Experiment 31: Gas Damping of Micromechanical Resonances

From PhysicsLab

Introduction

As a general rule, when tuning forks, guitar strings, drumheads and other mechanical resonators are made smaller, their notes go up. In this experiment, your resonator is a small drum, 5 mm on a side. The extremely thin (200 nanometre) "drumskin" is made of silicon nitride. The structure is a commercial product manufactured by the Edmonton microelectromechanical systems (MEMS) company Norcada. The nitride is supported by a crystalline silicon frame, 200 μm thick. The notes of this drum are beyond the frequency range of human hearing, but no matter -- you have to operate them in vacuum, so wouldn't be able to hear them anyway.

In order to drive vibrations of the membrane in this experiment, you will shake it up and down by its edges, rather than striking it with a tiny drumstick. At particular shaking frequencies, the amplitude of membrane vibration will grow to much larger values than at other frequencies (resonances; analogous to shaking the end of a string under tension to set up standing waves). In order to see these resonances clearly, the membrane must be in vacuum. Because the membrane is so light, the momentum of its motion cannot push much air out of the way.

The motion of the membrane is detected using optical interference. The membrane is easily incorporated into a simple optical interferometer, such that the reflected optical intensity changes when the membrane moves. Specifically, we will track the magnitude of the membrane motion, and its phase relative to that of the drive signal, using a lock-in amplifier. A voltage-controlled generator is used to allow you to sweep the frequency and record the response using a computer.

Object

You have two goals in conducting this experiment. The first goal is to understand the basic concepts underlying the sinusoidal drive, the resonator response, and the detection of the response. The second goal is to characterize the effect of gas in inhibiting the membrane vibrations, through measurements of one of the membrane resonances as a function of pressure, at low pressures (in this case about 1/20 th of an atmosphere and below).

Theory

This is an example of sinusoidally-driven, damped harmonic motion, as described in your Classical Mechanics textbook (Morin section 4.4, p. 113). A handout from the Classical Mechanics text by Marion is also with the apparatus, as it shows representative plots of the variation of the magnitude and phase of the resonator response as the drive frequency is swept through a resonance.

The "quality factor", *Q*, of a damped, driven resonance is, to a very good approximation when the damping is not too large, the resonance frequency divided by the frequency width, Δ*f*, measured between the points where the amplitude of response is $1/\sqrt{2} = 0.71$ of the peak amplitude:

