

In this case, the source function  $\Sigma$  is equal to  $J_\nu$ , the mean intensity.

Gives transfer equation  $\frac{dI_\nu}{ds} = -\alpha_\nu(I_\nu - J_\nu)$   
for pure scattering,  
but depends on all  $I_\nu$ .

So we don't solve this.

Just consider # of mean free paths, to escape from medium. For isotropic scattering photon does random walk. If each step same length  
how far do you get per  $N$  steps?

$$\vec{d} = \vec{l}_1 + \vec{l}_2 + \vec{l}_3 + \dots \vec{l}_N$$

We'll take the dot product  $\vec{d} \cdot \vec{d} = \vec{l}_1 \cdot \vec{l}_1 + \vec{l}_1 \cdot \vec{l}_2 + \dots$   
which can be summed as  $\sum_{i=1}^N \sum_{j=1}^N \vec{l}_i \cdot \vec{l}_j = l_i l_j / \cos \theta_{ij}$

(Assume mean free path constant)

Separate into  $i=j$  parts & rest,

$$\vec{d} \cdot \vec{d} = Nl^2 + l^2 \sum_{i=1}^N \sum_{j=1, j \neq i}^N \cos \theta_{ij}$$

Summing all the cosine terms just gives zero, so

$$\vec{d} \cdot \vec{d} = Nl^2, \text{ or}$$

$$N = \frac{d^2}{l^2}, \text{ or } \underline{\underline{d}} = \sqrt{N} l$$

If the typical # of scatters is  $< 1$ , there's little random walking, &  $N \sim \underline{\underline{d}}$ .

Consider both scattering & absorption.

Then

$$\frac{dI_\nu}{ds} = -\alpha_\nu(I_\nu - B_\nu) - \alpha_\nu(I_\nu - J_\nu)$$

The source function is

$$S_\nu = \alpha_\nu B_\nu + \sigma_\nu J_\nu$$
$$\frac{\alpha_\nu + \sigma_\nu}{\alpha_\nu + \sigma_\nu}$$

which is the sum of the absorption & scattering source functions (assuming the gas is in LTE, so  $J_\nu = \alpha_\nu B_\nu$ ) weighted by their absorption coefficients.

Thus we can rewrite

$$\frac{dI_\nu}{ds} = -(\alpha_\nu + \sigma_\nu)(I_\nu - S_\nu).$$

So we could talk about a "net" absorption coefficient  $(\alpha_\nu + \sigma_\nu)$ , better defined as the "extinction coefficient".

~~As before~~  
Now let's think about random walks again,

now with scattering  $\geq$  absorption.  
The free path of a photon is now set by the total extinction coefficient  $\alpha_\nu + \sigma_\nu$ , and is

$$l_\nu = \frac{1}{\alpha_\nu + \sigma_\nu}$$

The chance that a photon's free path will end in absorption (e.g. scattering) is  $\epsilon_\nu = \frac{\sigma_\nu}{\alpha_\nu + \sigma_\nu}$ ,

and for scattering must be  $(1 - \epsilon_\nu) = \frac{\alpha_\nu}{\alpha_\nu + \sigma_\nu}$ , the single-scattering albedo.

Consider a very large, optically thick medium.  
A random walk may end with an absorption after a number of scatterings; the prob. of absorption at the end of each free path is  $\epsilon$ , so the # of free paths is

$$N = 1/\epsilon$$

So the total distance traveled is  $\left( d = \frac{l}{\sqrt{\epsilon}} \right)$ .

We can use the above to write  $d_* = \frac{1}{\alpha_r(\alpha_s + \alpha_r)}$   
an "effective mean path."

This is the average distance  
between where a photon is created & absorbed.  
(Typically frequency dependent.)

For a finite medium, we've identified  
two possibilities; it's optically thick, or  
optically thin.

But we now see a third possibility,  
that the medium may scatter a typical  
photon but not absorb it.

Describe an effective optical thickness  
of a medium as

$$\Sigma_* = \sqrt{\Sigma_a(\Sigma_a + \Sigma_s)}$$

where  $\Sigma_a = \alpha_r L$ ,  $\Sigma_s = \alpha_s L$ , for a medium of size  $L$ .  
(So  $\Sigma_a$  is the normal  $\Sigma$  we've been discussing.)

Say  $\Sigma_s > 1$ , but  $\Sigma_* \ll 1$ . A typical photon  
will scatter, but still get through — we call this  
material translucent. Example is thin  
piece of colorless plastic.

But if  $\Sigma_* \gg 1$ , most photons will get absorbed,  
and  $I_\nu \rightarrow B_\nu$  as the radiation comes into equilibrium  
with matter.

The effective mean path  $d_* = L \Sigma_*$ ,  
and can also be called the thermalization length,  
since it gives the distance over which the  
radiation comes into thermal equilibrium.