

Gravitational Lensing

Bending of light by gravity is result of GR leading to multiple images, rings, distortions.

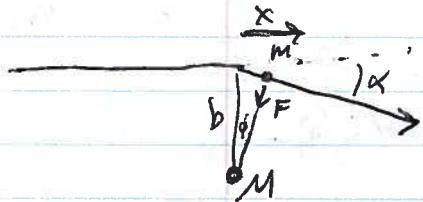
Extremely useful in astrophysics today,

Most renowned test of GR was

measurement of deflection of light by Sun (during Eclipse of 1919), which made Einstein a celebrity.

Grav. Lens Geometry

Can make simple estimate of deflection of light with Newtonian mechanics — GR gives value 2x larger.



Consider light as point mass m , with velocity v , passing at closest approach b to mass M .

$M \gg m$, & mv , is large, so α small. The force integrated over time can be approximated as impulse at closest approach $x=0$.

For a position x , the force on m is

$$\vec{F} = -\frac{GMm}{r^2} \hat{r},$$

The components of the

integrated force in $+x$, $-x$ directions cancel, so

consider only transverse (to velocity);

$$F_t = \frac{GMm}{r^2} \cos\phi$$

$$dv_t = a_t dt = \frac{GM}{r^2} \cos\phi dt$$

For small α , $r \approx (b^2 + x^2)^{1/2}$, $\cos\phi \approx b/r$, $dt = dx/v$, so

$$\Delta v_t = \int dv_t = \int_{-\infty}^{\infty} \frac{GMb}{(b^2 + x^2)^{3/2}} \frac{dx}{v} = \frac{2GM}{bv}$$

This could be derived if the F is assumed to be constant with $x=0$ for $\Delta x = 2b$,

$$\text{So } \alpha_{\text{Newton}} = \frac{\Delta Vt}{v} = \frac{2GM}{bv^2} \quad \text{for light, } \alpha_{\text{Newton}} = \frac{2GM}{bc^2}$$

~~Performing~~ Performing this calculation in GR gives an extra factor of 2,

$$\alpha_{\text{GR}} = \frac{4GM}{bc^2}$$

For a light ray grazing the Sun's surface,
 $\alpha_{\text{GR}} = 8.49 \times 10^{-6}$ radians = $1.75''$

Grav. lensing is strongest near the point mass so doesn't bring multiple rays to a true image.

What is the observed shift in position?

Consider Θ = offset of image from lensing mass;
 β (or θ_s in Bredt) = unlensed angle between background quasar & lensing mass.

Then

$$\Theta - \beta = \frac{(x-y)}{ds} = \frac{\alpha d_{ls}}{ds} \quad \text{Then use } \alpha = \frac{4GM}{bc^2}$$

$$\Theta - \beta = \frac{4GM}{bc^2} \frac{d_{ls}}{ds}, \quad \text{use } b = \Theta d_{\text{lens}}$$

$$\Theta - \beta = \frac{4GM}{\Theta c^2} \frac{d_{ls}}{ds d_{\text{lens}}}$$

Now define $\Theta_E = \sqrt{\frac{4GM}{c^2} \frac{d_{ls}}{ds d_{\text{lens}}}}$, Einstein radius,

$$\Theta - \beta = \Theta_E^2 / \Theta$$

Solving this quadratic equation gives

$$\Theta_{\pm} = \beta \pm \sqrt{\beta^2 + 4\theta_E^2}$$

Double quasar

Gives two lensed images; one farther than true distance, other closer (on far side).

MOT lensing illustration

← Example with normal lens.
Each dot has two distorted images.

A star directly behind the lens (~~circle~~) will have solution

$$\Theta_{\pm} = \frac{\pm \sqrt{4\theta_E^2}}{2} = \pm \theta_E$$

Since there is no unique direction, the image will appear at $\Theta = \theta_E$ in all directions — the Einstein ring.

Einstein rings

Einstein ring lenses are reasonably frequent, Einstein predicted the effect but didn't think it would be observed since he didn't know about galaxies & quasars was only thinking about stars,

In the case of ~~circle~~ very large, $\Theta_+ \sim \beta, \Theta_- \sim 0$; one image is the source, the other merges into the lens.

One can reconstruct the angles if can spot the lens & images;

$$\beta = \Theta_+ + \Theta_-$$

$$\theta_E = \sqrt{\Theta_+ \cdot \Theta_-}$$

Then, if you know the distances (e.g., have redshifts & a cosmology), can find lensing mass:

$$M = \frac{c^2 \theta_E}{4G} \frac{D_{\text{lens}} D_s}{D_{\text{ls}}}$$

In most lensing, the lens is extended, making the calculation complicated. E.g., elliptical galaxies or clusters, with sizes larger than the impact parameter b .

$$\text{Here, one must integrate } \theta_E = \int \frac{GM(r)b dx}{(b^2 + x^2)^{3/2}}$$

where $M(r)$ is the mass within that radius, for single sphere case.