$$Q = \frac{f_o}{\Delta f}$$

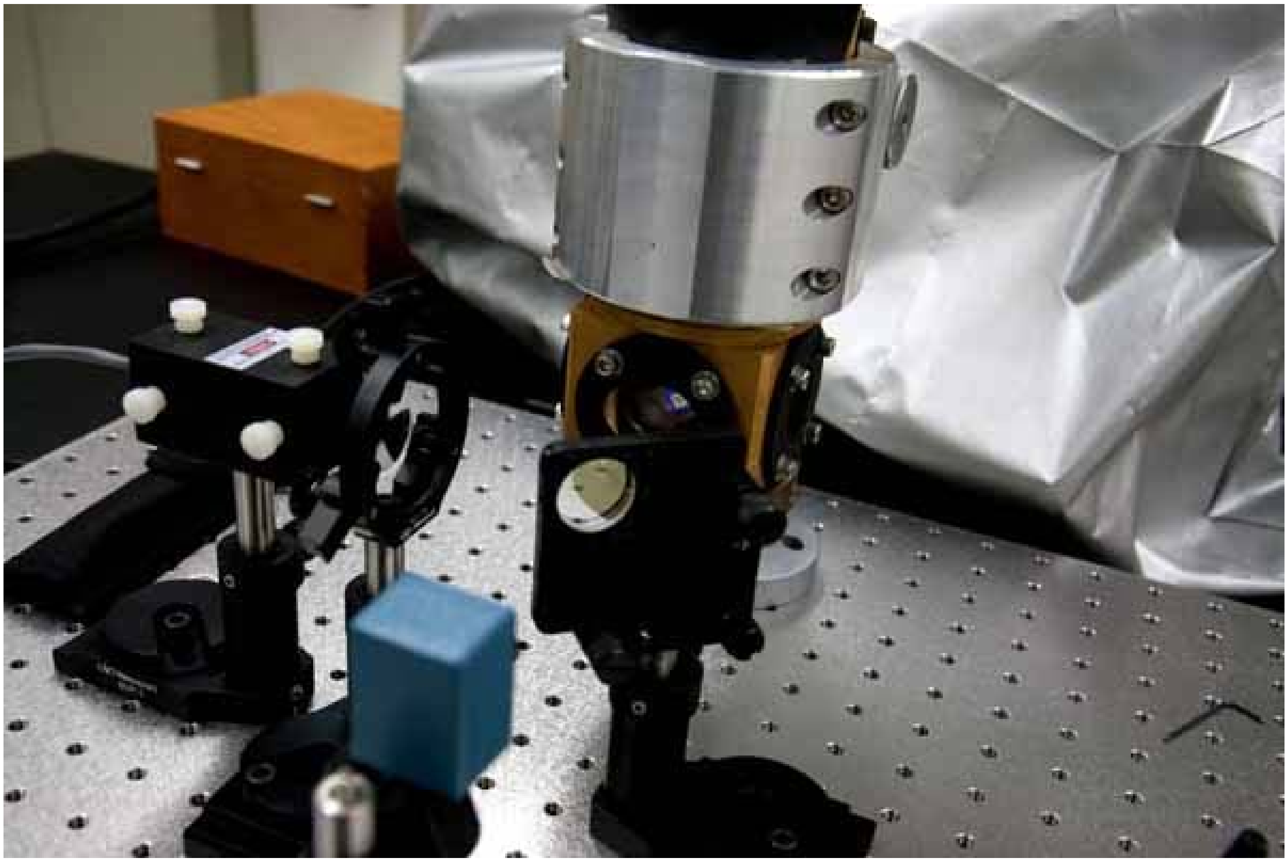
Damping of the resonance comes from numerous sources. Some of the mechanical energy of the resonator is converted to heat in the resonator, and some of it is transmitted into vibrations of the frame and supporting structure. In perfect vacuum one reaches the minimum damping we would observe, translating to the smallest Δf and maximum quality factor, which we term $Q_{intrinsic}$. When we add gas, and additional energy is lost in each cycle of oscillation to collisions with gas molecules, the damping rates add and we find

$$\frac{1}{Q} = \frac{1}{Q_{intrinsic}} + \frac{1}{Q_{gas}}$$

The expectation is that, at sufficiently low pressures, Q_{gas}^{-1} will be proportional to the gas density (and hence proportional to the gas pressure, if the temperature is constant). Beyond this lowest pressure regime the pressure dependence is expected to be weaker, eventually becoming proportional to the square root of pressure.

