

Relativistic Particles & Synchrotron

Crab Nebula

Crab Nebula observed long before relevant physics was calculated.

Seen to shine in bluish light, spectrally without features, but polarized. Filaments seen that emit fine radiation, but most of nebula doesn't.

Nature of blue radiation mystery until 1950s. Bremsstrahlung didn't match spectrum & would require too much mass. 1953 - I. S. Shklovsky suggested radiation from relativistic electrons in a strong magnetic field.

Crab, multi-

Explains radiation seen from Crab, from radio up to hard X-rays. Strong B field organizes electron motion & acceleration, inducing polarization.

Aberration of Light

Power emitted by relativistic particles

To calculate the power emitted by a relativistic particle, we use an instantaneous rest frame K' , such that at any moment the particle has zero velocity in that frame.

This allows us to use the dipole (Larmor) formula to find the emitted power & transform it into our frame.

We know energy transforms as $dE = \gamma dE'$ (for particle at velocity v), but time also transforms as $dt = \gamma dt'$, so $\frac{dE}{dt} = \frac{dE'}{\gamma dt'} \Rightarrow$ total emitted power is Lorentz-invariant.

Next we must deal with acceleration. We note that acceleration parallel to, $v \perp v_0$, the frame velocity will transform differently. Use our Lorentz transform equations, for computing how dt , dx , etc. transform from moving frame to observer frame; where frame moves w.r.t. v .

$$dt = \gamma (dt' + \frac{v}{c^2} dx')$$

~~If we set $dx' = 0$ then $dt' = \gamma dt$~~
 ~~$\sigma \equiv (1 + \frac{vu'_x}{c^2})$, where $u'_x = dx'/dt'$,~~
~~then $dt = \gamma \sigma dt'$~~

$$\text{Then for velocity, } u_x = \frac{dx}{dt} = \frac{\gamma(dx' + v dt')}{\gamma(dt' + v dx'/c^2)} = \frac{u'_x + v}{1 + vu'_x/c^2}$$

Rybicki & Lightman show that

$$a'_{||} = \gamma^3 a_{||}$$

$$a'_{\perp} = \gamma^2 a_{\perp}$$

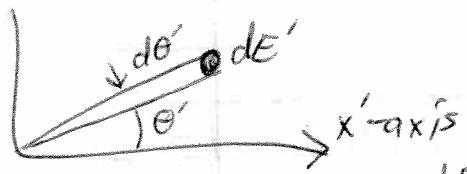
for components of acceleration \perp or $||$ to the frame velocity.

We calculate the power emitted by the electron under acceleration, in its own frame, using the Lorentz formula, $P = \frac{q^2 a^2}{6\pi \epsilon_0 c^3}$.

$$P = P' = \frac{q^2}{6\pi \epsilon_0 c^3} (a_{||}^2 + a_{\perp}^2)$$

$$P = \frac{q^2 \gamma^4}{6\pi \epsilon_0 c^3} (a_{\perp}^2 + \gamma^2 a_{||}^2)$$

Next question is angular distribution of light from these particles. Consider energy dE' emitted into solid angle $d\Omega' = \sin \theta' d\theta' d\phi'$, at angle θ' to the x-axis.



We'll use the Doppler shift formula to change E , as

$$dE = dE' \gamma / (1 + v/c \cos \theta).$$

Can write $d\Omega' = d\phi' d\cos \theta$.

We recently showed that $\cos \theta = \frac{\cos \theta' + v/c}{1 + v/c \cos \theta'}$.

Differentiated, we get $d(\cos \theta) = \frac{d(\cos \theta')}{\gamma^2 (1 + v/c \cos \theta)^2}$

ϕ is the coordinate going around the x-axis in our setup, γ isn't changed by a boost in x , so $\phi = \phi'$, $d\phi = d\phi'$.