Einstein Cross Here more than two images can be produced — for single sphere, up to three; more for asymmetric systems

Abell 1689 Distant galaxies seen, distorted & lensed by clusters. Allows estimation of mass of galaxy cluster, thus estimation of cluster dark matter content. Example of both lensing & distortion by tidal stripping.

Abell 2667 — lensed & stripped

Cluster maps of galaxies, DM

Weak lensing is slight lensing of galaxies far from the lens — seen as tiny elongations in galaxy shapes. Requires study of many galaxies to measure, but can give gross signal at large distances.

Weak lensing used to map full distribution of mass in clusters of galaxies, tracing dark matter vs. baryonic matter — stars & gas.

Galaxy cluster hot gas measured by X-ray bremsstrahlung radiation, &/or S-Z effect. Galaxies measured by optical light. Thus lensing can identify how dark matter, gas & galaxies behave.

Bullet Cluster

X-ray images of "Bullet Cluster" show evidence of major collision: gas is heated, compressed in shock fronts. Two clusters have collided — gas has slammed together, been slowed down, galaxies pass through without difficulties, so precede gas. Lensing shows dark matter behaves like galaxies, passing by other dark matter w/o interaction. Taken as proof that dark matter is not really just an incorrect model to explain anomalous accelerations, as modified Newtonian dynamics (MOND), which explains galaxy rotation curves well can't explain the clear separation between mass ~~gas~~ & baryons (gas holds most baryons).

$$\text{Magnification} = M$$

Specific intensity along a ray is conserved ~~in~~ in grav. lensing, so surface brightness is unchanged. Images will be magnified or demagnified by lensing — angular area will change, so total flux received can change.

Different magnifications in radial azimuthal directions,

Azimuthal magnification just $\propto \theta$, $M_{az} = \theta/\beta$.

(Magnification)

$$M_{red} = \frac{\Delta\theta}{\theta\beta} = \frac{d\theta}{d\beta}$$

where $\Delta\theta$ is size
in θ between edges

of source & $\theta\beta$ is size in β of (unlensed) source.
From quadratic lens equation,

$$\beta = \theta - \frac{\theta_E^2}{\theta} = \theta \left(1 - \frac{\theta_E^2}{\theta^2}\right).$$

$$\text{So } M_{az} = \frac{\theta}{\beta} = \left(1 - \frac{\theta_E^2}{\theta^2}\right)^{-1}.$$

$$M_{red}^{-1} = \frac{d\beta}{d\theta} = \frac{d}{d\theta} \left(\theta - \frac{\theta_E^2}{\theta}\right) = 1 + \frac{\theta_E^2}{\theta^2}$$

$$M_{red} = \left(1 + \frac{\theta_E^2}{\theta^2}\right)^{-1}$$

$$\text{So total magnification } \mu = M_{red} M_{az} = \left(1 - \frac{\theta_E^4}{\theta^4}\right)^{-1}$$

The ratio of flux densities is ~~not~~ increased equally.

We see that if the image is at the Einstein radius, μ is very large (we assumed the source was infinitesimal, causing the ∞ here).

MCA again Magnification of image further from lens is always positive, while closer image shrinks with increasing β , finally disappears.

Magnification allows use of clusters to magnify faint background galaxies, permitting detection of galaxies at $\approx \sim 6$ smaller than LMC.

Microlensing is lensing by stars; $\theta_E \sim 1$ millarcsec, so can't image lensing. To identify, monitor millions of stars for years history spot transient

brightening,

Microlensing

Look for symmetric brightening same in all wavelengths (no freq. dependence)

Can distinguish from novae, etc., which are typically bluer when brighter (since hotter).

- MACHO project; Microlensing search

targeting postulated dark matter in halo, if made of massive compact objects.

For given d_s , D_L largest if $d_{LS} \approx d_{lens}$, i.e. can get most lensing if lens is halfway to source.

Thus, to probe possible dark objects in Milky Way's halo, monitor nearby galaxies, LMC & SMC.

MACHO project has found some microlensing, but not enough to explain majority of dark matter. Thus, WIMPs required.

Microlensing can also catch planets (the planet can give very fast, small magnification.) Several planets have been found in microlensing surveys in our Galaxy, including one of only 5.5 Earth.

Microlensed planet

Multiple-image lensed quasars are seen to vary independently, both compared to models, & in time.

Explained as microlensing of individual images of background quasar, by individual stars in host galaxy.

Peale
PG 1155+080
lens

These flux ratio anomalies are larger in X-ray than optical. However, lensing is frequency-independent.

This is explained as the source size being larger in the optical than X-ray; evidence that the optically-emitting part of the quasar disk is much larger than the X-ray part. This allows the X-ray emission to fall entirely within one star's Θ_E , while only part of the optical light does.

Finally, the frequency of stellar microlensing tells us the fraction of mass along the path of the image, in a smooth vs. stellar component.

D. Pooley et al. have found, from studies of 61 ~~over~~ observations of 14 quasars, that only $\sim 10\%$ of the mass is in stars, with the rest in dark matter (& gas, a small proportion).