Apparatus

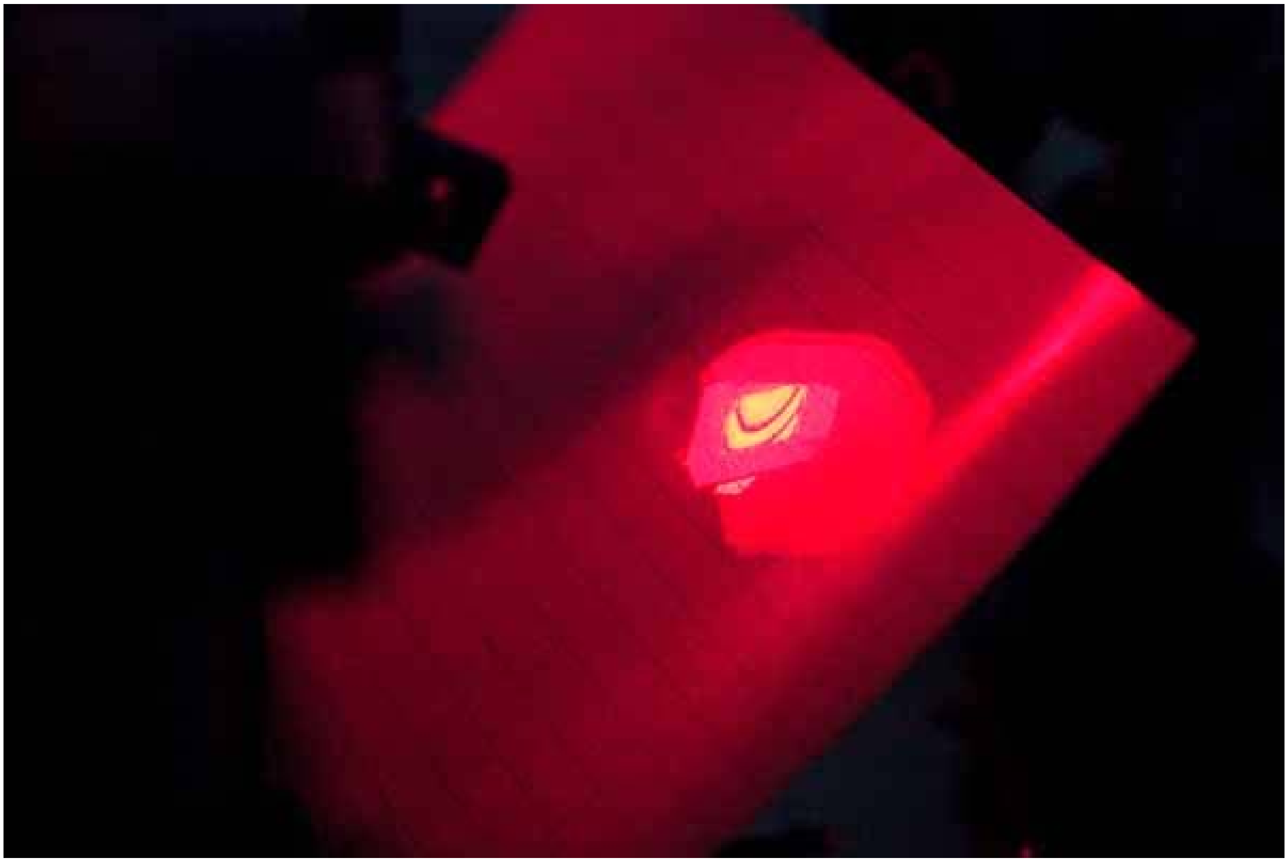
The layout is closely related to the Michelson Interferometer (see photo below: the membrane, in vacuum, is at the centre of the image; laser is at left, then lens, then beamsplitter; the blue box at the bottom of the image is the photodetector housing). In order to minimize unwanted relative motion between our "moving mirror" (the membrane) and the "reference mirror", we mount the semi-transparent membrane directly on top of another reflector, rather than having it in a separate arm after the beamsplitter.



Sample Geometry:

The small piezoelectric shaker is held with four 4-40 screws and number 4 washers against a copper surface in a small vacuum chamber (and suited for cooling the sample with liquid nitrogen, if desired). A small piece of silicon is placed on top of the piezo disk (held by a piece of Scotch double-sided tape), and serves as the back mirror of our simple optical interferometer. The silicon frame of the nitride membrane is attached to this back mirror, again held by small pieces of the double-stick tape at two corners. The static spacing between the membrane and the back mirror is therefore approximately 250 micrometres (thickness of silicon frame plus thickness of tape).

The membrane was set in place while viewing the setup under one of the sodium lamps from the Newton's Rings lab, to make sure that the membrane and backing silicon are "parallel enough" (we don't want too many interference fringes across the window). The parallelism can also be checked with the sample inside the vacuum chamber, by looking at the reflection of light from our favourite inexpensive, nearly ideal point-source of coherent light, the bare laser diode (see photo).



The sample mounting procedure is a bit too dicey to ask you to do it yourselves and to finish the experiment in a single afternoon (although the membranes are surprisingly robust; there is a second one you can inspect if you wish).

Optical Alignment:

The laser beam should travel in a plane approximately parallel to the surface of the breadboard. You can use an index card with a small hole (punched with a pen or pencil) as an alignment aid.

