

Curvature Radiation (Bradt 8.6)

Relativistic electrons in strong B fields may have $h\nu_{\text{syn}} \sim E_{\text{electron}}$, requiring quantum calculations

Obviously $\nu_{\text{syn}} \neq E_{\text{electron}}$!

For 10^8 T (typical NS field), limit is $\sim 15 \text{ MeV}$
before quantum effects ~~limit~~ synchrotron radiation
eliminate electron's \perp velocity.

If e^- velocity is along field line, electron won't be accelerated, & won't lose energy. If pitch angle very small, loses little energy.

If B field is slightly curved, it forces slight acceleration of relativistic e^- moving along field lines producing curvature radiation.

Pulsar schematic

Relevant when radius of curvature of B fields is much larger than electrons gyroradius (around B field).

Consider frequency as in synchrotron radiation, but use a fake B field (call it B_f) specified to give the radius of curvature R for a given energy $E = \gamma mc^2$.

The relativistic cyclotron frequency ($=$ orbital rotation frequency) is

$$\nu_{\text{orb}} = \frac{eB_f}{2\pi fm}$$

This is also the frequency with which the particle travels around its loop of radius R , so

$$\nu_{\text{orb}} = \frac{v}{2\pi R} = \frac{c}{2\pi R} \quad (\text{if } v \approx c)$$

$$\frac{c}{2\pi R} = \frac{eB_f}{2\pi fm}, \quad B_f = \frac{\gamma mc}{eR}$$

We use this inferred B_f to characterize the acceleration of the particle, & thus to estimate the peak frequency of its radiation, as the emission is produced by accelerating relativistic electrons as in synchrotron radiation.

Using $\nu_{\text{syn}} \propto \gamma^3 \nu_{\text{orb}}$, $\nu_{\text{syn}} = \frac{\gamma^2 e B_f}{2\pi m} = \frac{\gamma^2 e \gamma m c}{2\pi m e \gamma}$

$$\nu_{\text{curv}} = \frac{\gamma^3 c}{2\pi e}$$

Sometimes a critical frequency $\nu_c = 1.5 \nu_{\text{syn}}$ is given, both for synchrotron & for curvature.

Note that in synchrotron radiation, as energy (γ) increases, the orbit widens, so $\nu_{\text{syn}}(\text{synch}) \propto \gamma^2$, while $\nu_{\text{curv}} \propto \gamma^3$ since the orbital radius is fixed.

$$E_{\text{curv}} = h\nu_{\text{curv}} = \frac{\gamma^3 h c}{2\pi e}$$

The power emitted is similar to synchrotron; using $B_f = \gamma m c^2 / e R$, assuming $\phi = \pi/2$, $\beta = 1$, $\epsilon = 1$,

$$\frac{dU_{\text{curv}}}{dt} = \frac{-1}{6\pi\epsilon_0} \frac{e^4 \gamma^2 B_f^2}{m_e^2 c^4} \delta' = \frac{-1}{6\pi\epsilon_0} \frac{e^4 \gamma^2}{m_e^2 c^4} \left(\frac{\gamma m c}{e R} \right)^2 =$$

$$\frac{dU_{\text{curv}}}{dt} = \frac{-1}{6\pi\epsilon_0} \frac{e^2 \gamma^4 c}{R^2}$$

One can rewrite this as $\frac{1}{6\pi\epsilon_0} \frac{e^2 \gamma^4}{c^2} \left(\frac{c^2}{R} \right)^2$, which looks like the Larmor equation with c^2/e , equivalent to v^2/R , being the (centrifugal) a_\perp .

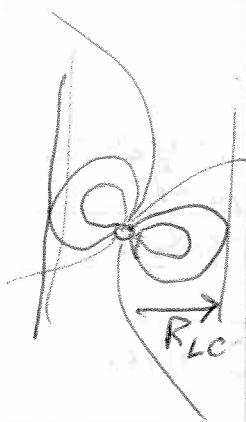
Note that $\frac{dU_{\text{curv}}}{dt} \propto \gamma^4$, vs $(\frac{dU}{dt})_{\text{syn}} \propto \gamma^2$, again since

electrons can't expand the size of orbiting radius to compensate.

[02]

Curvature radiation likely primary energy loss mechanism in pulsar magnetospheres.

Pulsars spin rapidly (< 10 s), and have large ($> 10^9$ T) B-fields, so produce large electric potentials, & accelerate electrons, etc.



Spectrum like synchrotron, with peak at ν_{curv} . How estimate ρ , curvature radius?

Pulsar B field rotates with star.

