

Consider a cosmic ray with  $\gamma \sim 10^5$ . These are common, about one per  $m^2$  per s on Earth.

$$\gamma^2 = 10^{10}$$

This can boost a  $10^9$ -Hz (30 cm) radio photon to  $10^{19}$  Hz = 40 keV X-ray.

Higher energy cosmic rays push photons farther, producing gamma-rays.

### Multiscale Milky Way

We can see the Compton scattering due to cosmic rays in images of the Milky Way in gamma-rays.

Cosmic ray locations are clearly identified in low-energy radio (due to synchrotron radiation) & high-energy  $\gamma$ -rays, principally boosting photons from starlight.

Note that the  $\gamma \sim 10^5$  electron has  $E \sim 10^{11}$  eV so only loses  $\sim 10^{-7}$  of its energy in one collision.

### Rate of energy loss

We can find how quickly electrons lose energy. Start with cross section.

If photon, in electron frame, has  $E \ll mc^2$  ( $\gamma \gamma h\nu \ll mc^2$ ) then can use Thomson cross-section,  $\sigma_T = 6.65 \times 10^{-29} m^2$ .

In this case, we showed  $\frac{dE}{dt} = \sigma_T c U_{\text{rad}}$  is the energy loss rate for an electron - in its frame. Converting to our frame is a little tricky, write this as

$$\frac{dE'}{dt'} = \sigma_T c U'_{\text{rad}}$$

Larmor formula radiated

~~Since~~ power in the emitted frame is symmetrical, so in ~~all directions~~ its overall momentum is zero.

Thus the  $dE$  transforms as

$$U^2 = \gamma(U' - \beta c p_x) \rightarrow U^2 = \gamma U'$$

or  $dE = \gamma dE'$

However, we have seen  $\Delta t = \gamma \Delta t'$ ,  
from relativistic time dilation.

$$\text{Thus } \frac{dE}{dt} = \frac{\gamma dE'}{\gamma dt'}.$$

So emitted power is Lorentz invariant for an emitter with front-back symmetry in its (instantaneous) rest frame.

$$\text{So } \frac{dE}{dt} = \alpha c U_{\text{rad}}.$$

To transform the radiation density, we need an argument using phase space volumes.

Bradt does this in chapter 7.8; we use a simpler argument from Rydberg & Lightman.

Consider a group of particles with a small spread in position & momentum around their average  $\bar{x}, \bar{p}$ .

They occupy a volume element  $d^3x' = dx' dy' dz'$  in their frame.

They also have a "momentum volume element"  $d^3p' = dp'_x dp'_y dp'_z$ . We choose these particles to all have the same energy, so  $dU' = 0$ ,

Thus they occupy an element of 6-dimensional phase space  $dV' = d^3p' d^3x'$ . This  $dV'$  element, we will see, is Lorentz invariant.

Let an observer have velocity  $\beta$ , & move along the  $x$  axis. The  $dy$  and  $dz$  distances are unchanged, but the  $x$  direction contracts,  $dx = \gamma^{-1} d\bar{x}$ , so

$$d^3x = d^3\bar{x} / \gamma.$$

$$d^3x = \frac{d^3x'}{\gamma}$$



Next consider momentum.  $p_x$  and  $p_z$  are still unchanged, but  $d\tilde{p}_x = \gamma(d\tilde{p}'_x + \beta dU/c)$ .

Since  $dU=0$ ,  $d\tilde{p}_x = \gamma d\tilde{p}'_x$ , so  $d^3\tilde{p} = \gamma d^3p'$

But  $dV = d^3p d^3x$ , so the  $\gamma$  dependence cancels &  $dV$  is a Lorentz invariant.

Now consider the photon phase space distribution function  $n(p)$ . Since the phase space volume is Lorentz invariant, the phase space density function will also be invariant.

The # of particles within each phase space volume is a countable quantity, & thus invariant, so  $\frac{dN}{dV} = n(p)$  is also invariant.

We saw  $d^3p$  transforms like energy, so  $d^3p/E$  is also Lorentz invariant.

The total power scattered, in the electron's rest frame, is

$$\frac{dE'}{dt'} = \alpha c U_{rad} = \alpha c S E'_p n' d^3 p'$$

where  $n' d^3 p'$  is the space number density of incident photons (phase space density times momentum space).

We assume Thomson scattering in the electron frame, so  $E'_f = E'_i$ .

