

Radiative Processes

9/8

Discuss syllabus.

Midterm Oct. 18 - Oct. 22,
between guest lectures rest of week.

- Fundamentals of radiative transfer
- Radiation from free electrons
- Key radiation mechanisms
- Radiation from bound electrons

cgs system of units:

	cgs	factor	SI unit
length:	cm	10^{-2}	m
mass:	gram	10^{-3}	kg
time	second	1	second
Energy:	erg	10^{-7}	Joule
Power:	erg/s	10^{-7}	Joule/s, or Watt
Temp:	Kelvin(K)	1	Kelvin
Force:	dyne	10^{-5}	Newton(N)
Pressure:	dyn/cm ²	0.1	N/m ² , or Pascal
Magnetic flux density:	gauss(G)	10^{-4}	tesla(T)
Angle	radian(rad)	1	radian
Solid angle	steradian(sr)	1	steradian
Charge	esu	3.33×10^{-10}	Coulomb(C)
Frequency	Hertz(Hz)	1	
Coulomb's Law:	$F = \frac{q_1 q_2}{r^2}$		$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$
Lorentz Equation:	$\vec{F} = q(\vec{E} + \frac{\vec{v}}{c} \times \vec{B})$		$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

Typical units of distance in astrophysics are

$$\text{parsec (pc)} = 3.0857 \times 10^{18} \text{ cm}, \text{ or } 206,265 \text{ AU}$$

$$1 \text{ AU (astronomical unit)} = 1.496 \times 10^{13} \text{ cm}$$

$$\text{Mass in solar masses, } M_{\odot} = 1.989 \times 10^{33} \text{ g}$$

$$\text{Luminosity in solar luminosities, } L_{\odot} = 3.826 \times 10^{33} \text{ ergs/s.}$$

Electromagnetic Radiation

Can be treated as a wave or a particle -
we will use both descriptions at diff. times.

For wave, $\lambda\nu = c$.

Energy of particle relates as $E = h\nu$.

A temperature can be associated; $T = E/k$ \leftarrow Boltzmann's constant

Ranges of wavelength (energy, freq.) of
EM radiation are given different names;

<u>radio</u>	for	$\lambda \gtrsim 0.03\text{ cm}$,	All ranges are only approximately defined
<u>infrared</u>	for	$0.03\text{ cm} \sim 8 \times 10^{-5}\text{ cm}$	
<u>optical</u>	for	$4 \times 10^{-5}\text{ cm} - 8 \times 10^{-5}\text{ cm}$	
<u>ultraviolet</u>	for	$4 \times 10^{-5}\text{ cm} - 9 \times 10^{-7}\text{ cm}$	
<u>X-ray</u>		$9 \times 10^{-7}\text{ cm} - 10^{-9}\text{ cm}$	
<u>gamma rays</u>		$\lambda \lesssim 10^{-9}\text{ cm}$	

Note the temperatures: at few 100 K, emit IR;
at ~~~~~ ~10,000 K, emit optical / light;
at ~million K, emit X-rays.

Luminosity of object is rate of radiating
away its energy.

$$dE = L dt$$

Same units as power.
Doesn't depend on your distance, intrinsic.

At distance of Earth, the Sun's luminosity
passes through surface of sphere, radius 1 AU.

Earth intercepts photons only over the cross-section

facing the Sun, πR_{\oplus}^2 .

So the fraction of Sun's luminosity intercepted is

$$\frac{A(\text{Earth, cross-section})}{A(\text{sphere of 1 AU})} = \frac{\pi R_{\oplus}^2}{4\pi (\text{AU})^2} = \frac{\pi (6.378 \times 10^8 \text{ cm})^2}{4\pi (1.496 \times 10^{13} \text{ cm})^2}$$

= only 4.5×10^{-10} of Sun's luminosity captured,
or 1.73×10^{24} ergs/s. (or 1.73×10^{12} J/s, or W —

compare to world energy usage,
in 2008, 1.5×10^{15} W, or $\sim 10^{-4}$
of available solar power.

(Although, plants & animals use it, etc.)

Science fiction concept;
what if we could collect all the L
from the Sun?

Construct a sphere of $R=1 \text{ AU}$ (made from what?)
around Sun, intercepting entire luminosity L_0 .

This concept is called a Dyson Sphere.

Luminosity over all wavelengths is bolometric L.

We can't measure this directly,

since we need different instruments for
diff. λ , & can't see some λ .

e.g. Measure L in a given waveband, ~~or freq.~~ or freq.,
the blue "B" filter, or the X-ray range of
the Chandra satellite, $L_x(0.5-10 \text{ keV})$.