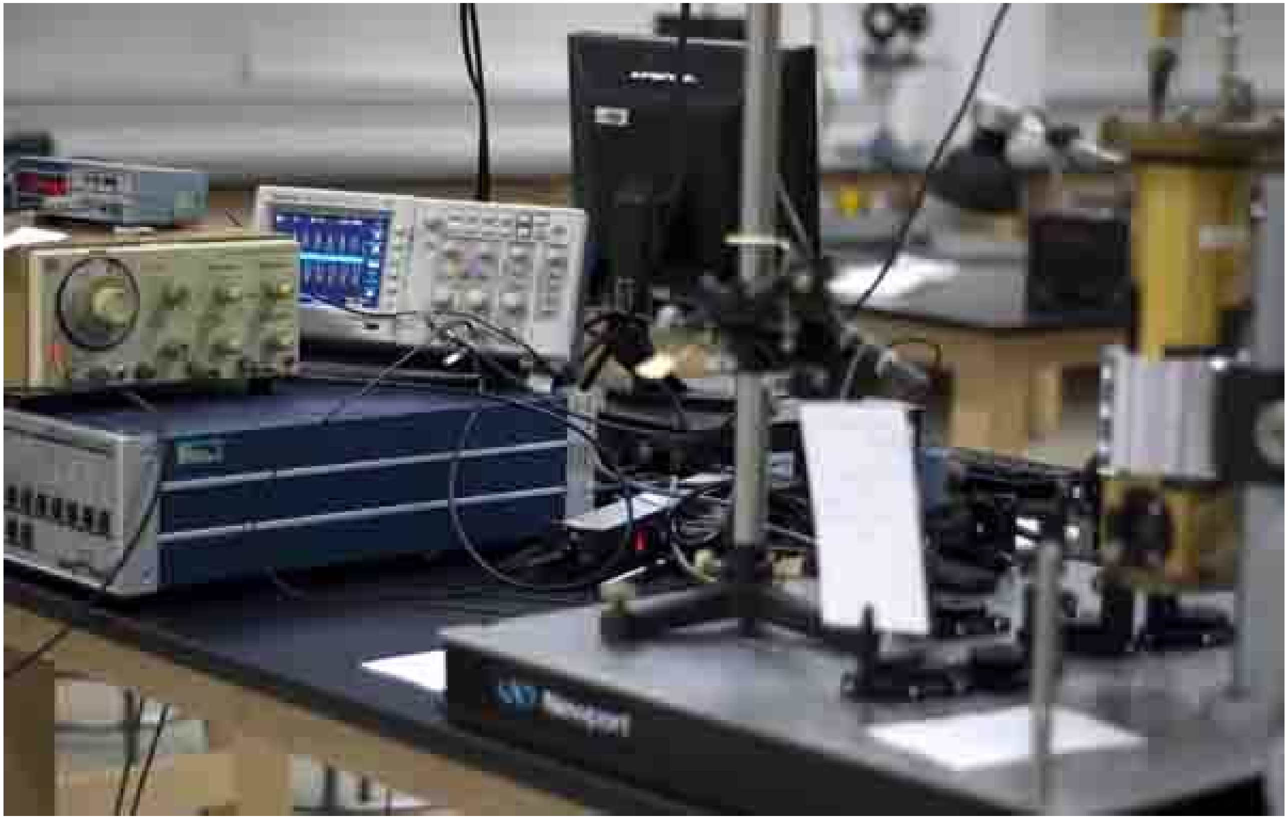
A 150 mm focal length plano-convex lens is placed near the output of the laser, to focus the beam on the sample. Ideally the interferometer spacing should not vary much across the size of the optical focus spot. After reflecting from the beamsplitter, the beam should arrive at the sample approximately at normal incidence. The beam reflected from the sample needs to return approximately on the same path, back through the beamsplitter, and onto the photodetector. There will be multiple reflection spots because of the window on the vacuum chamber. Roughly in order of brightness, the spots you should see are: main (desired) reflection from sample; direct reflection from window; secondary reflection (double bounce) between sample and window.

Procedure

Start with the vacuum chamber pumped down to the base pressure of the roughing pump (approximately 10 mTorr).

i) First "scope out" the situation using the oscilloscope to view the drive and signal voltages. (Please refer to the Appendix below for more oscilloscope information.) Set the load resistor switch of the photodetector to "1K". This will ensure that the photodetector response is linear and frequency independent for the purposes of this experiment. Connect the BNC cable out of the photodetector to the channel 2 input of the scope.