Since emitted power is Lorentz invariant,  $\frac{dE_f}{dt} = \frac{dE'_f}{dt'}$   
So

$$\frac{dE_f}{dt} = \alpha c S E'_i n' d^3 p' \left( \frac{E'_i}{E_i} \right) = \alpha c S (E'_i)^2 n \left( \frac{d^3 p}{E_i} \right)$$

where we've noted that  $n(p)$  and  $\frac{d^3 p}{E}$  are Lorentz invariant.

Now,  $E'_i = E_i \gamma (1 - \beta \cos\theta)$  in the general case, from our Doppler shift formulas. (We saw that for isotropic dist'n of angles,  $\langle E'_i \rangle = \frac{2}{\sqrt{3}} \gamma E_i$ .) Plugging this in,

$$\frac{dE_f}{dt} = \alpha_F c \gamma^2 \int (1 - \beta \cos\theta)^2 E_i n d^3 p \quad \text{all in the observer's frame!}$$

Now assume an isotropic photon distribution. Then

$$\begin{aligned} \langle (1 - \beta \cos\theta)^2 \rangle &= \langle -2\beta \cos\theta + \beta^2 \cos^2\theta \rangle \\ &= \langle 1 \rangle - \langle 2\beta \cos\theta \rangle + \langle \beta^2 \cos^2\theta \rangle \\ &= 1 - 0 + \frac{1}{3} \beta^2 \end{aligned} \quad \text{So}$$

$$\frac{dE_f}{dt} = \alpha_F c \gamma^2 \left(1 + \frac{1}{3} \beta^2\right) \int E_i n d^3 p$$

Finally, note that  $\int E_i$  times the space # density, gives the radiation energy density,  $U_{rad}$ . So

$$\boxed{\frac{dE_f}{dt} = \alpha_F c \gamma^2 \left(1 + \frac{1}{3} \beta^2\right) U_{rad}}$$

How much of this energy comes from the initial photons, & how much from the electrons?

In the Thomson scattering regime, the change in the initial radiation field is

$$\boxed{\frac{dE_f}{dt} (\text{photons}) = -c \alpha_F U_{rad}} \quad \begin{matrix} (\text{just the negative of}) \\ (\text{the scattered light}) \end{matrix}$$

so subtract this from above, getting

$$\frac{dE_f}{dt} (\text{electrons}) = c \alpha_F U_{rad} \left[ \gamma^2 \left(1 + \frac{1}{3} \beta^2\right) - 1 \right]$$

$$\text{We do a trick: } \gamma^2 - 1 = \cancel{\frac{1}{2} \beta^2 \cancel{(\gamma^2 - 1)}} \cancel{\beta^2}$$

$$= \gamma^2 - \gamma^2 (1 - \beta^2) = \gamma^2 \beta^2 \quad \text{So}$$

$$\boxed{\frac{dE_f}{dt} (\text{electrons}) = \frac{4}{3} c \alpha_F \gamma^2 \beta^2 U_{rad}}$$

If the  $E_i' \gtrsim m_e c^2$ , a longer derivation is needed, giving a result with an added factor of

$$\left[ 1 - \frac{6.3 \gamma \langle E_i' \rangle}{m_e c^2 \langle E_i \rangle} \right].$$

For the Thomson scattering case, we get in SI units

$$\frac{dE}{dt}(\text{electrons}) = -2.66 \times 10^{-20} \beta^2 \gamma^2 U_{\text{rad}}$$

in W, with  $U_{\text{rad}}$  in  $\text{J/m}^3$ , for a single electron.

We have distinguished between the total outgoing power in IC radiation ( $\frac{dE_F}{dt}$ ) and that contributed by the electrons ( $\frac{dE_F}{dt}(\text{electrons})$ ). In most cases, the electrons supply most of the energy, so one can neglect the distinction, & just use the simpler  $\frac{dE_F}{dt}(\text{electrons})$  form.

To get the volume emissivity we just multiply  $\frac{dE_F}{dt}$  by  $n_e$ , the electron # density, so

$$\hat{j}_{\text{IC}} = \frac{4}{3} \alpha c \beta^2 \gamma^2 n_e U_{\text{rad}} \quad (\text{volume emissivity, } \text{W/m}^3)$$

This is integrated over all  $\gamma$ , but assumes a single electron velocity. If the electrons are thermally distributed, they are probably nonrelativistic, so can assume  $\gamma \approx 1$ . Take  $\frac{1}{2} m v^2 = \frac{3}{2} k T_e$ ,  $\langle \beta^2 \rangle = \frac{3 k T_e}{m c^2}$ . Then

$$\hat{j}_{\text{thermal IC}} = \left( \frac{4 k T_e}{m_e c^2} \right) c \alpha n_e U_{\text{rad}}$$

When nonthermal electrons are involved, energies can be relativistic. Describe electron energies as power-law, typically,  $N_e(\gamma) d\gamma = N_1 \gamma^\alpha d\gamma (\text{m}^{-3})$

between  $\gamma_{\min}$  &  $\gamma_{\max}$ , zero elsewhere, with  $N_1$  &  $\alpha$  as constants.