In order to infer total luminosity need to
understand the spectrum.

$$L = \int L_\nu d\nu \quad \text{or} \quad \int L_\lambda d\lambda.$$

4

Flux Energy Flux F ,

energy passing through area is $F dA dt$.

Note that the energy flux can depend on orientation.

If isotropic emitter (equally in all directions), then
flux constant over spherical surface.
Choose 2 spheres, radii r_1 and r_2 , then

$$F(r_1) \cdot 4\pi r_1^2 = F(r_2) \cdot 4\pi r_2^2 = F(r) 4\pi r^2$$

$$F(r) = \frac{F(r_1) 4\pi r_1^2}{4\pi r^2} = \frac{F(r_1) r_1^2}{r^2} = \frac{\text{constant}}{r^2}$$

inverse square law, $L = 4\pi r^2 F$, if emission is isotropic.

As for luminosity, depends on the bandwidth: can speak of wavelength range $d\lambda$, freq. range $d\nu$, $\int f_\nu d\nu = F$

Flux is measure of energy carried by all rays \rightarrow passing through a given area.

Now consider just those rays within a limited solid angle.

This is energy per unit time, per unit solid angle, passing through unit area perpendicular to path of rays;
intensity, I ($\text{erg s}^{-1} \text{cm}^{-2} \text{sr}^{-1}$)

Specific intensity I_ν ($\text{erg s}^{-1} \text{cm}^{-2} \text{Hz}^{-1} \text{sr}^{-1}$)
or brightness. \boxed{I} don't eliminate!

$$\boxed{dA} \rightarrow dS \quad dE = I_\nu dA dt dS d\nu$$

Depends on location, direction & frequency.

(specify)

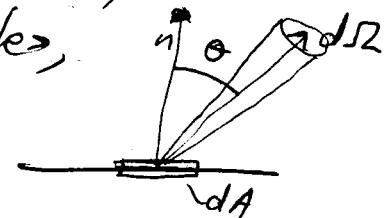
Relate flux, intensity; $dF_\nu = I_\nu \cos\theta d\Omega$

where θ is angle from normal of patch dA .
 $(\cos\theta dA = \text{area normal to rays})$

To get net flux in direction \vec{n} (unit vector)

integrate F over all solid angles,

$$F_\nu = \int I_\nu \cos\theta d\Omega$$



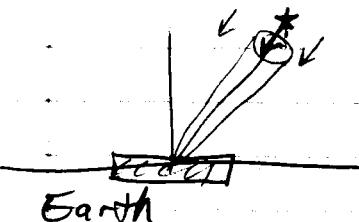
If radiation field is isotropic, then
net flux is zero, since $\int \cos\theta d\Omega = 0$

Just as much E going in $+\vec{n}$ as $-\vec{n}$ directions.

Consider emission from distant star,

which covers some solid angle Ω .

small!



Point a detector directly at this star, and detector is able to measure from Ω larger than star's Ω .

Then

$$F_\nu = \int_{\Omega} I_\nu \cos\theta d\Omega$$

$$= \int_{\Omega} I_\nu d\Omega = I_\nu \Omega.$$

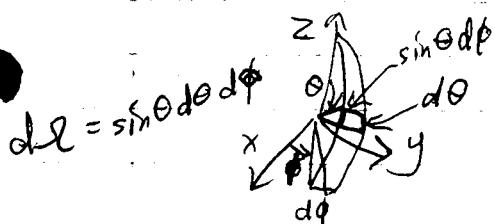
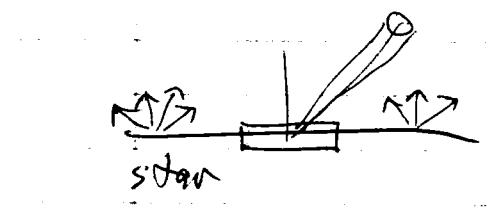
Consider emission at surface of star.

Now direction of radiation is reversed, emission going out, into all outward angles ($2\pi \text{ sr}$).

$$F_\nu = \int_{2\pi} I_\nu \cos\theta d\Omega$$

$$= \int_0^{\pi} \int_0^{2\pi} I_\nu \cos\theta \sin\theta d\theta d\phi \quad (\text{spherical coords})$$

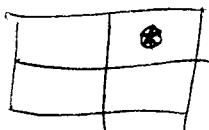
$$F_\nu = I_\nu \pi$$



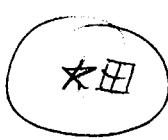
Can measure intensity of a source if you can measure its solid angle.

