

Thermal Radiation

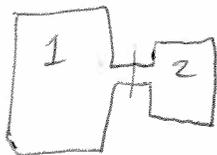
Thermodynamic equilibrium means that the matter & radiation are in a steady state at one temperature — that if left alone, the radiation will be absorbed & re-emitted in the same way by the matter (thus, ~~at~~ the matter & radiation are at one temperature.)

This condition generates blackbody radiation. If the matter & radiation aren't in equilibrium, then we have complexity (as seen above). We do, typically, assume that the matter in one location is in equilibrium with itself — meaning that the particle momenta, & electron states, follow Maxwell-Boltzmann distributions & the Saha equation.

This is the assumption of Local Thermodynamic Equilibrium or LTE. It is a common assumption in astrophysics but there are places it breaks down (we'll see some).

Blackbody radiation assumes that radiation & matter, after interacting enough times, come to equilibrium. Standard thought experiment: allow a closed space to reach equilibrium at temperature T , open tiny hole to measure the radiation.

I_ν will be independent of the space, except for T . To prove, consider two containers at same T ,



connected by a small aperture with a filter. If the filter only allows one frequency of light through, & I_ν is different, then heat will flow from one side to the other. That would violate the 2nd law of thermodynamics, so $I_\nu(1) = I_\nu(2)$ for any ν , if they have equal T . Thus, I_ν depends only on T .

We can thus define a blackbody intensity, sometimes called $B_\nu(T)$, or $I_\nu(T)$, called the Planck function.

In the volume at equilibrium, $I_\nu(T)$ is isotropic; don't use this fact outside that volume (e.g. starlight far from the star).

The source function S_ν (or I_s) of material in LTE is $I_\nu(T)$, the Planck function ($S_\nu = \frac{j_\nu}{4\pi\alpha_\nu} = I_\nu(T)$).

This doesn't mean that all matter in LTE radiates ~~the~~ with a blackbody spectrum — only such matter that is optically thick, so $I_\nu = I_\nu(T)$.

As we saw before, the radiation may be stronger at freq. of higher opacity, producing emission lines.

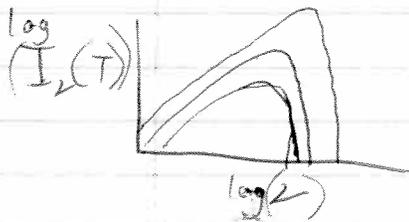
CMB Far-IR spectrum, COBE

The best-measured Planck function is from the cosmic microwave background, measured by satellites like COBE.

The average & peak photon energies are at about $kT = h\nu_{\text{peak}}$.

Particle & photon energies are often given in electron-Volts eV, where $1.0 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$ and 1 eV is about the energy of an optical photon.

~~So, in SI, $kT (\text{in eV}) \sim h\nu (\text{eV}) = T (\text{K}) / 11609$; one sees that typical photons from the sun ($T \sim 6000 \text{ K}$) are around $\frac{1}{2} \text{ eV}$.~~



Planck function: $I_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{(\exp(h\nu/kT) - 1)}$

This is the intensity in all directions measured inside a black body or the intensity of radiation from a distant source if your pixel is smaller than the source.

In wavelength units, $I_{\lambda}(T) = \frac{2hc^2}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{\lambda kT}\right) - 1}$

measured in $\frac{\text{W}}{\text{m}^2} \text{m}^{-1} \text{sr}^{-1}$
area ↑ wavelength ↑

Switching between freq. & wavelength units isn't trivial, & the peak λ_{peak} is not c/ν_{peak} use $d\lambda = -(c/\nu^2)d\nu$.

Key Features of Planck law:

If one integrates the blackbody $I_{\nu}(T)$ at a surface over frequency, one can find the emergent flux:

$$F_{\text{surf}}(T) = \pi \int \frac{2h\nu^3}{c^2} \frac{d\nu}{(e^{h\nu/kT} - 1)} = \sigma T^4$$

↑ Stefan-Boltzmann constant

Consider limit of Planck law in the low-frequency limit, $h\nu \ll kT$ (the Rayleigh-Jeans law).

Expand the exponential, $\exp\left(\frac{h\nu}{kT}\right) - 1 \approx \frac{h\nu}{kT} + 1 - 1$

So for $h\nu \ll kT$, $I_{\nu}^{\text{RJ}}(T) = \frac{2\nu^2}{c^2} kT$ (lacks h !)

Originally derived classically. Applies in, e.g. radio, or freq. well below blackbody peak.

If R-J law applied at all freq, total $E \rightarrow \infty$. This was the "ultraviolet catastrophe" that inspired Planck to invent h , & began quantum mechanics.

For $h\nu \gg kT$, simplifies to Wien law.

Here the "1" in denominator is $\ll \exp(h\nu/kT)$, so

$$I_{\nu}^{\text{Wien}}(T) = \frac{2h\nu^3}{c^2} \exp\left(-\frac{h\nu}{kT}\right)$$

This is a decaying exponential, which describes the high- ν extreme of blackbody spectrum.

Wien's Law (Wien's Displacement Law)

In between, we can identify the peak ν , ν_{\max} , for $I_{\nu}(T)$, by $\left. \frac{\partial I_{\nu}(T)}{\partial \nu} \right|_{\nu=\nu_{\max}} = 0$

Substitute $x = \frac{h\nu}{kT}$ into $I_{\nu}(T)$ expression, get (after algebra)

$$h\nu_{\max} = 2.82 kT \quad \text{or} \quad \frac{\nu_{\max}}{T} = 5.88 \times 10^{10} \text{ Hz/deg K}$$

Finding the peak for wavelength,

we get $\lambda_{\max} T = 2.9 \times 10^{-3} \text{ m} \cdot \text{K}$

Monotonicity with Temperature

For any 2 blackbody curves, the higher- T curve lies above the other everywhere.