So we find the total power by summing over the distribution,

$$J_{\text{nth, IC}} = \int \frac{dE}{dt}(\gamma) N_e(\gamma) d\gamma$$

$$= \int_{\gamma_{\min}}^{\gamma_{\max}} \frac{dE}{dt}(\gamma) N_1 \gamma^\alpha$$

$$J_{\text{nth, IC}} = \frac{4}{3} \alpha c \beta^2 U_{\text{rad}} \left( \frac{N_1}{\alpha + 3} \right) \left( \gamma_{\max}^{-\alpha-3} - \gamma_{\min}^{-\alpha-3} \right)$$

(note that  $\alpha$  is typically negative)

(added)

Spectra from ICS.

(Using Langair ch. 4.3)

Pure Thomson scattering changes direction, but leaves spectrum of radiation unchanged. Compton scattering off (relatively) cold electrons gives a small downward shift to incident spectrum, while scattering off hot electrons gives a positive shift to spectrum.

If the medium is optically thick to numerous Compton scatterings, the output spectrum will (to some extent) resemble a blackbody at the electron temperature. (Note that a true blackbody dist'n requires mechanism to create more photons, not available in pure Comptonization.)

Average energy gain by photons will be

$$\frac{\frac{dE}{dt}(\text{electrons})}{\frac{dE}{dt}(\text{photons})} = \frac{4}{3} \gamma^2 \beta^2 \left( = \frac{\Delta E}{E} \right) \text{In low-electron-energy limit, } \frac{4}{3} \beta^2.$$

If the electrons have a thermal distribution,

$$\frac{\Delta E}{E} = \frac{\frac{dE}{dt}(e^-)}{\frac{dE}{dt}(\text{photons})} = \frac{4kT_e}{m_e c^2}; \quad \frac{dE}{dt}(e^-) = \frac{4kT_e}{m_e c^2} \frac{dE}{dt}(\text{photons})$$

In order for Comptonisation to change the spectrum substantially, the electrons must be hotter than the radiation field, & many of the photons must interact with electrons.

First, how hot must the electrons be?

We see that the average energy gain by photons due to IC scattering (assuming Thomson scattering in e' frame) is  $4kT_e/m_e c^2$ .

But if we permit the photons to have high energies we should consider Compton scattering; in that case photons lose energy. We saw

$$h\nu_s = \frac{h\nu}{1 + \frac{h\nu}{m_e c^2}(1 - \cos\theta)} \quad \text{which we rearrange to}$$

$$\frac{h\nu}{m_e c^2}(1 - \cos\theta) = \frac{h\nu - h\nu_s}{h\nu_s} = \frac{\Delta E}{E}$$

If we take the average over  $\Theta$ , the dependence cancels out, so

$$\frac{\Delta E}{E} (\text{Compton effect}) = -\frac{h\nu}{m_e c^2}$$

Thus we see that the net  $\frac{\Delta E}{E}$  is  $\frac{4kT_e - h\nu}{m_e c^2}$  for photons

& that  $4kT_e \gg h\nu$  is the criterion for energy to be transferred to photons (if the reverse,  $E$  is transferred to electrons).

The average photon should also interact with 1 or more electrons, so optical depth should be  $\gtrsim 1$ .  $\Sigma_e$  for Thomson scattering is

$$\Sigma_e = N_e \alpha \frac{L}{\text{volume density}} \times \text{path length}$$

If  $\Sigma_e \gg 1$ , photons do random walk to escape, so travel a distance  $d = \sqrt{N}l$ , where  $l$  is the mean free path. So the # of scatterings is  $N = \Sigma_e^2$

If  $\Sigma_e < 1$ , then the average # of scatterings is just  $\Sigma_e$ .

We define the Compton  $\gamma$  parameter as the average  $\Delta E/E$  per scattering times the average # of scatterings.

So  $\gamma$  estimates how much a photon will change its energy while passing through.

