-false
#2	$\exists y\ y = \iota xFx$	Frege: logically true	Russell: i-false
#8	$\iota xFx = \iota xFx$	Frege: logically true	Russell: i-false
#9	$\iota x\ x{\neq}x = \iota x\ x{\neq}x$	Frege: logically true	Russell: logically false
#13	$(\iota xFx = \iota xGx) \supset G\iota xFx$	Frege: i-contingent	Russell: logically true
#14	$\forall x(Fx{\equiv}Gx) \therefore \iota xFx = \iota xGx$	Frege: i-valid	Russell: i-invalid
#16	i) $\forall x(Sxa \equiv x{=}b)$ ii) $b = \iota xSxa$	Frege: not inter-derivable	Russell: logically equivalent

Most of these differences are due to the fundamental #2. Given that different choice, it is clear that #1i and #1iii must differ as they do. And Russell's interpretation of a definite description as asserting that there exists a unique satisfier of the description (and his related "contextual definition") will account for all the cases where the formula (or argument) is i-false (or i-invalid) in Russell. However, since it is logically impossible that there be a non-self-identical item, #9 must be *logically* false. The two places where all three theories provide different answers are #3, which we have already discussed, and #13. #13 is logically true in Russell because if the descriptions are improper then the antecedent is false (and hence #13 is true), but if the antecedent is true then both descriptions must be proper…and hence the consequent must be true. As we mentioned before, in Frege-Carnap, when the descriptions are improper then the antecedent is true. But whether the consequent is true or not depends on whether the chosen-object has the property G or not. In Frege-*Grundgesetze*, if the descriptions are improper and the predicates F and G are co-extensive, then the antecedent is true, but in that case the predicate G could not be true of ιxFx, for then it would have to be true of ιxGx …which we showed above to be impossible. But #13 is not logically false in Frege-*Grundgesetze* because it is true when the descriptions are proper.

A supporter of Frege's singular term theory or of Russell's quantificational analysis should ask of themselves what they think of these differences in the logic between Frege and Russell. For example, if one is drawn to the idea that every *thing* is self-identical, and infers from this claim that every *statement* of self-identity must be true, then one will be in agreement with Frege on #8 and #9, and opposed to Russell. If, like Morscher and Simons (2001: 21) one thinks that #14 is an obvious truth that must be obeyed by any theory of definite descriptions, then you will side with Frege against Russell. And conversely, of course. There should be some discussion among the proponents of these two theories as to which way they prefer to tilt, and why, concerning the many formulas that are treated differently in Frege's theories vs. Russell's theory.

As we mentioned, most of the differences can be traced to the crucial #2. But this in fact is just a restatement of the difference between a singular term analysis of definite descriptions and a quantificational analysis. Once you have made the choice to adopt #2, the main further differences among singular term analyses concern whether or not to differentially treat descriptions where nothing satisfies the predicate from those where the predicate is true of more than one thing. If you choose *not* to treat them differently, then the most natural theory is Frege-Carnap. If you *do* wish to treat them differently, the Frege-*Grundgesetze* theory offers one direction (although as we mentioned earlier, there is a difficulty in mixing individuals in the domain with sets of these individuals, and making the result all be in the same domain). There are other ways to carry this general strategy out, some of them following from the epsilon-calculus, others looking like the "Russellian theory" of Kalish, Montague, and Mar (1980) Chapter 8. But we won't go into those sorts of theories now.

In any case, given that #2 is just a restatement of the fundamental difference between singular term theories and quantificational theories, and given that the remaining logical differences basically follow from this choice, it seems to follow that the only real way to favour one theory over the other is to see which of the other formulas and arguments are more in accord with one's semantic intuitions. Russell has not done this, despite his belief to the contrary. He focused basically on #3; but this formula is not treated differently in any important way between Russell and the Frege theories. Instead, it only separates the Meinongian singular term theory from both Frege's and Russell's theories.