Since $d\Omega' = d\phi' d(\cos \theta)$,

$$d\Omega' = \frac{d\phi' d(\cos \theta)}{\gamma^2 (1 + v/c \cos \theta)^2}.$$

Combining with dE transform,

$$\frac{dE}{d\Omega'} = \frac{dE'}{d\Omega'} \gamma^3 (1 + v/c \cos \theta)^3$$

To get the power we divide by dt' , in frame 5.
 However we have 2 choices for the stationary time interval.

- $dt = \gamma dt'$, the interval in which the emission occurs, gives emitted power, $P_e = dE/dt$.

- $dt_A = \gamma(1 - \beta \cos\theta) dt'$
 the interval in which the radiation is received (see Poppler derivation).

Gives $P_r = dE/dt_A$, power received.

For most applications second is preferred.

$$\frac{dP_r}{d\tau} = \gamma^4 \left(1 + \frac{v}{c} \cos\theta\right)^4 \frac{dP'}{d\tau'} \quad \text{or} \quad \frac{1}{\gamma^4 \left(1 - \frac{v}{c} \cos\theta\right)^4} \frac{dP'}{d\tau'}$$

Now consider an emitting accelerated particle.
 We showed (direction-dependent Larmor formula)

$$\frac{dP'}{d\tau'} = \frac{q^2 q'^2 \sin^2 \Theta'}{(4\pi)^2 E_0 c^3}$$

where Θ' is the angle between acceleration and emission.

Distinguish from θ' , between velocity & emission.



$$\frac{dP_r}{d\tau} = \frac{1}{\gamma^4 \left(1 - \frac{v}{c} \cos\theta\right)^4} \frac{dP'}{d\tau'} = \frac{1}{\gamma^4 \left(1 - \frac{v}{c} \cos\theta\right)^4} \frac{q^2 (q'^2_{||} + q'^2_{\perp}) \sin^2 \Theta'}{(4\pi)^2 c^3 E_0}$$

$$\frac{dP_r}{d\tau} = \frac{1}{\gamma^4 \left(1 - \frac{v}{c} \cos\theta\right)^4} \frac{q^2 (\gamma^6 q^2_{||} + \gamma^4 q^2_{\perp}) \sin^2 \Theta'}{(4\pi)^2 c^3 E_0}$$

$$\frac{dP_r}{d\tau} = \frac{q^2 (\gamma^2 q^2_{||} + q^2_{\perp}) \sin^2 \Theta'}{\left(1 - \frac{v}{c} \cos\theta\right)^4 (4\pi)^2 c^3 E_0}$$

Connecting Θ' to angles in observer frame is complicated.
 Note results for limiting cases:

$$\underline{q_{11} \text{ only}}; \Theta' = \theta'; \frac{dP_{11}}{dr} = \frac{q^2 q_{11}^2 \sin^2 \theta}{(4\pi)^2 \epsilon_0 c^3 (1 - \frac{v}{c} \cos \theta)^6}$$

q_1 only; $\cos \Theta' = \sin \theta' \cos \phi'$

$$\frac{dP_1}{dr} = \frac{q^2 q_1^2}{(4\pi)^2 \epsilon_0 c^3 (1 - \frac{v}{c} \cos \theta)^4} \left[1 - \frac{\sin^2 \theta \cos^2 \Theta}{v^2 (1 - \frac{v}{c} \cos \theta)^2} \right]$$

As $v \gg 1$, $(1 - \frac{v}{c} \cos \theta)$ becomes small for $\theta \approx 0$, causing radiation to be peaked strongly in this direction.

Radiation patterns for q_1 and q_{11} are shown in R&L.

