

of electron column density,

Free electron
model of Galaxy

Nearby supernova remnants
enhance density in some directions;
local voids decrease density.

We can then reverse process to estimate
distances for newly detected pulsars

Fermi
pulsars

Iterative process. Comparing DM tells
whether two pulsars could both be in the
same globular cluster. (tiny DM changes
between pulsars in same cluster allow measurement
of gas density outside globular clusters)

Faraday Rotation

(Also produces whistler phenomenon
in ionosphere.)

Pulsars, supernova remnants, & AGN jets are often
linearly polarized. Passing through ~~an~~ ionized
gas with B field will cause the polarization
angle to rotate. This is due to different indices
of refraction (& dielectric constants) for
left & right circularly polarized waves.
The linearly polarized wave can be described
as a sum of left & right circularly polarized
waves.

Bredt 11.8;
electron
motions

Easy way to illustrate; under EM wave
electrons shake in \vec{E} plane. With no B field,
electrons produce \vec{E} in some plane as original \vec{E} .

With B , electrons feel magnetic force as
they move, bending trajectory (if B is parallel
to propagation, will tilt to left). This rotates
electron's direction of oscillation, & thus the EM wave's
 \vec{E} field (which is partially reradiated by electron).

Note that EM wave's own B field reverses direction
each half cycle, so rotation cancels out; external B
field gives small rotation which can add up.

Circular polarization

Circularly polarized EM waves trace out spiral in space at a given time.
In time, corkscrew moves to right w/o rotation. (RCP)

Can describe a right-circular polarized wave as sum of two linearly polarized at 90° in angle, & 90° out of phase.

Example:

$$E_x = E_0 \cos(\omega t - kz)$$

$$E_y = E_0 \cos(\omega t - kz - \pi/2)$$

If $z=0$, $E_x = E_0$ at $\omega t = 0$, while $E_y = 0$ at $t=0$.
At $\omega t = \pi/2$, $E_y = E_0$, $E_x = 0$; etc.

Circularly polarized radiation common in astrophysics, if particle observed in circular motion, e.g., electron in B field; magnetic white dwarfs, stars with spots, etc.

Circular polarization

Now sum left & right-polarized (LCP, RCP) waves, \vec{E}_R & \vec{E}_L , $\vec{E} = \vec{E}_R + \vec{E}_L$.

If \vec{E}_R & \vec{E}_L have identical amplitudes, freq., & phases, \vec{E} will oscillate vertically — the horizontal components perfectly cancel.

The rotation angles are, at $z=0$

$$\psi_R = +\omega t, \quad \psi_L = -\omega t \quad (z=0)$$

Any given value of ψ_R will move down the z -axis, can write as

$$\psi_R = +\left(\omega t - \frac{2\pi z}{\lambda_R}\right) \quad \psi_L = -\left(\omega t - \frac{2\pi z}{\lambda_L}\right)$$

We allow different λ 's, with same ω ; ~~so it's scattering~~.
this implies different indices of refraction.

If the λ 's are different, then moving along \hat{z}
(direction of propagation) will rotate the \vec{E}_R
 $\in \vec{E}_L$ vectors by different amounts; here $\lambda_R > \lambda_L$,
so \vec{E}_R rotates less than \vec{E}_L ,
& the sum vector rotates from vertical ($\phi=0$)
to $\phi = \frac{(\phi_R + \phi_L)}{2}$.

This gives the net rotation
over distance D as
$$\boxed{\phi = \pi D \left(\frac{1}{\lambda_L} - \frac{1}{\lambda_R} \right)}$$

with no time dependence.

To find these λ 's, we need $\eta(\omega)$ for RCP vs. LCP
so need electron displacement, Polarization \vec{P} , & \vec{B}
for both. As the effects of the EM wave's \vec{B}
field cancel out, we refer only to the external \vec{B} .
Force on electrons:

$$\vec{F} = -e [\vec{E} + (\vec{v} \times \vec{B})]$$

First consider $\vec{B} = B_z$ only, & RCP or LCP waves;
electron moves in circle, at same rate
as rotating electric vector. For RCP,

Electron's circular motion

Both electric & magnetic forces constantly
 $\vec{F}_B = -e\vec{v} \times \vec{B}$ point to center of circle. For linearly polarized
wave, the electron shows no horizontal motion.
 $\vec{F}_B = e\omega B_z$ For LCP, \vec{F}_B points outward, so net \vec{F}
is reduced. For same ω , must have smaller radius.

Thus the polarization $\vec{P} = -n_e e \vec{r}$ is smaller
for LCP than RCP. We find the displacement,
using $a = \vec{\omega}^2 R$, $\vec{F} = m\vec{\omega}^2 \vec{R}$, $\vec{F} = (-e)\vec{E} \pm eB_z\omega R$, (+ for LCP, - for RCP)
 $-m\vec{\omega}^2 \vec{R} = -e\vec{E} \pm eB_z\omega R$ (+ for LCP, - for RCP)

~~ER~~

Hilroy

$$\text{Solve for radius, } \vec{R}_{L,R} = \frac{(e/m)}{\omega^2 \pm eB_z\omega/m} \vec{E}_{SR}$$

Generally $\omega^2 \gg \frac{eB_z\omega}{m}$, so for $+B_z$ LCP has the smaller radius. Put R into polarization,

$$\vec{P}_{L,R} = \frac{-ne e^2 (m\omega^2)}{1 \pm (eB_z/m\omega)} \vec{E}_{SR} \quad (\text{LCP+}, \text{RCP-})$$