V. Concluding Remarks

The fundamental divide in theories of descriptions now, as well as in Russell's time, is whether definite descriptions are "really" singular terms, or "really" not singular terms (in some philosophical "logical form" sense of 'really'). If they are "really" not singular terms then this might be accommodated in two rather different ways. One such way is Russell's: there is *no* grammatically identifiable unit of any sentence in logical form that corresponds to the natural language definite description. Instead there is a grab-bag of chunks of the logical form which somehow coalesce into the illusory definite descriptions. A different way is more modern and stems from theories of generalized quantifiers in which quantified terms, such as 'all men', are represented as a single unit in logical form and this unit can be semantically evaluated in its own right—this one perhaps as the set of all those properties possessed by every man. In combining this generalized quantifier interpretation of quantified noun phrases into the evaluation of entire sentences, such as 'All men are mortal', the final, overall logical form for the entire sentence becomes essentially that of classical logic. So, although quantified noun phrases are given an interpretable status on their own in this second version, neither does their resulting use in a sentence yield an identifiable portion of the sentence that corresponds to them nor does the interpretation of the quantified noun phrase itself designate an "object" in the way that a singular term does (when it is proper). It instead denotes some set-theoretic construct.

If we treat definite descriptions as a type of generalized quantifier, and thereby take this second way of denying that definite descriptions are "really" singular terms, the logical form of a sentence containing a definite description that results after evaluating the various set-theoretic constructions will (or could, if we made Russellian assumptions) be that which is generated in the purely intuitive manner of Russell's method. So these two ways to deny that definite descriptions are singular terms really amount to the same thing. The only reason the two theories might be thought different is due to the algorithms by which they generate the final logical form in which definite descriptions "really" are not singular terms, not by whether the one has an independent unit that corresponds to the definite description. In this they both stand in sharp contrast to Fregean theories.

These latter disagreements are pretty much orthogonal to those of the earlier generation. The contemporary accounts, which have definite descriptions as being "nearly" a classical quantifier phrase, agree with the Russellian truth conditions for sentences involving them. Although these truth conditions might be suggested or generated in different ways by the different methods (the classical or the generalized quantifier methods) of representing the logical form of sentences with descriptions, this is not required. For one could use either the Russellian or Frege-Strawson truth conditions with any contemporary account. It is clear, however, that we must first settle on an account of improper descriptions.

We remarked already on the various considerations that might move a theorist in one direction or another as they construct a theory of definite descriptions. We would like to point to one further consideration that has not, we think, received sufficient consideration.[29] It seems to us that a logical theory of language should treat designating and empty proper names in the same way, since there is no intuitive syntactic way to distinguish denoting from non-denoting proper names in natural language. We also find there to be much in common between empty proper names and improper descriptions, from an intuitive point of view—their semantics should be the same. And of course there is no intuitive way to distinguish (empirically) non-proper vs. proper descriptions in general. So, all these apparent singular terms should be dealt with in the same way. If definite descriptions are to be analyzed away à la Russell, then the same procedure should be followed for 'Pegasus' and its kin. And if for 'Pegasus', then for 'Benjamin Franklin' and its kin. (A strategy taken by Russell, in other works). If, on the other hand, these latter are taken to be singular terms, then so too should definite descriptions. And whatever account is given for non-denoting proper names should also be given for improper descriptions: if non-denoting names have a sense but no denotation in the theory, then we should adopt the Frege-Strawson theory of improper descriptions. If we think we can make meaningful and true statements using 'Pegasus' and its cohort, then we should adopt either the Frege-Carnap or the Frege-*Grundgesetze* theory of improper descriptions.

[29] Except from certain of the free logicians, who take the view that sentences which contain non-denoting names are neither true nor false, and this ought to be carried over to non-denoting definite descriptions as well.

In any case we should care about the present king of France.