R&L
Fig. 4.11

Relativistic Motion & Radiation in a Magnetic Field

The force equation for relativistic motion in a B field is

$$\frac{d\vec{p}}{dt} = \vec{F} = q(\vec{E} + (\vec{v} \times \vec{B})) \quad \text{where } \vec{p} = \gamma m \vec{v} \text{ relativistically. So}$$

$$\frac{d(\gamma m \vec{v})}{dt} = q(\vec{v} \times \vec{B}) \quad \text{for just a B field.}$$

Similarly, the work equation is $\frac{dE}{dt} = \frac{d(\vec{F} \cdot \vec{x})}{dt} = \vec{F} \cdot \vec{v}$

$$E = \gamma mc^2, \text{ so } \frac{dE}{dt} = \frac{d(\gamma mc^2)}{dt} = \vec{F} \cdot \vec{J},$$

Since \vec{F} from \vec{B} is always \propto to $\vec{v} \times \vec{B}$, $\vec{F}_B \cdot \vec{v} = 0$,

so $\vec{F}_E \cdot \vec{v}$ is the only component.

In a frame with only B , $\frac{d(\gamma mc^2)}{dt} = q \vec{v} \cdot \vec{E} = 0$.

This implies that γ is a constant, so \vec{v} is constant. So

$$\gamma m \frac{d\vec{v}}{dt} = q(\vec{v} \times \vec{B})$$

Separating velocity components $v_{||}$ & v_{\perp} to \vec{B} ,

$$\frac{dv_{||}}{dt} = 0, \quad \frac{dv_{\perp}}{dt} = \frac{q}{\gamma m} (\vec{v} \times \vec{B})$$

This gives uniform circular motion on the plane normal to \vec{B} , & constant motion (if any) along the field, so generally helical motion.

So far, similar to nonrelativistic case.

Frequency of rotation around \vec{B} is ω_B .

$$a = v^2/r = 2/\gamma m \cdot (\vec{v} \times \vec{B})$$

$$\Rightarrow v = 2\pi r/T = \omega_B r, \text{ so } a = \omega_B^2 r / T, \text{ so } \omega_B = \frac{2B}{\gamma m}$$

All acceleration is \perp to velocity, so total emitted radiation is

$$P_e = \frac{e^2 q^2 \gamma^4 (a_{\perp}^2)}{6\pi\epsilon_0 c^3} = \frac{e^2 \gamma^4}{6\pi\epsilon_0 c^3} \frac{e^2 B^2 v_{\perp}^2}{\gamma^2 m^2}$$

Substituting $r_0 = \frac{e^2}{4\pi\epsilon_0 m_e c^2}$, and $\beta_{\perp} = v_{\perp}/c$, $q = e$,

$$P_e = \frac{2e^4 \gamma^2 c B^2 v_{\perp}^2}{3 \cdot 4 \pi \epsilon_0 m_e^2 c^4} = \frac{e r_0^2 \gamma^2 c B^2 v_{\perp}^2}{3 (4\pi\epsilon_0)} = P_e$$

Alternatively, use $U = \gamma mc^2$ for the particle, getting

$$P_e = \frac{e^4 U^2 B^2 v_{\perp}^2}{6\pi\epsilon_0 m_e^4 c^7} \approx \frac{e^4 U^2 B^2}{6\pi\epsilon_0 m_e^4 c^5} \quad (v \approx c)$$

In SI

$$P_e = -2.37 \times 10^{12} U^2 B^2 \beta^2 \sin^2 \phi$$

where ϕ is pitch angle between \vec{p} and \vec{B} .

Averaging over $\langle \sin^2 \theta \rangle_* = 2/3$, $P_e =$

We've seen that $P_{\text{emitted}} = P'_{\text{emitted}}$ in different frames, so the power we see is the same as the emitted power (though the directionality will be different).

A relevant point of comparison is the magnetic energy density, $U_B = B^2/(2\mu_0)$, (J/m^3)

As the P_e formula depends on B^2 ,

we can rewrite as

$$P_{e,\text{synch}} = -2.66 \times 10^{-20} \beta^2 \gamma^2 U_B$$

Compare this to the power emitted in IC scattering from a radiation field, $P_{e,IC} = -2.66 \times 10^{-20} \beta^2 \gamma^2 U_{\text{rad}}$,

if U_{rad} in the electron frame $\ll m_ec^2$.

These are identical in form, illustrating that both processes can be considered as electrons interacting with an ~~radiation~~ electromagnetic energy density.