B-field can't stay connected past light cylinder, where particle must travel at c to rotate with star;

$$\text{velocity } = c = \frac{2\pi R_{LC}}{\text{Period}}. \quad R_{LC} = \frac{cP}{2\pi} = \frac{c}{2\pi \nu_{PSR}}$$

For Crab pulsar, $\nu_{PSR} = 30.3 \text{ Hz}$, so $R_{LC} = \frac{3 \times 10^8 \text{ m/s}}{2\pi \times 30.3}$,

$$R_{LC} = 1.6 \times 10^6 \text{ m},$$

Models of curvature radiation from pulsars expect majority from "outer gap". Use those field lines to estimate $\rho \sim R_{LC}/2$, so for Crab $\rho \sim 8 \times 10^5 \text{ m}$.

$$\text{So } \nu_{\text{curv}} = \frac{\gamma^3 c}{2\pi R} = 2\gamma^3 \nu_{PSR}$$

$$\nu_{\text{curv, Crab}} = 60 \text{ g}^3$$

If we measure the pulsed curvature-rad spectrum of Crab out to $\sim 25 \text{ GeV}$ (as seen) we can

find γ . $\nu_{\text{curv}} = E/h \sim \cancel{2.6 \text{ eV}} \times \frac{2.5 \times 10^9 \text{ eV} \times 1.6 \times 10^{-19} \text{ J/eV}}{6.6 \times 10^{-34} \text{ Js}}$

$$\gamma = \left(\frac{6 \times 10^{24}}{60} \right)^{1/3} = 5 \times 10^7$$

Crab pulses

Particles accelerated on open field lines will continue flying away from pulsar, creating pulsar wind such as Crab nebula. However, not clear that all acceleration takes place near pulsar - max γ seen in curvature radiation (5×10^7) is $\sim 100^{\times}$ less than seen in synchrotron radiation from Crab Nebula.

Fermi pulsars

Curvature radiation common from pulsars - Fermi has identified dozens of γ -ray pulsars, & many suspected pulsars w/o pulsations yet detected. γ -ray luminosity can be a few % of total energy loss from pulsar spinning down.

Rotational energy lost as magnetic dipole radiation & in form of accelerated particles.

Radio detection

Radio pulses from pulsars harder to explain. Total radio power small (10^{-5} of NS spin down), but inferred brightness temp $\approx 10^{11}$ K (compare Compton limit of $\sim 10^{10}$ K).

Can get around limit by coherent emission by large # of particles, radiating together. If electrons radiate in phase, emitted radiation grows rapidly.

Consider one radiating electron; $P_{\text{L,atom}} \propto e^2 q^2$. Multiple incoherent electrons sum, $P_L \propto N e^2 q^2$. But if electrons radiate in phase, act like one electron of charge $q = Ne$, so $P_{\text{L,coh}} \propto N^2 e^2 q^2$.

This applies in formulas for synchrotron, curvature rad., so clearly coherent emission drastically increases power.

Two questions; where do large #s of particles come from, & how do they radiate?

Acceleration of electrons from NS surface is done by strong E fields generated by moving B field of NS. Electrons follow B-field lines, & eventually emit γ -rays as curvature radiation. (We'll look at this acceleration in more detail soon.)

If curvature γ -rays have $E > 2 \times (511 \text{ keV})$, then they can produce a pair-electron & positron when they hit something (including another photon, or even just interaction with pulsar's own strong B field). The newly produced pair accelerate in opposite directions, emit γ -rays. Those γ -rays produce more pairs, etc. etc., populating the pulsar's magnetosphere with charged particles, until an equilibrium is established (charge density partly shorts out the accelerating E fields).

So the magnetosphere being populated requires acceleration to sufficient energy for pair production, $> 1.2 \text{ MeV}$. This constrains γ . The E of accelerated electrons depends on B and P_{spin} of pulsar; appropriate combination of B & P provides a "death line" below which pair production doesn't happen, & pulsar radio pulsations generally aren't seen.

How do electrons radiate radio waves? Evidence from pulse profiles that radio, γ -rays often arise from different locations.

Pulsar
P vs. B

γ -ray,
radio
pulse
profiles

Radio emission seems to come from directly over magnetic poles. Here, B-field lines are straighter, making curvature explanations difficult.

Problem not ~~solved~~, but current theories focus on MHD plasma waves in the out-flowing electrons.

~2000 pulsars currently known, wide range of B, spin period, companion stars, etc.

PSR 1913+16
Orbital Decay
(Weisberg+10)

One key result has been the confirmation of GR's prediction of gravitational radiation. Monitoring the two-NS binary, PSR 1913+16, allows very careful measurement of its very short (7.75 hours) orbital period. Its orbital period decays exactly as predicted if the NSs are emitting gravitational waves.

Astrophysical Jets & AGN

M87-wide
M87-Hubble
M87-Hubble, jet

M87-radio, optical
& X-ray jet

Jets observed in centers of galaxies

Emission on wide range of frequencies, with polarization, suggests synchrotron radiation

for most,

Some X-ray jets may be IC from CMB. This has the bonus that intensity of CMB increases with z , matching decrease of intensity with z for constant emission, so IC jets scattering CMB are equally bright at any z .