Bibliography

Carnap, R. (1956) *Meaning and Necessity*. 2nd Edition. University of Chicago Press, Chicago.

Frege, G. (1884) *Grundlagen der Arithmetik, enie logisch-mathematische Untersuchung über den Begriff der Zahl*. W. Koebner: Breslau.

Frege, G. (1892) "Über Sinn und Bedeutung" *Zeitschrift für Philosophie und philosophische Kritik* v. 100, pp. 25-50. Translated by M. Black (1952) *Selections from the Philosophical Writings of Gottlob Frege* , 3rd ed., (1980), (Blackwells, Oxford) as "On Sense and Meaning", pp. 56-78. Quotations and page references are to this translation.

Frege, G. (1893) *Grundgesetze der Arithmetik, begriffsschriftlich abgeleitet*. Vol. 1. Verlag Hermann Pohle, Jena. Translated M. Furth (1964) as *The Basic Laws of Arithmetic: Exposition of the System*, University of California Press, Los Angeles. Quotations and page references are to this translation.

Kalish, D. & R. Montague (1964) *Logic: Techniques of Formal Reasoning*. Harcourt Brace Jovanovich, New York.

Kalish, D., R. Montague, G. Mar (1980) *Logic: Techniques of Formal Reasoning*. 2nd Edition. Harcourt Brace Jovanovich, New York.

Kaplan, D. (1972) "What is Russell's Theory of Descriptions?" in D. Pears *Bertrand Russell: A Collection of Critical Essays*. New York, pp. 227-244.

Kleene, S. (1952) *Introduction to Metamathematics*. Van Nostrand, NY.

Lambert, K. & van Fraassen, B. (1967) "On Free Description Theory" *Zeitschrift für Mathematische Logik und Grundlagen der Mathematik* **13:** 225-240.

Lehmann, S. (1994) "Strict Fregean Free Logic" *Journal of Philosophical Logic* **23:** 307-336.

Linsky, B. (2004/5) "Bertrand Russell's Notes for Appendix A of *The Principles of Mathematics*", *Russell: The Journal of Bertrand Russell Studies*, n.s. **24**: 133-72.

Montague, R. & D. Kalish (1957) "Remarks on Descriptions and Natural Deduction (Parts 1 and 2)" *Archiv für mathematische Logik und Grundlagenforschung* v.8, pp. 50-64, 65-73.

Morscher, E. & P. Simons (2001) "Free Logic: A Fifty-Year Past and Open Future" in E. Morscher & A. Hieke (eds.) *New Essays in Free Logic: In Honour of Karel Lambert* (Dordrecht: Kluwer), pp. 1-34.

Neale, S. (2001) *Facing Facts*. Oxford Univ. Press, NY.

Pelletier, F.J. & B. Linsky (2005) "What is Frege's Theory of Descriptions?" in B. Linsky and G. Imaguire, *On Denoting: 1905-2005*, Philosophia Verlag: Munich, pp. 195-250.

Russell, B. (1903) *Principles of Mathematics*. Cambridge Univ. Press, Cambridge.

Russell,B. (1905a) "On Denoting", *Mind* **14:** 479-493. Reprinted in *The Collected Papers of Bertrand Russell, Vol.IV: Foundations of Logic 1903-05* (Routledge, 1994) A.Urquhart & A. Lewis (eds.), pp. 414-427. Page references to this reprint.

Russell, B. (1905b) "Review of Meinong and Others, *Untersuchungen zur Gegenstandstheorie und Psychologie*", *Mind* **14:** 530-538.

Strawson, P. (1950) “On Referring” *Mind* **59:** 320-344.

Strawson, P. (1952) *Introduction to Logical Theory*. Methuen, London.

Whitehead, A. & B. Russell (1910) *Principia Mathematica*. Cambridge: Cambridge University Press. Second Edition 1925-1927. Reprinted in paperback as *Principia Mathematica to *56*. Cambridge: Cambridge University Press 1962.