The relative emission rates are

$$\frac{P_{e,IC}}{P_{e,\text{sync}}} = \frac{U_{\text{rad}}}{U_B}$$

What is the spectrum of synchrotron radiation?

Start with nonrelativistic case, where radiation follows standard Larmor pattern.



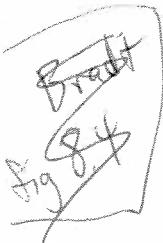
\propto radiation pattern

$$\omega = \frac{\omega}{2\pi} = \frac{qB}{2\pi fm}$$

E field vector varies sinusoidally at gyration frequency, so

$$\omega_B = \frac{qB}{fm} \quad (\text{Relativistic cyclotron frequency})$$

(Note that $\omega_B = \frac{qB}{m}$ is an approximation that works in nonrelativistic limit.)



Emission of this freq. is seen from cyclotron accelerators from particles with $v \ll c$, & is completely polarized.

It's also possible to see cyclotron absorption from electrons in strong B fields (& cyclotron emission) in astrophysical contexts.

Her X-1 Example of X-ray binary Her X-1, which shows abs. line near 40 keV.

If assume it's cyclotron, then

$$E = h\nu = \frac{hqB}{2\pi fm}, \quad B = \frac{2\pi fm E}{hq} \quad (\delta \sim 1)$$

$$B = \frac{2\pi (9.11 \times 10^{-31} \text{ kg})(40,000 \text{ eV} \times 1.6 \times 10^{-19} \text{ J/eV})}{(6.63 \times 10^{-34} \text{ Js})(1.60 \times 10^{-19} \text{ As})} = 3.5 \times 10^8 \text{ T} \quad (= 3.5 \times 10^{12} \text{ G})$$

This is a typical inferred B for young pulsars & seems appropriate for a young system like Her X-1. Similar lines are not seen in older CMXBs, with $B \sim 10^4$ times smaller since the e^- cyclotron line then lies in the UV where it is difficult to observe.

IE 1207 e^- cyclotron lines recently identified in very young neutron star (inside SNR), allowing an estimate of $B \sim 7 \times 10^6 \text{ T}$ ($7 \times 10^{10} \text{ G}$), with several harmonics clearly seen.

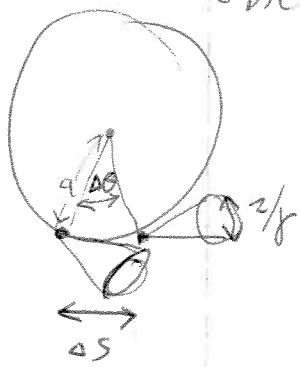
Magnetic white dwarfs with very low accretion rates can show optically thin cyclotron emission lines, typically in the optical or infrared.

RBS 0206

In the relativistic case, radiation is concentrated into a forward-opening cone of angle $\gamma/2 \sim \theta$.

Breit
Fig. 8.4

The pulse of observed radiation is over a time shorter than the full cycle, so the observed spectrum is more spread out,



The particle emits towards the observer in an angle $\frac{2\pi}{f}$.
In a fraction $\Delta\theta$ around the circle we see from one edge of the emitting beam to the other; we can show that this $\Delta\theta = \frac{1}{2}\frac{\pi}{f}$.

Then the distance traveled by the particle $\Delta s \sim 2a/\frac{\pi}{f}$. We find a , from $|\Delta \vec{v}| = v/\Delta\theta$, $\Delta s = v\Delta t$, so $\frac{|\Delta \vec{v}|}{\Delta t} = \frac{v^2 \Delta\theta}{\Delta s}$. Substituting into $\gamma m \frac{d\vec{v}}{dt} = \frac{q}{m}(\vec{v} \times \vec{B})$

$$\frac{\Delta\theta}{\Delta s} = \frac{q B \sin\phi}{\gamma m v} \quad \text{where } (\vec{v} \times \vec{B}) = v B \sin\phi$$

Then using $q\Delta\theta = \Delta s$, we get $a = \frac{v}{\omega_B \sin\phi}$.