Using the F34 function generator, apply a 250 mV peak-to-peak drive to the piezo disk (electrical connector is the BNC on the right, as seen looking at the chamber from the front). Using one of the BNC adapters with three ports, to route one output to two inputs, connect this signal also to the channel 1 input of the scope. The "sync out" from the function generator connects to the input of the Simpson Model 710 frequency counter, to allow you to read the frequency easily. This output should also connect to the Reference Input of the EG&G Model 5208 lock-in amplifier, using another one of BNC adapters. (The lockin will be used for the more sensitive, computer-controlled measurements below.)



Manually sweep through the frequency range from 10 kHz to 100 kHz. . You should find numerous resonances. (Note: the membrane resonances are quite sharp in vacuum, you have to adjust the frequency quite carefully.) With the frequency set to one of the resonance peaks, try adjusting the optical alignment slightly to see if you can increase the signal. For this, it is easiest to steer the beam slightly with the top right and bottom left adjustment knobs on the beamsplitter holder (called a "kinematic mount"). The AC interference signal, generated by small oscillations of the membrane height above the back mirror, depends sensitively on the position of the laser spot because the membrane and mirror are not perfectly parallel.

The photodetector output connects to the "A" input of the lockin. Now you are ready for data acquisition under computer control.

ii) use the labview program "MainProgram_MicroResonator.vi" to sweep the frequency under computer control, and record the magnitude and phase from the lockin. Note that you have to keep track of the start and stop frequencies by hand in order to rescale the data later -- the program will keep track of "frequency" as the voltage sent from analog output 0 ("AO0") to the "VCG 10V Span" input of the F34 function generator. For example, a 0-5V range of the voltage sweeps the frequency by approximately 145 kHz when you are in the x100K frequency range. Note that the EG&G 5208 lockin works for external reference frequencies between 5 Hz and 200 kHz. You will have to save two sweeps with different starting frequencies to cover this entire range (they can have some overlap). The frequency you have manually dialed the generator to sets the starting frequency of the sweep.

Settings corresponding to appropriate sweep rates are listed below (too fast and the lockin can't keep up. If you choose the option to scan forward and back, you an check if there is a lag due to the lockin time constant). in MainProgram_MicroResonator.vi --

Data points/run: 1000 (means 2000 if you select forward/backward). More than 1000 points for the 0-5 V range exceeds the resolution of the digital-to-analog converter (DAC) for AO0.

Time between data points: 1000 ms

ADC readings avgd per data pt: 1000

(for these settings, a forward-only sweep should take <6 minutes to complete)

