

Let's compute the mean intensity and energy density of sunlight on Earth.

First we need the intensity of rays from the Sun. Intensity doesn't change with distance, so compute at Sun's surface.

We showed that at the surface, $F_0 = I_0 \pi$, or $F = I \pi$. We'll use bolometric quantities (all energies). So first compute surface flux of Sun, from its L and R , $L_0 = 4\pi R_0^2 F_0$,

$$F_{0,\text{surf}} = L_0 / 4\pi R_0^2 = \frac{3.85 \times 10^{26} \text{ W}}{4\pi (6.96 \times 10^8 \text{ m})^2} = 6.33 \times 10^7 \left(\frac{\text{W}}{\text{m}^2} \right)$$

Then $I_0 = F_0 / \pi = 2.01 \times 10^7 \left(\frac{\text{W}}{\text{m}^2/\text{sr}} \right)$

So this is the solar intensity, no matter how far away from the Sun you are.

The flux, mean intensity, & energy density do decrease, via the inverse square law, so you can always calculate $F = L / (4\pi d^2)$. We can also calculate F without the distance, if we know the angle on the sky,

$$F = \int I \cos\theta d\Omega$$



where Θ is angle from normal

Take $\Theta \sim 0^\circ$ (the considered area faces the Sun - this angular dependence helps explain why Edmonton is cold), so $\cos\theta \sim 1$. Assume I constant over solar disk.

$$\text{Then } F_0 = I_0 S d\Omega = I_0 \Omega_0, \quad \Omega_0 = \pi \Theta_0^2 = \pi \left(\frac{1}{4}\right)^2 \frac{\pi}{360^\circ} = 4.36 \times 10^{-3} \text{ radians}, \text{ so } \Omega_0 = 6 \times 10^{-5} \text{ steradians}.$$

$$\text{So } F = (2.01 \times 10^7 \text{ W/m}^2/\text{sr}) \times (6 \times 10^{-5} \text{ sr}) = 1.21 \times 10^3 \text{ W/m}^2.$$

Similarly can calculate mean intensity $J_0 = \frac{1}{4\pi} \int I_0 d\Omega$,

$$J_0 = \frac{1}{4\pi} I_0 \Omega_0 = 96 \text{ W/m}^2$$

And energy density $u = \frac{4\pi}{c} J = 4.0 \times 10^{-6} \text{ J/m}^3$

Radiative Transfer

Intensity of a ray I doesn't change in free space (ignoring relativity).

But passage through matter can change it.
Can be emission, (add to ray, spontaneous or stimulated)
absorption,
or scattering (just direction changes).

Consider emission. Define spontaneous emission coefficient j_ν (following Bratt, define as energy emitted $(J \cdot m^{-3} s^{-1} Hz^{-1})$ per unit time, per unit volume, per unit frequency - Bratt omits ν subscript)

Can be directional - here, we assume isotropic. (Thus no angle dependence.)

Relates to energy emitted,

$$dE = j_\nu dV dt d\nu$$

Compare to specific intensity def., $dE = I_\nu dA d\nu dL d\nu$

These are the same if $dI_\nu dL = j_\nu dx$

Since we assume isotropy in j_ν ,

$j_\nu / 4\pi$ will be emitted in any given steradian.

So we can write

$$dI_\nu = \frac{j_\nu dx}{4\pi}$$

(dx is the path along the ray; so emission adds up)

This is the contribution to the intensity from emission.

Instead of per unit volume, can write per unit mass,

$$j_\nu = E_\nu \rho \quad \text{where } \rho \text{ is the mass density.}$$

Can also say $j_\nu = P_\nu$, the power radiated.

(In following Bratt's notation, except explicitly indicating freq. dependence, C&O define j_λ as λ -dependent emission per unit mass, R&L define j as angle-dependent, so differs by factor 4π .)

Absorption

Think of small particles with cross-section σ_ν (m^2) space density n (per unit volume).

The combination $\sigma_\nu n$ has units m^{-1} , and gives the fraction of the path that's blocked per m along the ray. (For small fractions.)
So we can write

$$dI_\nu = -\sigma_\nu n I_\nu dx$$

where the amount of intensity removed from the beam depends on its intensity & the path length dx .

Can write $\sigma_\nu n = \alpha_\nu$, an absorption coefficient.

Also $\sigma_\nu n = \rho K_\nu$, where K_ν is the mass absorption coefficient (m^2/kg), or opacity.