How do we know?

$$\text{Take } \frac{\partial I_{\nu}(T)}{\partial T} = \frac{2h^2\nu^4 \exp(h\nu/kT)}{c^2 k T^2 [\exp(h\nu/kT) - 1]^2}$$

We see this is always positive (for all T, ν), so increasing T always increases $I_{\nu}(T)$.

Definitions of temperature; 3 related to Planck spectrum.

- Brightness temperature: Specific intensity (brightness) at ν equals that of a blackbody at some temperature T . For any I_{ν} , define $T_b(\text{brightness})$ as $I_{\nu} = I_{\nu}(T_b)$, the brightness temperature.



This expression is useful in radio astronomy, where sources can often be resolved (so can measure I_ν), & where the R-J law is generally applicable, $I_\nu^{RJ}(T) = \frac{2\nu^2}{c^2} k T_b$

Note that T_b is linear with I_ν .
So this makes a simple transfer equation,

$$\frac{dT_b}{dz_\nu} = -T_b + T \quad \begin{array}{l} \leftarrow \text{material of interest} \\ \leftarrow \text{starting ray} \end{array}$$

Get $T_b = T_b(0)e^{-z_\nu} + T(1 - e^{-z_\nu})$ for $h\nu \ll kT$.
↑
measured

T_b is not useful near peak of BB spectrum, so this concept used mostly in radio astronomy.

Note that $I_\nu \leq I_\nu(T)$ always, so the brightness temperature is the minimum temperature of the emitting material.

- Color temperature - If you can measure the spectrum but don't know Ω (& thus not I_ν), and the spectrum looks like a blackbody, you can estimate the temperature by fitting a blackbody. In simplest case, just measure peak of spectrum & apply Wien's Law.

We can try to use color temperature to measure the size of neutron stars. Need to know the distance, & understand atmosphere (will alter spectrum). If measure flux, temperature, can estimate $I_\nu(T)$ from temp, then calculate Ω , & then radius.

- Effective temperature: Derive from total flux integrated over all λ , radiated at surface.

Equate the actual flux F to flux of a Blackbody at ~~some~~ temp T_{eff} , $F = \sigma T_{\text{eff}}^4$.

This is how we define the Sun's temperature, based on its total luminosity. — it's the temp of a BB that gives that L .

Gray bodies: Opaque bodies at one T_{surf} are blackbodies if they absorb all radiation. However, most solids reflect some fraction of light (depends on freq.) Fraction reflected is albedo, A , in range 0-1. True blackbody has $A=0$, a perfect mirror has $A=1$. (Earth has $A=0.3$, Venus has $A=0.75$.) "Gray" bodies are in-between. Absorbed light heats the object, which also emits a Planck-like spectrum.

Temp of a gray body comes to equilibrium with the temp appropriate for absorbed fraction of the incoming radiation. Can trap some heat by having different opacities at different λ .

So, optical light reaches Earth, but emitted infrared radiation is trapped by the opacity of H_2O , CO_2 , CH_4 , etc.

Earth temp that balances absorption, emission is higher than if no "greenhouse effect" existed. Adding CO_2 increases opacity of atmosphere, requiring a higher temperature for Earth.

We can look for extrasolar planets through either reflected light (optical) or their thermal emission (infrared).

Consider a Jupiter around another star (like the Sun), with albedo 0.5. It reflects a fraction of star's light $\frac{\pi R_J^2 A}{4\pi d_{J-\star}^2}$ ← albedo

$$\text{So } \frac{F_{\text{Jup}}}{F_{\odot}} = \frac{\pi R_J^2 A}{4 d_{J-\star}^2} = \frac{(11.2 \times 6.4 \times 10^4 \text{ m})^2 \times 0.5}{4 \times (5.2 \times 1.5 \times 10^{11} \text{ m})^2} = 10^{-9}$$

Fomalhaut

So to detect such a planet requires a contrast of 10^9 , at a very small angular spacing — very difficult!

Consider IR light, produced from thermal emission of planet.

Take Jupiter & Sun, use $I_{\lambda}(T)$ Planck function,

$$I_{\lambda}(T) = \frac{2hc^2}{\lambda^5} \frac{1}{(e^{hc/\lambda kT} - 1)} \quad \text{Take } \begin{matrix} T_{\text{Jup}} \sim 200 \text{ K} \\ T_{\odot} \sim 6000 \text{ K} \end{matrix}$$

$$\frac{F_{\odot}}{F_{\text{Jup}}} = \frac{I_{\lambda}(6000 \text{ K}) R_{\odot}^2}{I_{\lambda}(200 \text{ K}) R_{\text{Jup}}^2} = \frac{I_{\lambda}(6000 \text{ K}) R_{\odot}^2}{I_{\lambda}(200 \text{ K}) R_{\text{Jup}}^2}$$

Consider two choices of λ ; 500 nm (green optical light),
 $\frac{I_{\lambda}(6000 \text{ K}, 500 \text{ nm})}{I_{\lambda}(200 \text{ K}, 500 \text{ nm})} \approx 10^{60}$ (Explains why it's dark when you close your eyes.)

For $\lambda = 10 \mu\text{m}$,
 $\frac{I_{\lambda}(6000 \text{ K})}{I_{\lambda}(200 \text{ K})} \sim 5000$



$$\text{Thus } \frac{F_{\odot}}{F_{\text{Jup}}} = 5000 \times \frac{R_{\odot}^2}{R_{\text{Jup}}^2} \sim 5 \times 10^5$$

much smaller ratio than 10^9

HR 8799

GG Lupi

Young, cooling planets are even hotter, easier to see.