This differs by the $\sin\phi$ term (the pitch angle - how much of the particle's velocity is parallel vs \perp to the B field) from the radius of the projected circle.

$$\Delta s = \frac{2v}{\gamma \omega_B \sin\phi} = v(t_2 - t_1)$$

Positions & times on circle where we see emission beam appear/disappear.

$$\text{So } t_2 - t_1 = \frac{2}{\gamma \omega_B \sin\phi}$$

Those are the ~~the~~ time differences in the emitted light, but we'd like to know

what the time difference is in receiving the beginning & end of the pulse,

$t_2^A - t_1^A$ will be the difference in arrival times, which differs from $t_2 - t_1$ by ω/c , the time for light to travel. So

$$\Delta t_A = t_2^A - t_1^A = \frac{z}{\gamma w_B \sin\phi} (1 - \frac{v}{c})$$

$$\text{For } \gamma \gg 1, 1 - \frac{v}{c} \approx \frac{1}{2\gamma^2} = \frac{(1 - \beta^2)}{2}, \text{ so}$$

$$\Delta t_A \approx \frac{1}{\gamma^3 w_B \sin\phi}$$

So the width of the observed pulses is a factor γ^3 smaller than the gyration period.

Shrinking Δt increases ω , so this broadens the spectrum, cutting off at frequencies $\sim 1/\Delta t_A$.

We can define a critical frequency

$$\omega_{\text{syn}} = \frac{1}{\Delta t_{\text{syn}}} = \gamma^3 w_B \sin\phi$$

$$\nu_{\text{syn}} = \frac{\omega_{\text{syn}}}{2\pi} = \frac{\gamma^3 e B \sin\phi}{2\pi m} = \left(\frac{1}{2\pi} \left(\frac{e}{mc^2} \right)^2 \frac{e B}{m} \sin\phi \right) \nu_{\text{syn}}$$

This is roughly the maximum, & also the dominant frequency of emission for synchrotron radiation.

How does the spectrum change with γ ?

For $\gamma \sim 1$, nonrelativistic, we have cyclotron radiation; electric field is a perfect sinusoid, & freq. is cyclotron freq. only.

As γ factor increases, radiation becomes beamed, appearing first as harmonics of w_B ($2w_B$, $3w_B$, etc).

R&L

6.8

Fig. 6.9,

R&L

As γ becomes very large, most of power goes into harmonics. Although technically, each electron radiates only in these harmonics, radiating spectrum usually perceived as continuum due to poor freq. resolution or some range of electron energies.

Fig. 6.10, R&L

Fig. 8.7, Bradt

Fig. 18.8, 18.3 Longair

In most cases, the electrons producing synchrotron don't have a thermal velocity distribution, but a power law dist'n is typical,

$$\text{e.g., } J(U)dU \propto U^p dU$$

where $J(U)$ is the number specific-intensity, particles $s^{-1} m^{-2} sr^{-1}$, so

$J(U)dU$ is # particles in interval dU at U .

p is often steeply negative; for cosmic rays, $p \sim -2.5$ over large energy ranges.

We can describe the # of particles per m^3 as $n(U)$. If the particles are moving isotropically then $J(U) = n(U)v/4\pi$.

For relativistic particles, $v \sim c$, so $n(U) \propto J(U)$, & $n(U)dU = U^p dU$.

Ok, next consider radiation per unit volume, assuming relativistic near-circular motion ($\beta \sim 1$, $\phi \sim \pi/2$)

$$\text{For a single electron } P_e \propto U^2 B^2$$

Total radiated power $j(U)dU$ is the power lost by one electron, multiplied by the # of electrons at $U - U + dU$ in a cubic meter.

$$j(U)dU = P_e \times n(U)dU$$

We find the proportionality

$$j(U)dU \propto U^2 B^2 U^p dU$$

$$j(U) \propto B^2 U^{p+2}$$

Now consider the frequency dependence, $j_\nu(\nu)$.

