

Inside R_{LC} , emitting energy via curvature radiation, while outside R_{LC} energy loss drops, due to just synchrotron (lower \propto dependence).

What's the acceleration a particle feels at R_{LC} ? We'll calculate E field there, & the change in \vec{v} over a relevant time interval (say $1/\omega$, or $P/2\pi$)

First, how does B field fall with distance? Magnetic dipole strength goes as

$$B(r, \lambda) = \frac{m}{r^3} \sqrt{1 + 3 \sin^2 \lambda} \quad \text{where } m = \text{dipole strength}, \quad \lambda = \text{magnetic latitude}$$

Only important bit is $B \propto r^{-3}$.

$$\text{So } \frac{B_{LC}}{B_{NS}} = \left(\frac{r_{LC}}{r_{NS}} \right)^{-3} \quad \text{For } \sim 10 \text{ km NS,} \\ R_{LC, \text{crab}} \sim 1.6 \times 10^6 \text{ m,}$$

$$\frac{B_{LC}}{B_{NS}} \sim (160)^{-3}, \quad B_{LC} \sim (160)^{-3} / (3 \times 10^8 \text{ T}) \sim 10^{-2} \text{ T}$$

At light cylinder, B & E are in form of outflowing EM waves at pulsar frequency, $E_0 = c B_0$ in SI, so E_0 (max E field) is $3 \times 10^{10} \text{ V/m}$.

$$\text{Now, } \frac{dp}{dt} = \frac{d(\gamma m v)}{dt} = q(\vec{E} + (\vec{v} \times \vec{B}))$$

We'll take the peak electric field for $\frac{1}{2\pi}$ of the period, as the maximum force all pointing in one direction. (Ignores screening effects from other charges).

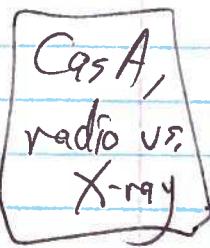
$$\text{So } dt = \frac{1}{2\pi} P = \frac{1}{2\pi} \left(\frac{1}{30} \text{ s} \right) \sim 0.005 \text{ s.}$$

For relativistic particles, $v \approx c$, so $\frac{d(\gamma mv)}{dt}$

$$\sim \frac{(df)_{mc}}{dt}. \text{ Set } \frac{(df)}{dt}_{mc} = qE, \Rightarrow$$

$$df = \frac{qE dt}{mc} = \frac{(1.6 \times 10^{-19} C)(3 \times 10^{10} V m^{-1})(0.005 s)}{(9.1 \times 10^{-31} kg)(3 \times 10^8 m/s)}$$

Gives $d\gamma \sim 9 \times 10^{10}$ above the $\gamma \sim 3 \times 10^9$ required to explain Crab's synchrotron. Fantastic.



Not all relativistic particles are accelerated by pulsars. Cas A, youngest nearby SNR, has neutron star that isn't a pulsar. Similar NSs have $P \sim 0.1$ second and $B < 10^7 T$, so $B_{LC} < 0.1 T$.

But Cas A is strongest known source of synchrotron in Galaxy, synch rad. from radio to X-rays.

Suggests that shocks (SNR forward shock) are accelerating particles.

Cas A:
radio & X-ray

However, pulsars can't explain all relativistic phenomena.

Consider Cas A, the youngest nearby SN

& brightest radio source in our Galaxy.
Neutron star at center is not a pulsar -
no evidence of electrons accelerated by a
neutron star, similar-looking NSs shown
have $P \approx 0.1$ second & $B < 10^8$ Gauss (so $B_{lc} <$

But Cas A is strongest source of synchrotron radiation
at radio wavelengths, & of synchrotron up to X-rays, req
highly relativistic particles (the "blue" edges of X-ray ems).

Suggests that shocks are capable of accelerating
charged particles. Let's study this..

We've talked about a power-law spectrum of
electron energies (e.g. derived from synchr. spectrum,
 $\alpha = (p-1)/z$). How can such a spectrum be produced?
Many synchr. sources show $\alpha \approx 0.7$ ($F_\nu \propto \nu^{-0.7}$), so $p(N_f) \propto f^{-2.5}$.