α_ν is positive if energy is removed;
a negative α_ν in this equation describes stimulated emission, which we see resembles absorption.

Radiative Transfer Equation Put emission & absorption together,

$$dI_\nu = -I_\nu \alpha_\nu dx + \frac{j_\nu}{4\pi} dx$$

$$\text{or } \frac{dI_\nu}{dx} = -I_\nu \alpha_\nu + j_\nu / 4\pi$$

This isn't the most general solution, because can have scattering — then emission into $d\Omega$ depends on I_ν in other angles. Such problems require numerical solutions.

Consider limiting cases: • Emission only, $\alpha_\nu = 0$:

$$\frac{dI_\nu}{dx} = j_\nu / 4\pi.$$

Solution $I_\nu(x) = I_\nu(x_0) + \frac{1}{4\pi} \int_{x_0}^x j_\nu(x') dx'$

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So the increase in brightness is just the integrated emission coefficient along the path.

- Absorption only: $j_2 = 0$. $\frac{dI_2}{dx} = -I_2 \alpha_2$

$$\text{Solution: } I_\nu(x) = I_\nu(x_0) \exp\left[-\int_{x_0}^x \alpha_\nu(x') dx'\right]$$

Brightness decreases by the exponential of the abs. coefficient integrated along the path.

Can rewrite more simply by introducing optical depth ζ ,
 defined $\int_{\text{bottom}}^{\text{top}} \frac{dx}{n(x)}$

$$d\Sigma = \alpha_x dx \quad \text{or} \quad \Sigma_x(x) = \int_{x_0}^x \alpha_x(x') dx'$$

Here we defined \mathcal{E} measuring along the path of the ray, so \mathcal{E} increases.

Can think of λ as the number of mean free paths (since the start point).

If $\epsilon_2 > 1$, the medium is optically thick, or opaque.

If $\Sigma < 1$, it's optically thin, or transparent.

Typical photons pass through without being absorbed.

Divide the "transfer" equation by α_x ($= d\bar{v}_x/dx$), get

$$\frac{dI_L}{d\tau_L} = -I_L + S_L$$

where $S_\nu \equiv \frac{ji}{4\pi d_\nu}$, the source function

Bradt calls this I_S (source intensity).

An observer embedded in an optically thick cloud would see intensity Σ ($= I_s$) in all directions.

Solve the equation of radiative transfer in terms of ϵ ,

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)} S_\nu(\tau'_\nu) d\tau'_\nu$$

initial intensity,
 diminished by absorption Integrated source emission,
 decreased by absorption.

Easier if S_r is constant, solution simplifies;

$$I_r(z_r) = I_r(0)e^{-z_r} + S_r(1 - e^{-z_r})$$

or

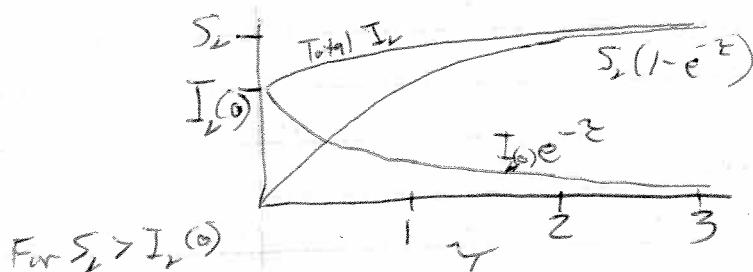
$$I_r(z_r) = S_r + e^{-z_r}(I_r(0) - S_r)$$

Take limits $z \rightarrow 0, z \rightarrow \infty$:

$$z \rightarrow 0, e^{-z} \rightarrow 1, \text{ so } I_r(z_r) = I_r(0). \quad \text{good.}$$

$$z \rightarrow \infty, e^{-z} \rightarrow 0, \text{ so } I_r(z_r) = S_r.$$

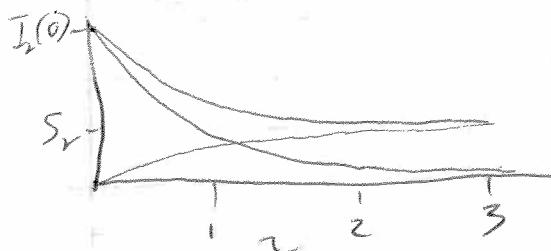
So I_r relaxes to S_r , with increasing z .
We can illustrate this, for increasing optical depth z .



As the thickness of the layer increases the background intensity of the ray diminishes as ~~e^{-z}~~ $I_r(0)e^{-z}$.