After one scattering  $\frac{E_f}{E_i} = 1 + \frac{4kT_e}{m_e c^2}$

so after  $N$  scatterings,  $\frac{E_f(N)}{E_i} = \left(1 + \frac{4kT_e}{m_e c^2}\right)^N$

We'll use the Thomson approximation, so  $4kT \ll m_e c^2$   
we can approximate

~~$\left(1 + \frac{4kT}{m_e c^2}\right) \approx \exp\left(\frac{4kT}{m_e c^2}\right)$~~  This (didn't like)

~~$\exp(\beta)$~~

$\gamma = \left(\frac{4kT}{m_e c^2}\right) \text{Max}\{\Sigma, \Sigma^2\}$ , the Compton optical depth.

After the photon is boosted a few times, it comes into thermal equilibrium with the electrons.

This photon distribution won't be a blackbody, but rather a Bose-Einstein distribution, of form

$$u_\nu d\nu = \frac{8\pi h\nu^3}{c^3} \left( \exp\left(\frac{(h\nu)}{kT}\right) + \mu \right)^{-1} d\nu$$

where  $\mu$  is the chemical potential, describing how the number density of photons falls below that required for a black body.

Longair figure In a Planck dist'n, photons can be created or destroyed. But here, scattering conserves photon #, allowing a deficit of photons. Setting  $\mu=0$  reproduces the Planck dist'n. Increasing  $\mu$  decreases intensity by factor  $\exp(-\mu)$ .

Equation relating how radiation field transforms toward Bose-Einstein, for any value of  $\mu$ , is the Kompaneets equation. Calculates interchange of energy among photons & electrons, & also effects of chemical potential & of occupation number (# of photons per state), typically defined per unit volume ( $n(z)$ ).

Start XSPEC

Can see how Comptonizing hot gas changes its spectrum by changing  $\zeta, kT_e$  in XSPEC.

Use CompTT model (by Lev Titarchuk, 1994.5) which is Comptonization of soft black body spectrum.

Photons Note scales;  $\log E$  on x-axis,  $\frac{E}{\text{photons}} \text{ s}^{-1} \text{ cm}^{-2} \text{ keV}$  on y-axis. Multiplying by  $E$  gives  $\frac{E^2}{\text{photons}} \text{ s}^{-1} \text{ cm}^{-2} \text{ keV}$  so gives energy flux.

To get total  $E$  under curve on  $\log(E)$  scale, plot  $E^2 \times \frac{\text{photons}}{\text{s} \cdot \text{cm}^2 \cdot \text{keV}} \times \underline{2F_E}$ .

At low optical depths, Comptonized spectrum looks linear (in log-log space), could be fit with power-law spectrum.

### Applications of Comptonization: X-ray binaries

MAXI all-sky image - 2-10 keV, majority of X-ray sources are accreting BHs or NSs.

Cyg X-1 illustration Prototype is Cygnus X-1, black hole accreting from wind of high-mass star. X-ray binaries go through "spectral states" with different spectral distributions.

Cyg X-1 spectra "Soft" dominated by accretion disk. Optically thick material, heated by viscous interactions (radiating away its potential energy as descends to black hole). Accretion spectrum a "disk black body". Series of annuli emitting BB radiation, higher T closer in.

XSPEC: BB vs. disk Effect is to broaden BB spectrum.



BB spectrum dominates at low energy, in soft state.

At high ( $\gtrsim 3$  keV) energies, emission dominated by Comptonized photons; in "hard" state

soft component disappears, & Comptonized part stronger.

Other X-ray binaries show a soft disk black body moving out to lower energies. Suggested that the inner edge of the disk recedes.

in the low/hard state, & that the mass transfer interior to this point is not via an optically thick disk, but an optically thin flow. This material can adve<sup>c</sup>t into the black hole without losing all its kinetic energy, so is more radiatively efficient flow.

Such a flow is associated with collimated outflows and hard X-ray emission, which is well-fit by Comptonized X-rays. Indicates a population of very energetic electrons, of KE up to 100s of keV ( $\sim m_e c^2$ )

### Compton Shoulders

Can study series of Fe K-shell lines in X-ray binaries. High-energy X-ray hitting neutral Fe will knock out K-shell (lowest E) electron. Higher electrons will fall in from L or M shells, giving  $K\alpha$ ,  $K\beta$  lines, etc. (6.4, 7.1 keV). The energy above which an inner-shell electron is ejected is the Fe K edge (7.125 keV), thus fewer photons above this E can escape. (Similar to H line & edge structure.)

### GX 301-2 X-ray grating spectrum

High-res X-ray spectrum of high-mass X-ray binaries clearly show these lines & edge, showing neutral gas (dense stellar wind from massive star). Also show hump at lower energy than bright line, "Compton shoulder". Line photons strike cold electrons, & lose energy. Shows existence of ionized gas with cooler electrons — part of stellar wind is ionized.