Basic concept is a scattering process in which
the energy of the particle changes by a constant
fraction $\Delta E/E$ on each scatter, but that each scatter
becomes less likely — because there's a chance of
escape after each scatter. Set $B = \frac{\Delta E}{E}$, P_{esc} = prob of
and average time between collisions t_c . Then
the particle energy with time goes as

$$\frac{dE}{dt} = \frac{EB}{t_c}, \text{ or } E = E_0 e^{Bt/t_c}, \text{ so } \frac{dE}{dt} = \frac{BE_0}{t_c} e^{Bt/t_c}$$

The number of particles drops by $\frac{dN}{dt} = \frac{d-N P_{esc}}{t_c}$, so

$$N = N_0 e^{-P_{esc} t/t_c}, \quad \frac{dN}{dt} = \frac{-N_0 P_{esc}}{t_c} e^{-P_{esc} t/t_c}$$

Divide ~~$\frac{dN/dt}{dE/dt}$~~ $\frac{dN/dt}{dE/dt} = \frac{dN}{dE} \times \frac{N_0}{E_0} e^{-P_{esc} t/t_c} e^{-Bt/t_c}$

$$\frac{dN}{dE} \propto e^{-(Bt/t_c)(1+P_{esc}/B)} \propto \left(\frac{E}{E_0}\right)^{-(1+P_{esc}/B)}$$

So what kind of scattering are we talking about?
 (Note: also applies to photons in IC case, & naturally gives power-law in optically thin case.)

Longair
21.4)
Compton

Here, let's consider relativistic ~~non-relativistic~~ particles scattering back & forth across a shock, called Fermi acceleration. First, consider 2nd-order acceleration.

Consider the collision of a particle with a cloud, cloud at velocity V and particle velocity v (near c).
 The particle makes an angle θ to the normal direction of the cloud surface ($\theta=0$ gives head-on).

We use the frame of the cloud to analyze this; the scattering will change the particle's momentum but not its energy. (Cloud doesn't change)
 (E.g., bending by B fields) The incoming E, p in the cloud's rest frame are

$$E' = \gamma_V (E + pV \cos\theta) \quad (\text{where } \gamma_V = (1 - \frac{V^2}{c^2})^{-1/2})$$

and $p'_x = \gamma_V (p \cos\theta + VE/c)$ (in both cases we just add the additional component from the cloud's motion)

Then in the collision, we assume elastic, so E doesn't change but the p_x component reverses.

Now transform back, $E'' = \gamma_V (E' - Vp'_x)$, or combining both transforms,

$$E'' = E \gamma_V^2 \left[1 + 2 \frac{pV}{E} \cos\theta + \frac{V^2}{c^2} \right] \quad (p = \gamma m u, E = \gamma m c^2)$$

$$E'' = E \gamma_V^2 \left[1 + \frac{2 Vu \cos\theta}{c^2} + \frac{V^2}{c^2} \right] \quad (p = \frac{u}{c^2})$$

$$\boxed{\frac{E''}{E} = \frac{\left(1 + \frac{2 Vu \cos\theta}{c^2} + \frac{V^2}{c^2} \right)}{\left(1 - \frac{V^2}{c^2} \right)}}$$

Expanding this, & keeping only terms up to 2nd order in V/c ,

$$\frac{E'' - E}{E} = \frac{\Delta E}{E} = 2 \frac{Vu \cos\theta}{c^2} + 2 \left(\frac{V}{c} \right)^2$$

The question is what averaging over θ does. There will be a greater probability of randomly encountering clouds head-on than in following collisions proportional to their relative velocities, $\propto \frac{1}{c} + \frac{v}{c} \cos\theta$ or for $v \approx c$, $\propto (1 + \frac{v}{c} \cos\theta)$. Thus, the average $\langle \frac{\Delta E}{E} \rangle$ comes out to $\frac{8}{3} \left(\frac{v}{c}\right)^2$.

This leads to an exponential increase in energy of particle. Fermi's original suggestion was that cosmic rays were boosted by collisions with dense clouds in interstellar space. This has several problems; if v is rather small, $\ll 10^4 c$; the collision rate is low since clouds are widely distributed in the Galaxy (mean free path for cosmic rays is ~ 1 light-year) and the $(v/c)^2$ dependence makes it hard to bring particles up to speed.

If all collisions were head-on, so $\theta = 0$ then we would see a (v/c) dependence, $\propto v$ faster acceleration. This is first-order Fermi acceleration.