Meanwhile, as the layer thickens it contributes a larger amount of light to the ray.
When the layer is optically thick ($z \gg 1$), the ray ~~for~~ approaches S_r .

S_r is the balance between absorption & emission from the layer. If the radiation is in thermal equilibrium (to be defined soon) S_r is blackbody radiation, & determined only by temperature.



We can use these illustrations to understand the concept of spectral lines, i.e. frequencies at which the intensity is increased or decreased (emission lines, or absorption lines).

Consider five cases:

$I_2(0) = 0$ — no background radiation

(i) $\tau \ll 1$, gas is optically thin

(ii) $\tau \gg 1$, gas is optically thick

$I_2(0) > 0$ — there's background radiation

(iii-a) $\tau \ll 1$, ($I_S > I_0$, or $S_2 > I_2(0)$)

(iii-b) $\tau \ll 1$, ($I_S < I_0$, or $S_2 < I_2(0)$)

(iv) $\tau \gg 1$, gas is optically thin, source $<$ bg intensity

(v) $\tau \gg 1$, gas is optically thick.

For case (i), we know $I_2(0) = 0$. Since $\tau \ll 1$, we can expand the exponential, $e^{-\tau} \approx 1 - \tau$.

So the general solution $I_2(\tau_2) = I_2(0)e^{-\tau_2} + S_2(1 - e^{-\tau_2})$ becomes $I_2(\tau_2) = S_2(1 - 1 + \tau_2) = S_2 \tau_2$.

The emission is proportional to the optical depth τ . Since the cloud is optically thin, absorption of photons is rare, so all emission contributes ~equally.

Consider variation in frequency. At some freq., atoms have an atomic transition, so at that freq., their cross-section to absorb photons is high, & so is τ_2 .

Display

Fig. 19,

Bredt

Orion Nebula

In situation (i), we see that at these frequencies, we see emission lines.

This is the standard scenario in astrophysics for "emission nebulae" such as the Orion Nebula, which are bright in specific colors.

In situation (ii), the gas is thick, so $e^{-\tau} \rightarrow 0$, & $I_\nu(\tau_2) = 0 + S_\nu$

If the source function is a blackbody, then this produces the blackbody spectrum. Differences in optical depth make no difference to the spectrum, if the source intensity is the same everywhere.

To first-order, this is a good description of stars, which have basically blackbody-like spectra. However, the surfaces of stars are of varying temperature with depth, making them have different S_ν .

In situation (iii-a) a source lies behind an optically thin cloud, with $S_\nu > I_\nu(0)$.

Freq. with higher τ will show emission lines, added on to the background source spectrum.

In situation (iii-b), $S_\nu < I_\nu(0)$, so freq. of higher τ show as absorption lines. Mathematically,

$$I_\nu(\tau_2) = I_\nu(0)e^{-\tau_2} + S_\nu(1 - e^{-\tau_2})$$

$$= I_\nu(0)(1 - e^{-\tau_2}) + S_\nu \tau_2$$

$$I_\nu(\tau_2) = I_\nu(0) + \tau_2(S_\nu - I_\nu(0))$$

We can see that if S_ν is higher, the 2nd term is positive, & the reverse. As we will see shortly, blackbody S_ν functions ~~are monotonic~~ always increase with temperature. Thus, if the cloud is hotter the spectrum has emission lines vs. a cool cloud giving absorption lines. These situations describe real stars, which have atmospheres that

Stellar Spectra

affect their spectra.

In visible light, we see into stars to a depth where the temperature is falling with radius. Thus cooler gas lies above the background blackbody radiation, creating absorption-line spectra.

An alternative (& also valid) explanation is that the depth that we see to ($\approx z \sim 1$) is at different physical heights at diff r . Where the cross-section is high, our vision stops at high altitudes, where temperature is low,

Stars' atmospheres do not uniformly drop in T with height. Above the region where the visible spectrum forms, the corona reaches to ~1 million K (vs. 6000 K at the "surface").

Transitions in multiply-ionized atoms occur in the UV — observations in these lines see emission from atoms in the corona, since hotter than Sun's surface.

Sun
in UV

Situation (iv), gas is thick. So we again get $I_r(z_r) = S_r$

& it doesn't matter what's behind.

If we look at a dust cloud in the optical, it may be optically thick; while in other wavelengths (e.g. infrared) it's optically thin, & stars on the other side are visible.

Barnard 68

A final case is scattering, where light is changed in direction. Example of thin dust clouds before bright stars.

Pleiades