As a simple model, assume all energy is radiated at ν_c , where $\nu_c \propto U^2 B$.

This allows a simpler calculation without difficult integration, ν is correct enough for much astrophysics.

$$j_\nu(\nu) d\nu = j(U) dU \quad \text{so}$$

$$j_\nu(\nu) = j(U) \frac{dU}{d\nu}$$

Set $\nu_c = \nu$ (the only emitting freq. under assumption)
 $U \propto (\nu/B)^{1/2}$

$$j(U) \propto B^2 \left[(\nu/B)^{1/2} \right]^{p+2} = B^{(2-p)/2} \nu^{(z+p)/2}$$

$$\text{We find } dU \propto B^{-1/2} \nu^{-1/2} d\nu.$$

$$j_\nu(\nu) \propto B^{(2-p)/2} \nu^{(z+p)/2} B^{-1/2} \nu^{-1/2}$$

$$\boxed{j_\nu(\nu) d\nu \propto B^{(1-p)/2} \nu^{(1+p)/2} d\nu}$$

the synchrotron
photon-energy
spectrum,

The logarithmic slope of the photon energy spectrum
is the exponent of ν , $(p+1)/2$, called α ,

$$\alpha = \frac{p+1}{2}$$

So, if measure spectral slope of Crab to
be $\alpha = -1.1$ in X-ray, then $p = 2\alpha - 1 = -3, 2$, i.e.
 $n(U)dU = kU^{-3/2} dU$. (where k is some constant).

We can compute the typical energy of particles emitting at some frequency

$$\text{using } \nu_{\text{syn}} = \frac{1}{2\pi} \left(\frac{U}{mc^2} \right) \frac{eB}{m} \sin \phi.$$

If the average of $\sin \phi \sim \frac{2}{\pi}$ for $\phi = 0 - 180^\circ$,

$$\nu_{\text{syn}} \approx \left(\frac{U}{mc^2} \right) \frac{eB}{m \pi^2} = \frac{\gamma^2 e B}{m \pi^2}$$

So we can connect ν , γ & B . Considering synch. emission at one freq., we could find the particle energy if we knew B .

Several ways to estimate B in astrophysics:

- Easiest is to measure splitting of spectral lines (we'll see that later); but the Crab lacks narrow absorption or emission lines.
- Equipartition arguments.

Common astrophysics assumption that, if energy can be stored in multiple ways & easily transferred among them, that E tends to be equally distributed among them.

In simplest form: each degree of freedom for particle gets equal KE (so E of v_x, v_y, v_z are equal, each $\frac{1}{2} kT$). For monatomic gases, heavier atoms have lower speeds, $v \propto \sqrt{m}$.

In molecules, rotational E (& vibrational) gets equivalent E .

In astrophysics often extended to B energy density, if B fields are important to energetics. If B fields can accelerate particles (more later), then it is oft assumed that B-field energy density & energy density in fast particles are similar.

$$\int U n(U) dU$$

Since those particles emit synchrotron radiation, it's possible to estimate their density & work backwards to estimate B assuming equipartition. (Will do on homework.) Estimate $\sim 5 \times 10^{-8} T$.

- If one also measures IC radiation, can infer v_B , & thus B . Estimates $\sim 2 \times 10^{-8} T$; may point to difference over how nebula B is averaged between methods.

So cyclotron frequency ν_B of Crab is (for $\gamma \sim 1$)

$$\nu_B = \frac{eB}{2\pi m} = \frac{(1.6 \times 10^{-19} A \cdot s)(5 \times 10^{-8} T)}{2\pi(9.1 \times 10^{-31} \text{ kg})} = 1400 \text{ Hz}$$

The ratio $\frac{\nu_{\text{syn}}}{\nu_B} \sim \gamma^2$. So we can solve for the γ of electrons producing observed radiation. For the optical band, $\nu \sim 5 \times 10^{14} \text{ Hz}$, so $\gamma = (\nu_{\text{syn}}/\nu_B)^{1/2}$,

$$\gamma = \left(\frac{5 \times 10^{14}}{1400}\right)^{1/2} = 6.0 \times 10^5$$

The rest energy of an electron is 0.5 MeV, so these have $U = 3 \times 10^5 \text{ MeV}$, 300 GeV.