at the lockin --

output mode: R,theta

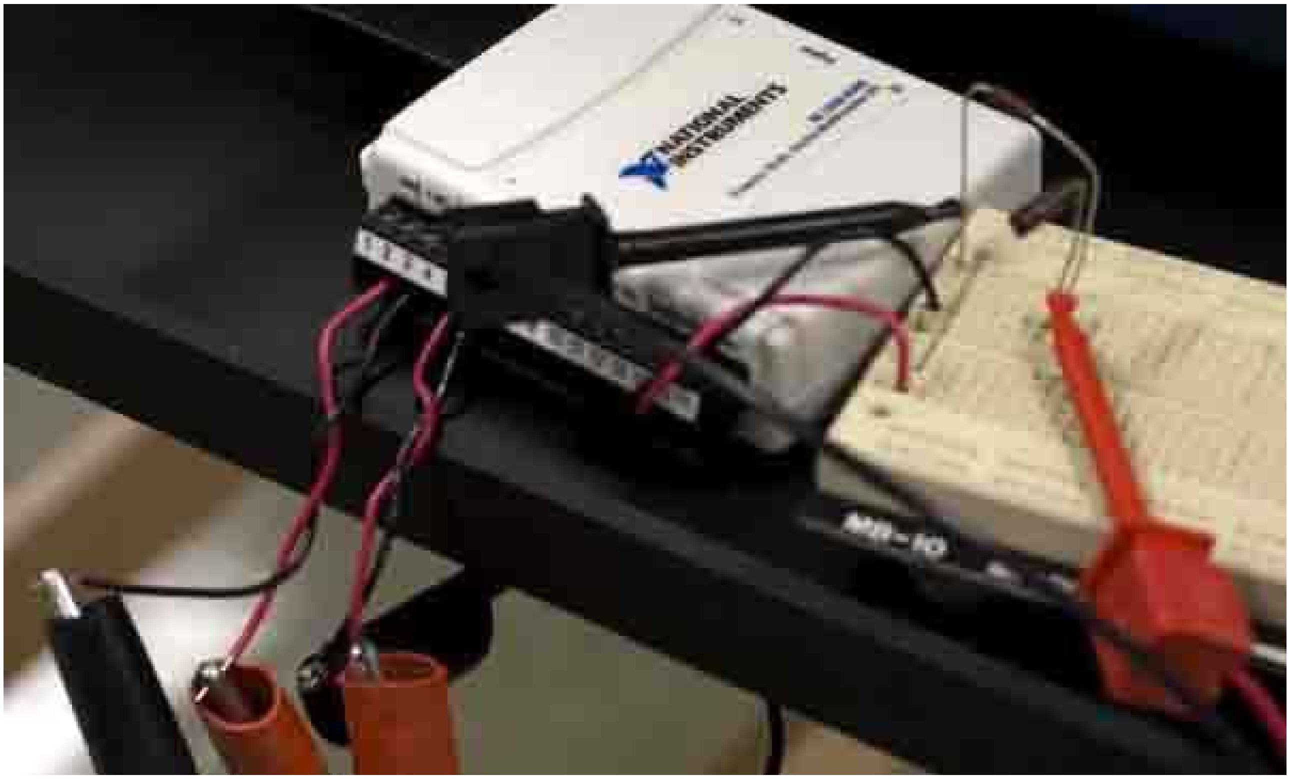
modes: normal, broadband

phase: 0 deg

time constant: 100 ms, 12 dB/oct

Save the data. Three files are stored when you save: i) a text file with three columns of data points (the AO0 setting in the first column, the voltage corresponding to the lockin phase in the second, and the voltage corresponding to the lockin magnitude in the third); ii) a png image of the magnitude plot; iii) a png image of the phase plot. Important note: you have to make note of the start and end frequencies of these scans in your logbook, so you can re-plot the data vs. frequency later on.

iii) Now it is time to measure the quality factor of a resonance as a function of gas pressure. Zoom into the freq. range around a particular high frequency resonance (higher frequencies are better for this test as they are expected to damp away less slowly with increasing pressure; see theory). To obtain higher frequency resolution for these sweeps, use a resistive voltage divider to limit the frequency sweep range without unduly restricting the range of voltages requested from AO0 (min step size ~ 7 mV). Recommended resistive voltage divider: 50 kOhm in series with 5 kOhm. Gives greater than 10X voltage division because the 5 kOhm is in parallel with the impedance of the VCG input. Translates to approximately 2 kHz scan range per 1 V out of AO0.



With this setup it is easier to control the pressure while pumping down, so you will measure the highest pressures first. Vent the chamber to a pressure of approximately 30 Torr (ask for help if you are unsure).

Recommended starting configuration: F34 generator set to 168 kHz, amplitude 200 mV peak-to-peak. vi: scan 0-5 V, 250 points, 1000 ms, 1000 averages per point, forward only. Lockin time constant 100 ms, 12 dB/octave. Lockin sensitivity 1 mV (change if max signal too small or too big).

Save a sweep (make sure it has captured enough of the peak that you can measure Δf. Carefully crack the pump valve open to reduce the pressure ~ 2x (to 15 Torr if you started at 30).

Repeat the process, taking ~2x steps down in pressure, until you reach the base pressure of the pump. You may need to "feather" the valve during the freq. sweep to keep the pressure reading ~ const. Occasionally reduce the drive amplitude at a pressure reduction step, to keep the signal at about the same peak strength as the damping decreases. (If the oscillation amplitude becomes too large, both the interferometric detection and the mechanical response will become nonlinear, and you don't want to venture there in this particular measurement sequence.) About 20 mV p-p should be appropriate at the lowest pressures. You can also narrow the voltage (and hence freq.) sweep range as the *Q* increases.