Thus, depends on your resolution. If you can't measure its solid angle, you only know the total flux.

Source is unresolved:



light is spread out over the pixel.
(Common in astrophysics!)



Or, if star solid angle is large, can measure I_ν at various points on sky.

If total Ω is small, $\cos\theta \approx 1$,

$$\text{so } F_\nu \approx \int I_\nu d\Omega \approx \sum I_\nu; \Omega;$$

Add up intensities of each pixel to get flux.

Intensity & specific intensity are independent of distance. (It no matter in way)

Easy to see, for somewhat distant star, where

$$F_\nu = I_\nu \Omega. \quad \Omega \approx \frac{\pi R_*^2}{d^2}, \text{ so } \Omega \text{ drops as } d^{-2}.$$

But F_ν also drops as d^{-2} , so I_ν must be const.

(Alternate, more general proof in next.)

Can also study momentum carried by light's radiation

$$p_\nu = E/c \text{ for photon.} \quad \xrightarrow[\text{per unit area}]{\text{per unit time}} \text{pressure (Force/area)}$$

So momentum flux along a ray at angle θ is $\frac{dp_\nu}{c}$.

We need the component of p_ν normal to dA , so

$$p_\nu = \frac{1}{c} \int I_\nu \cos^2 \theta d\Omega \quad \text{If photon reflected, double shot}$$

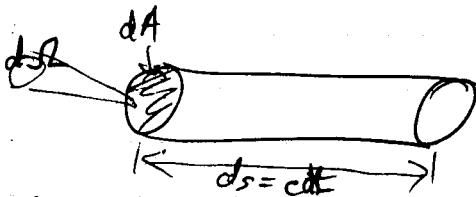
An isotropic radiation field will have a pressure but no net force.

A radiation from one direction gives a net force; this gives the concept of the solar sail, a large thin sheet to catch Sun's rays & sail around solar system.

Energy density $u = \int u_r d\Omega$

• Energy per unit volume (per unit freq. range, if " ") specific energy density is

To relate to intensity, consider specific energy density per unit solid angle, $u_r(\Omega)$. By definition $dE = u_r(\Omega) dV d\omega d\Omega$. dV = volume element.



For cylinder of length $ds = cdt$, $c ds$
 $dE = u_r(\Omega) dA c dt d\omega d\Omega$

Relate this to intensity of rays passing through cylinder.
As we wrote before,

$$dE = I_r dA dt d\omega d\Omega$$

Equate these two, find $u_r(\Omega) = I_r/c$

Then integrate over all solid angles, $u_r = \int u_r(\Omega) d\Omega = \frac{1}{c} \int I_r d\Omega$

Define mean ^{specific} intensity $\bar{I}_r = \frac{1}{4\pi} \int I_r d\Omega$, so $u_r = \frac{4\pi}{c} \bar{I}_r$; integrating over Ω gives

$$u = \frac{4\pi}{c} \int \bar{I}_r d\omega$$

In space, \bar{I}_r is not constant with Ω , although CMB comes close.
Olbers' Paradox: if universe is infinite & stars spread equally, any sight line will eventually intersect a star.

Thus night sky should be uniformly bright,
bright as Sun.

(Typical stars have same intensity as the Sun. But their light is spread out, in our eye, over a "pixel" or rod, so less concentrated. If we couldn't resolve the Sun, might be able to look at it directly — requires distance $\gtrsim 30 \text{ AU}$, beyond Neptune.)

But night sky isn't uniformly bright — which says universe isn't infinite. Indeed, universe had beginning, so our vision is limited.

Radiation Pressure

Consider reflecting enclosure containing isotropic radiation field ($I_\nu = J_\nu$).

Reflected photons transfer ~~one~~ twice momentum,

so $P_\nu = \frac{2}{c} \int I_\nu \cos^2 \theta d\Omega$ where we only integrate over 2π sr.



For isotropy, can take I_ν out of S .

$$P_\nu = \frac{2}{c} I_\nu \int \cos^2 \theta d\Omega \quad \text{Integral} = \frac{2\pi}{3}, \text{ so}$$

$$P_\nu = \frac{4\pi}{3c} I_\nu = \frac{4\pi}{3c} J_\nu = \frac{4\pi u_\nu}{3}$$

or $P = \frac{u}{3}$ Radiation pressure = $\frac{1}{3}$ energy density
for isotropic radiation.