Then consider the Crab's X-rays: at $\sim 4 \text{ keV}$, or $10^4 \times 10^{18} \text{ Hz}$, these have $\gamma = 2.7 \times 10^7$, $U = 14 \text{ TeV}$, about twice the LHC's maximum acceleration.

Crab Spectrum

Crab spectrum continues up to $4 \times 10^4 \text{ eV}$, or $\nu \sim 10^{22.5} \text{ Hz}$, due to synchrotron; which gives $U \sim 1400 \text{ TeV}$, over 100 times more powerful than the LHC.

Electron lifetimes

We have the power emitted by an electron, so dividing by its energy can estimate a "lifetime", τ .

This isn't the actual lifetime for particle to radiate its energy, since τ gets longer as γ drops. It does indicate roughly the timescale at which a particle can keep a certain energy, & thus radiate up to a certain ν_{syn} .

$$\tau = \frac{-U}{dU/dt} = \frac{U}{2.37 \times 10^{12} U^2 B^2 \beta^2 \sin^2 \phi}$$

Simplify by allowing $\beta^2 \approx 1$, $\sin^2 \phi \approx 1$.

Also use $U = \gamma m c^2 = (8.2 \times 10^{44})$ for electrons; so

$$\tau = \frac{5}{B^2 \gamma}, \text{ in seconds.}$$

Since $\nu_{\text{syn}} \propto \gamma^2$, we see that $\tau \propto \frac{1}{\sqrt{\nu}}$.

One might thus expect the nebula size to scale with $(\text{frequency})^{1/2}$, if the size is determined by particle lifetimes. At low frequencies, lower- E particles are trapped; but at high freq., yes.

Consider image of Crab, around 4 keV X-rays.

So $\tau \sim \frac{5}{(5 \times 10^{-8} \text{ Hz})(2.7 \times 10^7)} \sim 7 \times 10^5$ $\gamma \sim 2.7 \times 10^7$, about 2 years.

After this, electrons will cool, radiate in UV.

Size of Crab about 1' (this image includes some softer X-rays), 60", or $3e^{-4}$ radians.

~~so $x = \theta d$~~ ; $d \approx 2000 \text{ pc}$ to Crab,

so $x = (3e^{-4})(2000 \text{ pc}) = 0.6 \text{ pc}$,

or $0.6 \text{ pc} \times 3.26 \text{ ly}/\text{pc} = 2 \text{ light-years}$.

So electrons are traveling near light-speed from Crab pulsar through nebula.

Requires that electrons are being continuously accelerated in nebula (since 2 years \ll 950 years since SN). Major puzzle when synchrotron nature of Crab figured out, before Crab pulsar discovered, since total Crab $L \sim 10^{31} \text{W}$, $10^5 \times L_\odot$.

Now understand what energy source is rotation of NS, which is rapidly slowing down. Implies that we may see rapid changes in Crab at high energies, since electrons must be replenished — indeed!

Crab movies

Detailed structures of Crab not well-understood,

In gamma-ray, lifetime even shorter; 40 MeV electrons have $\tau \sim 10$ days.

Recently flares in γ -ray regime seen with Fermi, AGILE, on this timescale. Nature not yet known.

Synchrotron absorption

For every emission there's an absorption process.

Synchrotron radiation can also become optically thick, again at long wavelengths.

The source function of synchrotron is $S_\nu \propto \nu^{5/2}$, which is not the R-J blackbody slope.

At each ν , we do see a blackbody emission, but at each ν we see different particles with different energies, & thus diff. temperatures.

Synchrotron absorption relevant in dense, synchrotron-emitting locations, e.g. young SNRs.

SN 1993J

— Young SNR, emitting synchrotron radio emission.