To get this, consider particles scattering back and forth across a strong shock. Gas flows in a shock front at velocity v_1 (use the shock's frame) and flows away with "downstream" velocity v_2 . By mass conservation, $\rho_1 v_1 = \rho_2 v_2$. Conservation of momentum energy for fluids — the Euler equations of fluid dynamics — give $\rho_2/\rho_1 = (\gamma + 1)/(\gamma - 1)$ for a strong (high-velocity) shock. γ is the adiabatic index — the ratio of specific heats, $\gamma = 5/3$ for monatomic gases, in which case $\rho_2/\rho_1 = 8/5$. So $v_1/v_2 = \rho_2/\rho_1 = 4$; the post-shock gas flows at $1/4$ the pre-shock velocity.

On either side of the shock, gas sees the other side of the shock approaching at $3/4 v_1$.

The key to this mechanism is that a relativistic particle, on crossing the shock, scatters to isotropic velocities on the other side, and thus gains energy. Each crossing gives an energy boost, vs. only a slight majority boost vs. deboost in 2nd-order acceleration.

Quantitatively, in this case we assume that, on average, p_x' goes to zero when the particles motion becomes isotropic in each frame, so each scatter gives the same energy change. So at each scatter, $E' = \gamma_v(E + p_x V)$. For a nonrelativistic shock, $\gamma_v \sim 1$, so $\Delta E \sim p V \cos \theta$, $\frac{\Delta E}{E} \sim \frac{V}{c} \cos \theta$.

Now the key point; the # particles within $\Theta \times \theta + d\theta$ is $\sin \theta d\theta$, & the rate at which they approach the shock is $c \cos \theta$ (x-component of velocity), so

$$p(\theta) = 2 \sin \theta \cos \theta d\theta \quad (\text{normalized so the integral converges}).$$

Then

$$\langle \frac{\Delta E}{E} \rangle = \frac{V}{c} \int_0^{\pi/2} 2 \cos^2 \theta \sin \theta d\theta = \frac{2}{3} \frac{V}{c}$$

for one trip across the shock. For a round trip

$$\langle \frac{\Delta E}{E} \rangle = \langle \Delta E \rangle \frac{4}{3} \frac{V}{c} = \frac{V_1}{c}.$$

Then, for escape probability P_{esc} , note that any particle going upstream will (if scattering slows it enough) cross again, but some downstream particles will be advected away. The rate of particles crossing the shock is $\frac{1}{4} N v \sim \frac{1}{4} N c$ ($N \neq \# \text{ density}$).

Similarly, the particles are removed from the shock at rate $\frac{1}{4} N V_2 = \frac{1}{4} N V_1$.

So the fraction of particles lost per unit time is $\frac{1/4 N v_i}{1/4 N c} = \frac{v_i}{c}$, and $P_{\text{esc}} = \frac{v_i}{c}$.

We can use this to predict the energy spectrum of the particles;

$$\frac{dN}{dE} \propto \left(\frac{E}{E_0}\right)^{-\left(1 + \frac{P_{\text{esc}}}{B}\right)} = \left(\frac{E}{E_0}\right)^{-\left(1 + \frac{v_i/c}{v_i/c}\right)} = \left(\frac{E}{E_0}\right)^{-2}$$

which is quite close to the inferred energy spectrum of synchrotron radiation in SNRs, AGN jets, etc., at $p \approx 2-3$. Several 2nd-order corrections can influence P , tending to steepen it slightly, as does the radiative cooling of high-energy particles.

This suggests that Fermi acceleration is responsible for the majority of cosmic ray acceleration in our galaxy, through the shocks of SNRs. The highest-energy CRs cannot be accelerated in SNRs, due to their very large Larmor (gyration) radius which will not keep them in the SNR. These must be accelerated by pulsars, or in jets from (e.g.) AGN. There have been recent suggestions that the highest-energy cosmic rays are coming from the directions of nearby AGN.

(For lower- E cosmic rays, their direction is changed by the Galactic B field.)

Auger map

Propagation of EM waves in a plasma: dispersion

How does the interstellar medium affect EM waves?

First we consider unmagnetized plasma. (Effects of dust - extinction - are covered in standard astro texts, & Astro 320, 322.)

EM waves in nonmag. plasma are only able to propagate if their freq. is larger than the plasma freq.

Hilary