Now use $\vec{P} = \epsilon_0(K_e - 1)\vec{E}$ to get K_e ,

$$K_e(L,R) = 1 - \frac{\omega_p^2/\omega^2}{1 \pm \frac{\omega_{Bz}/\omega}{\omega}} \quad (L+, R-)$$

where we use $\omega_p = \left(\frac{ne e^2}{\epsilon_0 m}\right)^{1/2}$, & define

a (modified) cyclotron freq., $\omega_{Bz} = eB_z/m$. This will be less than the true cyc. freq. $\omega_B = eB/m$.

$$\text{Then } \eta_{L,R}^2 = K_e(L,R) = 1 - \frac{\omega_p^2/\omega^2}{1 \pm \left(\frac{\omega_{Bz}/\omega}{\omega}\right)}, \text{ and}$$

$$\text{we find } \lambda_R = \frac{c}{2\eta_R}, \lambda_L = \frac{c}{2\eta_L}.$$

$$\text{Typically, } \omega_{Bz} \ll \omega; \quad \nu_{Bz} = \frac{\omega_{Bz}}{2\pi} = 2.8 \times 10^{10} B_z,$$

$$\text{for typical interstellar } B \approx 0.3 \text{ nT, } \nu_{Bz} = \left(\frac{B_z}{10^{-9} \text{ T}}\right) 280 \text{ Hz.}$$

So indices of refraction, & speeds, are affected little, but rotation can be substantial.

To get actual rotation angles, first use $(1+x)^n = 1 + nx$,

$$\eta_{(L,R)} \approx 1 - \frac{\omega_p^2}{2\omega^2} \left(1 \mp \frac{\omega_{Bz}}{\omega}\right) \quad (\text{LCP-}, \text{RCP+})$$

Then write $\frac{1}{\lambda_R} = \frac{2\eta_L}{c}$, $\frac{1}{\lambda_L} = \frac{2\eta_R}{c}$

& plug into ψ . $\psi = \pi D \frac{\lambda}{c} (\eta_L - \eta_R)$.

$$\eta_L - \eta_R = \frac{c \omega_p^2 \cos B_z}{\omega^3} = \frac{c \omega_p^2 \cos B_z}{(2\pi\omega)^3}, \quad \text{so}$$

$$\psi = \frac{D}{8\pi^2 c} \frac{c \omega_p^2 \cos B_z}{\lambda^2} = \frac{1}{8\pi^2 \epsilon_0 m c} \frac{e^3 n_e B_z D}{\lambda^2}$$

This has assumed uniform n_e & B_z ,
but the general solution integrates
along the path, $\int_0^D n_e B_z ds$ for $n_e B_z \propto D$.

In SI

$$\psi = 2.36 \times 10^{-4} \left(\frac{1}{\lambda}\right) \int_0^D n_e B_z ds, \quad \text{(radians)}$$

(or $2.36 \times 10^{-13} \left(\frac{1}{\lambda^2}\right) \int_0^D n_e B_z ds$) using $\nu \approx c/\lambda$

In order to use this, one can't just measure polarization angle, one needs pol. angle at several λ .

Then one gets rotation measure

$$RM = \frac{\psi}{\lambda^2}$$

$$(RM = \left(\int_0^D n_e B_z ds \right) \times 2.63 \times 10^{-13} \text{ radians/m}^2)$$

This tells us about the average $B_z (x_{n_e})$ along the path. B in other directions (x, y) isn't measured, & reversals will reduce RM.

Example: Crab Nebula.

Crab
pol. angle

Plotting pol. angle vs. λ^2 gives straight line.

Y-intercept gives original pol. angle, so at $\lambda = 0.3 \text{ m}$, $\psi = -130^\circ$ or $RM = -25 \text{ radians/m}^2$.

Some of this RM comes from Crab Nebula itself.

Negative RM indicates rotation of \vec{E} is clockwise, so \vec{B} field points away from us on average.

$$\text{Then } \langle n_e B_z \rangle_{\text{average}} = \frac{RM}{2.6 \times 10^{-13} D} = \frac{-25 \text{ rad/m}^2}{(2.6 \times 10^{-13})(2000 \times 3.08 \times 10^{16} \text{ m})}$$

$$\langle n_e B_z \rangle_{\text{avg}} = -1.7 \times 10^{-6} \text{ T m}^{-3}$$

RM only gives a combination of B and n_e , but if we have dispersion in same direction we can get n_e — for Crab, $\langle n_e \rangle \sim 3 \times 10^4 \text{ m}^{-3}$, so $\langle B_z \rangle_{\text{avg}} \sim -0.06 \text{ nT}$.

Even better, we can find $\langle B_z \rangle_{\text{avg}}$ without knowing distance, if we have RM & DM.

$$\frac{\int_0^D B_z n_e ds}{\int_0^D n_e ds} = 3.8 \times 10^{12} \frac{RM}{DM} = \langle B_z \rangle_{\text{avg}}$$

Putting RM measurements from pulsars together, one can map the B field of the Galaxy.

We get integrated B values, so complex to interpret, but clear reversals of B direction seen in spiral arms.

Galactic B-map

Sgr A*

Also used to measure n_e (using assumptions of equipartition to estimate B) near Sgr A*, by using polarization of Sgr A* itself. The measured n_e is much less than predicted by simplest radiatively inefficient models of accretion onto Sgr A*.

Cerenkov Radiation

High-energy particles moving in a medium can travel faster than ~~the~~ speed (phase velocity, here) of light in that medium.