

Define $\gamma = \frac{\omega}{\omega'}$, boost factor,

Consider what we see from jet, from different angles. Take $\gamma = 10$, $\beta = 0.995$.

Within angle $\frac{1}{\gamma}$ ($\frac{1}{10}$ rad or 6°) is half emitted radiation, blueshifted.

Can find angle inside which radiation is blueshifted, vs. redshifted;

$$\text{set } \omega/\omega' = 1.0, \text{ so } 1 = \frac{1}{\sqrt{1-\beta \cos \theta}}$$

$$\cos \theta = \frac{1 - \frac{1}{\gamma}}{\beta} \quad \text{For } \gamma = 10, \cos \theta = 0.90$$

$$\theta = 0.44 \text{ or } 25^\circ$$

So angles within 25° of forward see blueshifts.

To get intensity of radiation, compare solid angles in emitted, received frames.

Solid angle in moving frame is

$$d\Omega' = \sin \theta' d\theta' d\phi' = -d(\cos \theta') d\phi'$$

where θ' is polar angle. Note $d\phi = d\phi'$. Showed how to relate θ, θ' ;

$$\text{rearrange to get } \cos \theta' = \frac{\cos \theta - \beta}{1 - \beta \cos \theta}.$$

After some algebra & differentiation, can get

$$d\Omega' = \frac{1 - \beta^2}{(1 - \beta \cos \theta)^2} \sin \theta d\theta d\phi.$$

But we know $\gamma = \frac{(1 - \beta^2)^{1/2}}{1 - \beta \cos \theta}$, so $d\Omega' = \gamma^2 \sin \theta d\theta d\phi$,

$$\boxed{d\Omega' = \gamma^2 d\Omega}$$

So the freq. change in beaming also gives the solid angle compression or expansion.

Next, we'll consider how specific intensity I_ν transforms.

For Comptonization we discussed the phase space volume $dV = d^3\vec{p}d^3\vec{x}$, & saw it was Lorentz invariant, & that the # of photons per phase space volume is also invariant. Call this $\frac{dN}{dV} = f$, the phase space density.

Let's write ~~the~~ energy density per solid angle, (per freq.) in two ways;

using f , as

$$\left(\frac{\text{energy}}{\text{photon}}\right)\left(\frac{\text{photons}}{dV}\right) \cdot (d^3\vec{p})d\Omega = (h\nu) f p^2 dp d\Omega$$

$$\begin{aligned} \text{Transform } p = \frac{h\nu}{c}, \text{ so } &= \frac{h^3 \nu^3}{c^2} f dp d\Omega \\ &= \frac{h^4 \nu^3}{c^3} f d\nu d\Omega \end{aligned}$$

Also write it in terms of u_ν , energy density as $u_\nu(\nu) d\nu d\Omega$.

$$u_\nu(\nu) = I_\nu^{(R)} / c, \text{ so } = \frac{I_\nu(\nu)}{c} d\nu d\Omega.$$

Equating these two forms,

$$\frac{h^4 \nu^3}{c^3} f d\nu d\Omega = \frac{I_\nu(\nu)}{c} d\nu d\Omega$$

$$\text{So } \frac{I_\nu(\nu)}{\nu^3} \propto f \quad (\text{since } h, c \text{ are constants})$$

Since f is Lorentz invariant, so is $\frac{I_\nu(\nu)}{\nu^3}$.

Then $\frac{I_\nu}{\nu^3} = \frac{I'_\nu}{(\nu')^3}$. Since $\delta = \frac{\nu}{\nu'}$,

~~$I_\nu = \delta^3 I'_\nu$~~ Considering that
intensity is angle-dependent,

$I(\nu, \theta) = \delta^3 I'(\nu', \theta')$ Differences in ν, θ

complicate interpretation.

Understand this factor δ^3 as sum of four factors (assume $\delta > 1$, so boost, but applies in reverse for $\delta < 1$),

- (i) Solid angle reduced by factor δ^{-2} , so power per unit solid angle increases by δ^2 .
- (ii) Frequencies of individual photons are boosted by δ , so their energy & intensity are boosted by δ .
- (iii) The interval in which a group of photons arrives is compressed by δ , $\Delta t_{\text{obs}} = \Delta t' / \delta$. This increases the rate of photon arrival, thus intensity, by δ .
- (iv) The bandwidth $\Delta\nu'$ increases proportionally as ν' , by factor δ . As I_ν is per frequency, specific intensity drops as δ^{-1} .

→ Putting these together gives factor δ^3 in I_ν .
(Note that $S I_\nu$ is boosted by δ^4 .)

However, we don't measure the same frequency for ν, ν' . So must account for measuring different part of spectrum.

Take simple example power-law spectrum,
 $I_\nu \propto (\nu')^{-\alpha}$ e.g., synchrotron spectrum.

When we measure I_ν , refer to lower value of ν' (by factor δ), so see change of $\delta^{-\alpha}$.

$I(\nu) = \delta^{3-\alpha} I'(\nu)$ (at same freq.)

Bredt
fig 7.11