Analysis

Analyze the data by making measurements of Δf from printouts of the saved images, to save time. You can analyze the magnitude traces and/or the phase traces. Extract $\frac{1}{Q_{gas}}$ versus pressure and make a plot to check for possible linearity at low pressures and sub-linearity at higher pressures.

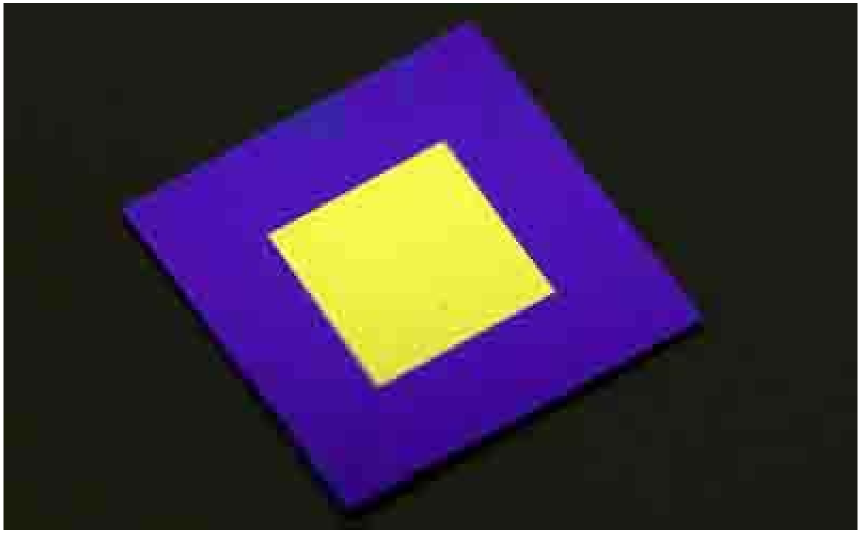
Question

1. Discuss how the AC signal, for a given amplitude of mechanical oscillation of the membrane height above the back mirror, should vary as a function of the static spacing between the membrane and the mirror (equal to the time-averaged spacing when the membrane is moving). This is an important consideration because the membrane and mirror are not perfectly parallel.

Additional points to ponder:

What are the phases corresponding to the 0.71 amplitude points on either side of the resonance maximum? (Theoretically phase = 0 far below resonance, $\pi / 2$ on resonance, π far above resonance; there can be additional fixed phase shifts between drive and detection in the measurement, but the overall phase shift should be the same.)

Can you explain the "interference colour" of the silicon nitride membrane, using the knowledge that it is 200 nm thick and has a refractive index n ~ 2.25?



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Introduction to Classical Mechanics by Charles Morin

Classical Dynamics of Particles and Systems by Jerry Marion

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Further Reading

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Acknowledgements

This experiment was made possible through generous donations: by an anonymous donor of a Newport Optics Education Kit, and by Norcada, Inc., of silicon nitride membranes. The micromechanics labs would never have gotten off the ground without the generous work of Tom Clement. Many individuals have also contributed to the development of the lab, including John Davis, Vince Sauer, Dave Fortin, Lorne Roth, Taylor Rogers, Dave Leonard, and Terry Singleton.

APPENDIX

"Digital Oscilloscope information (Tektronix TDS 2022B and similar)"

Use the "MEASURE" functions to read out amplitudes and frequencies of signals that are large enough to register.

Frequency components of a signal that are hard to spot in a regular trace of amplitude versus time (getting down into the noise) can be easier to identify using the FFT (fast Fourier transform) feature in the MATH MENU. In this experiment, with the photodiode signal connected to Ch 2, select (within the math menu) the FFT operation for Ch 2. In FFT mode, the SEC/DIV knob controls the frequency range displayed. For example, set this to 12.5 kHz (/div) and you view a 125 kHz span on the screen. Now an AC component at a particular frequency shows as a peak in the display. (The horizontal scale is linear but the vertical scale is logarithmic in this mode.) The CURSOR button activates the knob to the top right of the display, for dialing to cursor to different positions for reading the frequencies. Select the CH 2 MENU button again (for example) to get out of MATH mode.

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